Joint Equalization and Gaussian Sums Particle Filtering Phase Noise Estimation

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Abstract—With this paper we aim to show that Gaussian sums particle filtering (GSPF) combined with iterative frequency-domain equalization (IFDE) for single-carrier (SC) modulations offer an appropriate solution for data transmission over highly frequency-selective channels with strong phase noise and linear drift. We present a promising IFDE scheme dubbed iterative-block decision feedback equalization (IB-DFE) and propose a post-equalization phase noise estimation scheme within the iteration chain. In fact, by exploiting the Gaussianity of the equalizer output we are able to track the phase noise value by updating the state posterior distribution using the GSPF algorithm. Additionally, simulation results are presented considering QPSK, 16-QAM, and 64-QAM signalling in highly frequency-selective channels and different values for the phase noise variance and linear drift.

I. INTRODUCTION

Block transmission techniques, with appropriate cyclic extensions and employing frequency-domain equalization (FDE) techniques, are known to be excellent candidates for broadband wireless systems [1], [2]. Orthogonal frequency division multiplexing (OFDM) schemes [3] and single-carrier combined with FDE (SC-FDE) [4] are the most popular modulations based on this concept. The overall implementation complexities for OFDM and SC-FDE schemes are similar, although the OFDM receivers are slightly simpler and the transmitters more complex. Since OFDM signals have larger envelope fluctuations that lead to amplification difficulties, SC-FDE schemes are clearly preferable for the uplink (i.e., the transmission from the mobile terminal to the base station) [1], [2]. Actually, this is exactly what happens for the 3GPP long term evolution (LTE) systems [5] where the adopted modulation format is OFDM for the downlink, and single-carrier frequency division multiple access (SC-FDMA), which is a form of SC-FDE with resource allocation flexibility, for the uplink. Moreover, the performance of SC-FDE can be further improved if the conventional linear FDE is replaced by an iterative FDE (IFDE) [6]. A promising solution based on this concept is the iterative block-decision feedback equalizer (IB-DFE), which can be regarded as a turbo equalizer implemented in the frequency domain.

With IB-DFE schemes both the feedforward and feedback chains of the receiver are implemented in the frequency domain. Since the feedback chain takes into account not only the hard decisions for each block but also the overall block reliability, the error propagation problem is significantly reduced. Consequently, the IB-DFE techniques offer much better performance than non-iterative methods. In fact, IB-DFE techniques can be considered as low complexity turbo equalization schemes since the feedback chain uses the equalizer output instead of the channel decoder output [7]. To improve the performance and to allow truly turbo FDE implementations, IB-DFE schemes with soft decisions were proposed [8], [9].

The presence of frequency offsets and phase noise in a digital communications system can compromise the transmission quality. In fact, the SNR performance loss introduced by carrier frequency offset (CFO) and Wiener phase noise in OFDM and SC modulations has been quantified by Pollet et al. in [10]. There, it was proven that for OFDM this performance degradation results from two different contributions: common phase error and inter-carrier interference, while for SC signals it results from a common phase error only. Moreover, Pollet et al. also argue that OFDM is orders of magnitude more sensitive to CFO and phase noise than SC. The effect of phase noise on the performance of digital communications systems was further investigated in [11]–[14] for OFDM and in [15], [16] for multi-carrier code division multiple access (MC-CDMA) systems.

While the CFO is modeled by a linear drift\(^1\) phase noise is modeled by a random process where we may consider the Wiener-Lévy process or the Ornstein-Uhlenbeck process depending on whether the phase noise results from a free-running oscillator or a phase-locked loop (PLL) oscillator, respectively [18], [19]. While in PLL driven oscillators, the closed-loop control mechanism tracks the variations of the carrier signal, and consequently, the phase noise has limited variance, the so-called free-running oscillators operate without a PLL and the generated phase noise results from the accumulation of random frequency deviations leading to unlimited variance. Moreover, the Wiener-Lévy or random-walk model has been widely accepted in the literature [11], [20]–[24] and, although there are models that represent more accurately some of the characteristics of slow-varying phase noise processes (see [25])

\(^1\)An efficient low-complexity solution for joint equalization and CFO estimation in SC-FDE schemes have been presented by the authors in [17].
and references therein), we will consider the case of Wiener phase noise only.

Sabaghian and Falconer in [26] proposed a synchronization method to compensate for the effect of frequency offset and phase noise for Turbo FDE based on the maximum a posteriori (MAP) criterium which results in the form of a decision-directed (DD) PLL using the soft decisions of the decoder output. In [27] the combination of DFE and digital PLL is investigated for inter-symbol interference (ISI) impaired channels and phase noise.

Further phase noise compensation solutions in SC modulations may be found in the literature. In [21], [25] non-data aided (NDA) and decision-directed (DD) maximum-likelihood (ML) estimators for time-varying phase noise are proposed. Nevertheless, these estimators assume either high-SNR or slow-varying phase noise while no solution for the low-SNR and fast-varying phase noise case is presented. Moreover, the available results are limited to the AWGN channel. Similarly, Amblard et al. in [20] and the authors in [28] propose, respectively, a particle filtering and Gaussian sum filtering (GSF) solution for phase noise estimation but again limited to PSK modulations and the AWGN channel. Nevertheless, the GSF was after combined with the IB-DFE to withstand ISI in [29], [30].

Particle filtering has been proposed as a powerful solution for a variety of nonlinear and/or non-Gaussian problems [31]–[33]. Kotecha and Djurić introduced in [34] a particle filtering which approximates the posterior distributions by single Gaussians. It was argued there that, the so-called Gaussian particle filter (GPF) presents much-improved performance and versatility than other Gaussian filters like the extended Kalman filter and its variants and that, since GPFs require no resampling stage during operation, they present reduced complexity while compared with sequential importance sampling with resampling.

In a companion paper of [34], Kotecha and Djurić introduce the Gaussian sum particle filter (GSF) [35] where the stochastic recursive filtering and prediction distributions are approximated by weighted sums of Gaussian functions and consequently posterior distributions other than Gaussian can be described by Gaussian mixtures and have their moments propagated by a bank of GPFs.

The paper organization is as follows. After this introductory section we proceed to Sec. II where we describe the phase noise estimation problem in terms of its Bayesian formulation and propose the GSPF as solution. Afterwards, in Sec. III, we present a simple IB-DFE and the modified version for joint equalization and phase noise estimation. Eventually, in Sec. IV, we present some performance results and finally in Sec. V we end the paper by drawing some conclusions.

II. PHASE NOISE ESTIMATION

A. Non-linear Filtering

Phase noise estimation, as addressed in this paper, is an instance of a wider class of problems called non-linear filtering problems. These consist of estimating a dynamic state of a non-linear stochastic system, based on a set of noisy observations. Many of these problems are written in the form of a so-called dynamic state-space (DSS) model [20], [31]–[35]. This model represents the time-varying dynamics of an unobserved state variable and the observation or measurement, which is usually a noisy and transformed version of the state. In fact, the DSS model is given by the process and observations equations pair.

In our estimation problem the state is the phase noise \( \theta_n \) with distribution \( p(\theta_n|\theta_{n-1}) \) and the observation \( y_n \). It follows that \( p(y_n|\theta_n) \) represents the observation conditioned on the unknown state variable. The process equation is the well known Wiener-Lévy model for the phase noise whereas the observations equation corresponds to the measurement of the phase noise within a digital modulation affected by additive white Gaussian noise (AWGN) with power spectral density \( N_0/2 \). Thus, the DSS model is given by

\[
\theta_n = \theta_{n-1} + w_n, \quad \text{(process equation)} \tag{1}
\]
\[
y_n = s_n e^{j\theta_n} + v_n, \quad \text{(observation equation)} \tag{2}
\]

where \( w_n \) and \( v_n \) are independent random noises with known distributions. The prior knowledge of the initial state is given by the probability distribution \( p(\theta_0) \). Our aim is to estimate the unknown value of \( \theta_n \) given all observations up to time instant \( n \), \( y_{0:n} \equiv \{y_0, y_1, \ldots, y_n\} \). In a Bayesian context this represents estimating recursively in time

- the filtering distribution \( p(\theta_n|y_{0:n}) \) at time instant \( n \) given all observations up to time instant \( n \);
- the predictive distribution \( p(\theta_{n+1}|y_{0:n}) \) at time instant \( (n+1) \) given all observations up to time instant \( n \).

From these distributions an estimate of the state can be determined for any performance criterion, e.g., minimum mean-squared error (MMSE). The filtering distribution at time instant \( n \) can be written as

\[
p(\theta_n|y_{0:n}) = C_n p(\theta_n|y_{0:n-1}) p(y_n|\theta_n) \tag{3}
\]

where \( C_n \) is the normalizing constant given by

\[
C_n = \left( \int p(\theta_n|y_{0:n-1}) p(y_n|\theta_n) d\theta_n \right)^{-1}. \tag{4}
\]

As for the prediction distribution it can be expressed as the convolution

\[
p(\theta_{n+1}|y_{0:n}) = \int p(\theta_{n+1}|\theta_n) p(\theta_n|y_{0:n}) d\theta_n. \tag{5}
\]

Assuming that the prediction and filtering distributions can be approximated as Gaussians or sums of Gaussians provides the means for propagating the posterior distribution and thus track the evolution of the state.

B. Computation of the Filtering Density

Assuming that at time instant \( n \) we have the prediction distribution

\[
p(\theta_n|y_{0:n-1}) = \sum_{i=1}^{G} \tilde{w}_{ni} N(\theta_n; \mu_{ni}, \Sigma_{ni}). \tag{6}
\]
After receiving $y_n$, the filtering distribution is given by

$$p(\theta_n | y_{0:n}) = C_n \sum_{i=1}^{G} \tilde{w}_n p(y_n | \theta_n) \mathcal{N}(\theta_n; \bar{\mu}_{ni}, \bar{\Sigma}_{ni}).$$  \hspace{1cm} (7)

Notice that, in (7) each term, given by $p(y_n | \theta_n) \mathcal{N}(\theta_n; \bar{\mu}_{ni}, \bar{\Sigma}_{ni})$, is approximated as a Gaussian since the sensor factor $p(y_n | \theta_n)$ is described as a sum of Gaussians and the product of Gaussians is still a Gaussian.

The update filtering distribution can now be represented as

$$p(\theta_n | y_{0:n}) \approx \sum_{i=1}^{G} \tilde{w}_n \mathcal{N}(\theta_n; \mu_{ni}, \Sigma_{ni}).$$  \hspace{1cm} (8)

The algorithm for the filtering step as implemented by the GSPF is described in Sec. II-E.

C. Computation of the Prediction Density

Assume that at time instant $n$, we have the filtering distribution

$$p(\theta_n | y_{0:n}) = \sum_{i=1}^{G} w_{ni} \mathcal{N}(\theta_n; \mu_{ni}, \Sigma_{ni}).$$  \hspace{1cm} (9)

Then, from the prediction step (6) results

$$p(\theta_{n+1} | y_{0:n}) = \int p(\theta_{n+1} | \theta_n) p(\theta_n | y_{0:n}) d\theta_n$$

$$\approx \int \sum_{i=1}^{G} w_{ni} \mathcal{N}(\theta_n; \mu_{ni}, \Sigma_{ni}) p(\theta_{n+1} | \theta_n) d\theta_n$$

$$= \sum_{i=1}^{G} w_{ni} \int \mathcal{N}(\theta_n; \mu_{ni}, \Sigma_{ni}) p(\theta_{n+1} | \theta_n) d\theta_n$$  \hspace{1cm} (10)

The last integral on (10) is approximated by a Gaussian using the time update algorithm of the GPF.

The time updated (predictive) distribution is now approximated as

$$p(\theta_{n+1} | y_{0:n}) \approx \sum_{i=1}^{G} \tilde{w}_{(n+1)i} \mathcal{N}(\theta_{n+1}; \bar{\mu}_{(n+1)i}, \bar{\Sigma}_{(n+1)i}).$$  \hspace{1cm} (11)

The algorithm for the prediction step as implemented by the GSPF is described in Sec. II-F.

D. Inference

An advantage lent by the fact that we approximate the posterior distribution by sums of Gaussians is that MMSE estimates of the hidden state and its error covariance can be obtained straightforwardly. The estimate of $\theta_n$, $\hat{\theta}_n = E[\theta_n | y_{0:n}]$ and $\hat{\Sigma}_n = E[(\theta_n - \hat{\theta}_n)^2]$ can be computed from

$$\hat{\theta}_n = \sum_{i=1}^{G} w_{ni} \mu_{ni},$$  \hspace{1cm} (12)

$$\hat{\Sigma}_n = \sum_{i=1}^{G} w_{ni} (\Sigma_{ni} + (\hat{\theta}_n - \mu_{ni})^2).$$  \hspace{1cm} (13)

E. GSPF Algorithm for the Filtering Step

In this section we enumerate the steps required by the GSPF to perform the computation of the filtering distribution [35].

1) For $i = 1, \ldots, G$, draw $P$ samples from the importance function $\pi(\theta_n | y_{0:n})$ and denote them as $\{\theta_{ni}^{(j)}\}_{j=1}^{P}$.

2) For $i = 1, \ldots, G$, and $j = 1, \ldots, P$, compute weights by

$$\gamma_{ni}^{(j)} = \frac{p(y_n | \theta_{ni}^{(j)}) \mathcal{N}(\theta_{ni}^{(j)}; \bar{\mu}_{ni}, \bar{\Sigma}_{ni})}{\pi(\theta_{ni}^{(j)} | y_{0:n})},$$  \hspace{1cm} (14)

3) For $i = 1, \ldots, G$ estimate the mean and covariance as

$$\mu_{ni} = \frac{\sum_{j=1}^{P} \gamma_{ni}^{(j)} \theta_{ni}^{(j)}}{\sum_{j=1}^{P} \gamma_{ni}^{(j)}},$$  \hspace{1cm} (15)

$$\Sigma_{ni} = \frac{\sum_{j=1}^{P} \gamma_{ni}^{(j)} (\theta_{ni}^{(j)} - \mu_{ni})^2}{\sum_{j=1}^{P} \gamma_{ni}^{(j)}}.$$  \hspace{1cm} (16)

4) For $i = 1, \ldots, G$, update the weights as

$$\tilde{w}_{ni} = \tilde{w}_{(n-1)i} \frac{\sum_{j=1}^{P} \gamma_{ni}^{(j)}}{\sum_{i=1}^{G} \sum_{j=1}^{P} \gamma_{ni}^{(j)}}.$$  \hspace{1cm} (17)

5) Normalize the weights according to

$$w_{ni} = \frac{\tilde{w}_{ni}}{\sum_{i=1}^{G} \tilde{w}_{ni}}.$$  \hspace{1cm} (18)

Remarks: Notice that, while other importance functions were possible, we do $\pi(\theta_n | y_{0:n}) = \mathcal{N}(\theta_n; \bar{\mu}_{ni}, \bar{\Sigma}_{ni})$ for convenience [34]. Moreover, considering our choice for the importance function, the weights in (14) simplify to $\gamma_{ni}^{(j)} = p(y_n | \theta_{ni}^{(j)})$ which is simply the sensor factor conditioned on $\theta_{ni}^{(j)}$.

F. GSPF Algorithm for the Prediction Step

We now describe how the GSPF algorithm evaluates the prediction density. Notice that, this is possible since the prediction and filtering distribution can be approximated as Gaussians or sums of Gaussians.

1) For $i = 1, \ldots, G$ obtain samples from $\mathcal{N}(\theta_n; \mu_{ni}, \Sigma_{ni})$ and denote them as $\{\theta_{ni}^{(j)}\}_{j=1}^{P}$.

2) For $i = 1, \ldots, G$, and $j = 1, \ldots, P$ obtain samples from $p(\theta_{n+1} | \theta_n = \theta_{ni}^{(j)})$ and denote them as $\{\theta_{ni+1}^{(j)}\}_{j=1}^{P}$.

3) For $i = 1, \ldots, G$, update weights $\tilde{w}_{(n+1)i} = w_{ni}$.

4) For $i = 1, \ldots, G$, obtain $\bar{\mu}_{(n+1)i}$ and $\bar{\Sigma}_{(n+1)i}$ by taking sample means and covariances respectively.

G. Sensor Factor

A central element in our filtering solution is the definition of the sensor factor. Notice that the sensor factor is not a distribution and therefore does not need to integrate to one. In fact, in our case the sensor factor is not even integrable as it is a periodic function of the conditioning variable, $\theta_n$.

For an observation model (2) the sensor factor is given by

$$p(y_n | \theta_n, s_n) = \frac{1}{\pi \sigma_v} \exp \left( - \frac{|y_n - s_n e^{i\theta_n}|^2}{\sigma_v^2} \right).$$  \hspace{1cm} (19)
Defining
\[ \zeta_n = \frac{|y_n|^2 + |s_n|^2}{\sigma_v^2}, \]  
and
\[ \beta_n = \frac{|y_n s_n|}{\sigma_v^2}, \]  
and noticing that \[ |y_n - s_n e^{j\phi_n}|^2 = |y_n|^2 + |s_n|^2 - 2|y_n s_n| \cos(\phi_n + \theta_n - \eta_n), \] with \( \eta_n = \arg\{y_n\} \) we may write the sensor factor (19) as
\[ p(y_n|n, s_n) = \frac{1}{\pi \sigma_v} \exp\left[2\beta_n \cos(\phi_n + \theta_n - \eta_n) - \zeta_n\right]. \]  

Next, we marginalize (22) in order to remove the dependency on \( s_n \), i.e.,
\[ p(y_n|\theta_n) = \sum_{s \in \mathcal{L}} p(y_n|\theta_n, s)p(s), \]  
where \( \mathcal{L} \) is the signalling constellation alphabet with cardinality \( \#\mathcal{L} = M \). Assuming equiprobable symbols, i.e., \( p(s) = 1/M \) for all \( s \in \mathcal{L} \), (23) results
\[ p(y_n|\theta_n) = \frac{1}{M \pi \sigma_v} \sum_{s \in \mathcal{L}} \exp\left[2\beta_n \cos(\phi(s) + \theta_n - \eta_n) - \zeta_n(s)\right], \]  
where \( \phi(s) = \arg\{s\}, \zeta_n(s) = (|y_n|^2 + |s|^2)/\sigma_v^2, \) and \( \beta_n(s) = |y_n s_n|/\sigma_v^2 \).

Having expressed the sensor factor as a periodic function on \( \theta_n \) simply concludes the derivation of the GSPF for the estimation of phase noise. In the following we show how the combined use of this estimator with an IFDE offers an appropriate solution for data transmission over highly frequency-selective channels with strong phase noise and linear drift. This is achieved through a modified IB-DFE scheme where a GSPF estimates the phase noise within each iteration of the equalizer.

III. IB-DFE RECEIVERS

Having a much better performance than that of a linear FDE, the IB-DFE is a promising non-linear receiver structure for SC modulations with cyclic prefix (CP). With performances close to the optimum sequence detector, as implemented, e.g., by the Viterbi algorithm, and for a much lower complexity, the IBDFE receivers can be regarded as frequency-domain turbo equalizers.

Assume that at the receiver side we have
\[ Y_k = S_k H_k + N_k, \quad k = 0, 1, \ldots, N - 1. \]  
In (25) \{\( Y_k; k = 0, 1, \ldots, N - 1 \} = \text{DFT}\{y_n; n = 0, 1, \ldots, N - 1\} \) results from applying the DFT to the time-domain received signal after that we remove the CP, samples \{\( S_k; k = 0, 1, \ldots, N - 1 \} \) are the frequency-domain counterparts of \{\( s_n; n = 0, 1, \ldots, N - 1 \} \), which on their side are the data symbols, \{\( H_k; k = 0, 1, \ldots, N - 1 \} \) is the channel frequency response, and \{\( N_k; k = 0, 1, \ldots, N - 1 \} \) are the data symbols obtained in the last iteration and which is assumed to be zero for the first iteration.

One can show that
\[ B_k = F_k H_k - 1, \quad k = 0, 1, \ldots, N - 1, \]  
are the optimum feedback coefficients (see [36], [37]). Similarly, the optimum feedforward coefficients are
\[ F_k = \frac{\tilde{F}_k}{\gamma}, \quad k = 0, 1, \ldots, N - 1. \]  
With
\[ \tilde{F}_k = \frac{H_k^*}{\alpha (1 - \rho^2)|H_k|^2}, \quad k = 0, 1, \ldots, N - 1, \]  
and where
\[ \gamma = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{F}_k H_k, \]  
and \( \alpha = N_0/E_s \), with \( E_s \) denoting the symbol energy. As for the correlation factor, \( \rho \), it is defined as
\[ \rho = \frac{E[\hat{s}_n s_n^*]}{E[|s_n|^2]}, \]  
where \( \hat{s}_n; n = 0, 1, \ldots, N - 1 \) are the decisions on the data symbols obtained in the last iteration.

Depicted in Fig. 1 is the structure of the IB-DFE just described and which is usually denoted IB-DFE with hard decisions.

Notice that, contrarily to [36] and [37] we consider normalized equalizers, i.e., \( \frac{1}{N} \sum_{k=0}^{N-1} F_k H_k = 1 \).
B. Joint Equalization and Phase Noise Estimation

Significantly, the time-domain output of the FDE, \( \{ \tilde{s}_n; n = 0, 1, \ldots, N - 1 \} \), can be described as the sum of the data symbol with a noise plus ISI component, resulting

\[
\tilde{s}_n = s_n + \tilde{v}_n
\]

where the samples comprising the noise plus interference, \( \{ \tilde{v}_n; n = 0, 1, \ldots, N - 1 \} \), have the overall power of their frequency-domain counterparts, \( \{ \tilde{V}_k; k = 0, 1, \ldots, N - 1 \} \), given by [38]

\[
E[|\tilde{V}_k|^2] = E[|F_k H_k - 1 - \rho^2 B_k|^2] E[|S_k|^2] + E[|B_k|^2] \rho^2 (1 - \rho^2) E[|S_k|^2]
\]

(33)

Consequently, \( \tilde{s}_n = \tilde{s}_n e^{j \theta_n} + \tilde{v}_n \).

As we can see (34) is formally equal to the observation equation (2) of our DSS model. Thus, we are in conditions of estimating the phase noise at the output of the FDE resorting to the GSPF proposed in Sec. II.

IV. PERFORMANCE RESULTS

In this section we present the performance results for the proposed system. In Fig. 2, Fig. 3, and Fig. 4 we depict the BER curves for QPSK, 16-QAM, and 64-QAM signalling, respectively.

Since sensitiveness to phase noise increases with the order of the signalling constellation, the value for variance of the phase noise is selected according to the signalling order. Therefore, we have \( \sigma_\theta^2 = \{0.02, 0.01, 0.005\} \) for QPSK, 16-QAM, and 64-QAM signalling, respectively.

Furthermore, in order to show the robustness of the proposed solution to CFO we added a linear drift to the phase noise process. Accordingly, the phase noise process with a linear drift is given by \( \theta_n = \theta_{n-1} + 2\pi \Delta f T/N + w_n \), where \( \Delta f T \) is a carrier frequency offset normalized with respect to the symbol duration, \( T \), and where, for the simulation results presented here, we assumed \( \Delta f T = 0.05 \). The data block for every signalling order has \( N = 256 \) symbols with each symbol lasting \( T = 4\mu s \). The propagation channel is frequency-selective characterized by the power delay profile type C for HIPERLAN/2 (High PERformance Local Area Networks) [39], with uncorrelated Rayleigh fading on different paths. Similar results could be obtained for other severely time-dispersive channel models with rich multipath propagation.

By inspecting Fig. 2, Fig. 3, and Fig. 4 we can see that the use of GSPF estimation of the phase noise guarantees BER performance levels close to those obtained when there is no phase noise even when strong levels of phase noise are present. In addition to the BER performance curves we traced the matched-filter bound (MFB) which gives the system performance if there were no ISI [40]. It is clear that as the iterative process proceeds, the system performance approaches that of the MFB, for all signalling constellations. Also remarkable is that even for the largest signalling order used in simulations, \( i.e., \) 64-QAM, the use of just \( P = 100 \) particles and \( G = 3 \) Gaussians sufficed in bringing the systems performance from a catastrophic performance level to operational performance levels.
Fig. 4: BER performance for block data transmission over a frequency-selective channel in presence of phase noise (PHN) with $\sigma_w = 0.005 \text{rad/s}$ and a linear drift $\Delta fT = 0.05$ with the GSPF algorithm, without phase noise or linear drift estimation, and without phase noise or linear drift, where $N = 256$ 64-QAM symbols, $P = 100$, and $G = 3$.

Fig. 5: BER performance for block data transmission over a frequency-selective channel in presence of phase noise with a given $\sigma_w$ and no linear drift. Each signalling constellations has $E_b/N_0 = 21 \text{dB}$, $E_b/N_0 = 15 \text{dB}$, and $E_b/N_0 = 10 \text{dB}$, respectively, for 64-QAM, 16-QAM, and QPSK. In order to compensate for the phase noise we use the GSPF algorithm with $P = 100$, and $G = 3$.

Depicted in Fig. 5 we can see the evolution of the BER performance with respect to the value of the phase noise variance $\sigma_w$ while using the GSPF algorithm to estimate the phase noise. By inspecting the figure one may observe the performance gains enforced by the iterative process.

V. Conclusions

This paper has two distinct parts. The first part, corresponding to Sec II, in which we propose the GSPF as a tracking solution for random-walk phase noise observed within a digital modulation and AWGN. The latter part, corresponds to Sec III, where the IB-DFE and its modified version for joint equalization and phase noise estimation are described. In this manner a novel solution for data transmission over highly frequency-selective channels with strong phase noise and linear drift has been presented and its effectiveness substantiated by simulation results. Furthermore, these results show that a few Gaussian functions and a few dozens sample particles are enough to implement a computationally sustainable yet effective filter.

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