Bayesian anisotropic denoising in the Laguerre Gauss domain

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ABSTRACT

In this contribution, we propose an adaptive multiresolution denoising technique operating in the wavelet domain that selectively enhances object contours, extending a restoration scheme based on edge oriented wavelet representation by means of adaptive surround inhibition inspired by the human visual system characteristics. The use of the complex edge oriented wavelet representation is motivated by the fact that it is tuned to the most relevant visual image features. In this domain, an edge is represented by a complex number whose magnitude is proportional to its “strength” while phase equals the orientation angle. The complex edge wavelet is the first order dyadic Laguerre Gauss Circular Harmonic Wavelet, acting as a band limited gradient operator. The anisotropic sharpening function enhances or attenuates large/small edges more or less deeply, accounting for masking effects induced by textured background. Adapting sharpening to the local image content is realized by identifying the local statistics of natural and artificial textures like grass, foliage, water, composing the background. In the paper, the whole mathematical model is derived and its performances are validated on the basis of simulations on a wide data set.

Keywords: Image denoising, contour detection, Bayesian estimation, circular harmonic wavelet thresholding

1. INTRODUCTION

The main causes of noise in images are due to the imaging process, the analog-to-digital conversion, the coding, and transmission. The goal of image restoration is to reconstruct an acceptable estimate of the original image from the noisy observation. The design of an efficient algorithm for noise removal from images is still a challenging task for researchers. In the last two decades, great effort has been put in the direction of image enhancement and restoration. Restoration assumed a crucial role not only in improving visual quality, but also in increasing performances of subsequent tasks of image processing, such as segmentation, feature extraction, coding; the ability of the restoration process to remove irrelevant information (i.e. noise) is put in practice to restore the original spatial correlation structure of the signal, improving the efficiency of further tasks as compression or protection.

A prior probability model for both noise and original image is crucial for the purpose of this application. Two simple assumptions are commonly made for this aim. The first is that the probability structure may be defined locally. Typically, one makes the hypothesis that the probability density of a pixel, when conditioned on a set of neighbours, is independent of the pixels outside the corresponding neighbourhood. The second is the assumption of spatial homogeneity: the distribution of values in a neighbourhood is the same for all such neighbourhoods, regardless of absolute spatial position. These two assumptions give origin to a Markov random field model which is usually simplified if we assume that the aforementioned distributions are conditionally Gaussian. Some problems for image modelling arise from this last consideration, since the complexity of local structures is not properly described by Gaussian densities.

In recent years, models have been developed to account for non-Gaussian behaviours of image statistics. One can see from casual observation that individual images are highly inhomogeneous: they typically contain many regions that are smooth, mixed with “features” such as contours, or surface markings. This behaviour allows one to remove noise using a pointwise nonlinearity. Such approach has become quite popular in the image denoising literature, and is typically chosen to perform a type of thresholding operation, suppressing low-amplitude coefficients while maintaining high-amplitude values.

To this purpose, there has been considerable interest, over the last decade, in wavelet-based denoising methods. In these schemes, the basic idea consists in projecting the noisy image on a specific orthogonal set of basis functions and, before reconstruction, in finding some kind of threshold that will help to remove more noise (mostly represented by low wavelet...
coefficients) than significant image edge information (mainly described by high-value coefficients). Many variations on the basic procedure of wavelet-based denoising have been investigated and developed so far, and with some notable improvements, including shrinking instead of thresholding, translation invariant transforms, Bayesian thresholding, level-dependent or adaptive choice of thresholding functions, overcomplete decompositions (e.g., curvelet transform [1]), etc., to name some.

From the traditional tools for signal and image denoising such as Wiener’s [2] and Kalman’s [3] linear filters, passing through Donoho and Johnstone’s nonlinear denoisers [4], [5], to Strela et al. Gaussian mixture models [6], the amount of work and literature in between is extensive, and it is hard to present an exhaustive list of references.

More recently, non-linear techniques applied to wavelet based representations have been introduced. Basically, these techniques consist of thresholding the wavelet coefficients, setting them to zero if they are at noise level, and keeping them unaltered if they represent significant structures. This technique is also called “wavelet shrinkage”. Since the pioneering paper of Donoho [4] much work has been done for setting and optimizing thresholds under different criteria and using adaptive strategies (see for instance Chang et al. [7], Mihcak et al. [8]). These approaches are based on general wavelet image representations, such as Daubechies finite support wavelets. In the recent past, schemes taking also into account statistical dependencies between wavelet coefficients have been devised.

In this contribution, we propose an iterative, adaptive multiresolution restoration technique operating in the complex edge wavelet domain that selectively smoothes object textures and contours, extending a denoising scheme based on edge oriented wavelet representation by means of adaptive surround inhibition inspired by the human visual system characteristics. The complex edge wavelet employed here is the dyadic Laguerre-Gauss Circular Harmonic Wavelet of the first order acting as a band limited gradient operator. Its use is motivated by the fact that it is tuned to the most relevant visual image features. In this domain, an edge is represented by a complex number whose magnitude is proportional to its “strength” while phase equals the orientation angle.

The anisotropic smoothing function enhances or attenuates large/small edges more or less deeply, accounting for masking effects induced by textured background. Adapting smoothing to the local image content is realized by identifying the local statistics of natural and artificial textures like grass, foliage, water, composing the background, as well as the local statistics of object contours. In principle, noise, textures and contours statistics evaluation would require the knowledge of both noisy and original image. Here, the original image is replaced by a restored version obtained by means of general purpose noise and contour models. The estimated local statistics is then used to refine restoration. The whole process is further iterated, until convergence.

The paper is organized as follows. In Section 2 the Laguerre-Gauss decomposition is described. In Section 3 the iterative denoising scheme is detailed. Finally, experimental results and conclusions are drawn in Section 4.

### 2. THE COMPLEX EDGE FEATURE DOMAIN

Classical approaches to feature extraction are based on the Gabor family filters, directional filters, gradient operators [10], [11], and different kinds of wavelets in multiresolution schemes [12], [13]. Here we refer to a multiscale feature decomposition based on Harmonic Angular Filters (HAFs). The HAFs, also referred in literature to as CHFs (Circular Harmonic Filters) are complex, polar separable filters defined by a Point Spread Function (PSF) with a polar representation of the kind

\[ h^{(k)}(r, \theta) = v^{(k)}(r) e^{-j k \theta}, \quad k = 0, 1, 2, \ldots \]  

where \( r \) and \( \theta \) are polar coordinates, \( k \) is the order of the HAF and \( v^{(k)}(\cdot) \) is the radial profile. The output of a \( k \)-th order filter at site \([n,m]\) is the \( k \)-th term of the "angular harmonic expansion" of the image around \([n,m]\). It fact, it is easily shown that it represents the \( k \)-th harmonic coefficient of a periodic signal obtained by radial integration weighted by \( v^{(k)}(\cdot) \) of the image around \([n,m]\) [14]. A zero-order HAF is a simple real smoothing filter with PSF \( v^{(0)}(\cdot) \). A first-order HAF acts as a complex differential operator suited for edge extraction. Its output is a complex image whose magnitude is proportional to edge strength, and phase is equal to edge orientation. Likewise, higher order HAFs are tuned to other symmetric patterns, such as lines (\( k=2 \)), crosses (\( k=4 \)) etc. [15].

Let us now refer to the specific family of HAFs with Laguerre-Gauss radial profiles. It has been shown in [16] that each element of this family defines a dyadic Circular Harmonic Wavelet (CHW) suited for multiscale representation. In
particular, let us refer to a mother wavelet constituted by the following first-order member of the HAF Laguerre-Gauss family:

$$h^{(1)}(r, \theta) = 2\pi^{1/2} (ar)^{e^{-\pi (ar)^2} e^{i\theta}},$$

(2)

whose Fourier transform in polar frequency coordinates \((\rho, \gamma)\) is

$$H^{(1)}(\rho, \gamma) = -\frac{i}{\sqrt{\pi}} e^{-\rho^2/4\pi} e^{i\gamma}.$$ (3)

For notational convenience, let us denote with \(h^{(1)}[n,m]\) our mother wavelet in discrete Cartesian coordinates, and let us denote with \(h_s[n,m]\) the function obtained by dilation of \(h^{(1)}[n,m]\) by a scaling factor \(s\), i.e.

$$h_s[n,m] = \frac{1}{s} h^{(1)}\left[\frac{n}{s}, \frac{m}{s}\right].$$

(4)

Let \(\{s_1, s_2, ..., s_L\}\) be a finite sequence of scale factors corresponding to increasing resolutions. The set of the complex images \(\{z_{\gamma_i}[n,m] = y[n,m] * h_{s_i}[n,m], \quad k = 1, 2, ..., L\}\) yields the wavelet representation of the image in the multiscale complex edge feature domain. Since the frequency response (3) is zero at the origin, recovery of \(y[n,m]\) may be unstable when using a finite range of scales. Therefore, we include in the reconstruction a coarse approximation at very low resolution obtained with the help of a low pass zero order HAF filter \(h^{(0)}[n,m]\). Resorting to usual notations, let us denote with \(H_{\gamma_i}(\omega_1, \omega_2)\) the frequency response of the filter \(h^{(0)}[n,m]\) and with \(H^{(0)}(\omega_1, \omega_2)\) the frequency response of the filter \(h^{(0)}[n,m]\). Provided that for Laguerre-Gauss CHWs the stability condition

$$0 < A \leq \sum_{k=1}^{L} |H_{\gamma_i}(\omega_1, \omega_2)|^2 \leq B < \infty \quad \omega_1 > 0, \omega_2 > 0, \quad a.e.$$ (5)

holds, given the coarse image approximation \(y_{\gamma_i}[n,m] = y[n,m] * h_{s_i}[n,m]\) and the complex edge images \(\{z_{\gamma_i}[n,m] = y[n,m] * (1 - h^{(0)}[n,m]) * h_{s_i}[n,m], \quad k = 1, 2, ..., L\}\), we have

$$y[n,m] = \sum_{k=1}^{L} z_{\gamma_i}[n,m] * g_{s_i}[n,m] + y_{\gamma_i}[n,m],$$ (6)

where the reconstruction filters \(g_{s_i}[n,m]\) are defined in terms of their Fourier transform as follows

$$g_{s_i}(\omega_1, \omega_2) = \frac{H^{*}\gamma_i(\omega_1, \omega_2)}{\sum_{k=1}^{L} |H_{\gamma_i}(\omega_1, \omega_2)|^2}, \quad k=1,2, ..., L.$$ (7)

3. ITERATIVE BAYESIAN DENOISING IN THE LAGUERRE-GAUSS DOMAIN

The idea presented in this work is to perform an iterative noise removal in the Laguerre-Gauss wavelet domain via a Bayesian approach. As shown in the scheme of Fig. 1, and described in Section 3.1, the noisy observation is initially filtered with a bank of first order Laguerre-Gauss filters at different scales, then the iterative Bayesian denoising procedure we propose is applied and, finally, the restored image is reconstructed through inverse Laguerre-Gauss filtering.

For each subband of the multiresolution analysis an iterative Bayesian denoising procedure is applied as detailed in Section 3.4 and briefly sketched hereafter.

Since a first order Laguerre-Gauss wavelet is actually a bandlimited gradient operator, we need to address the problem represented by the presence of noise over highly textured areas of an image. This problem can be efficiently solved using the approach followed in [9] where a biologically motivated surround inhibition algorithm is applied to the complex edge images in order to selectively suppress noise components, in flat and textured areas. Therefore, after having performed the multiresolution decomposition in the wavelet domain by using the Circular Harmonic Functions with Laguerre Gauss radial profile, an anisotropic smoothing driven by a biologically motivated surround inhibition (see Section 3.2) is
considered. Based on psychophysical and neurophysiologic findings, to avoid edge’s self masking, the inhibition factor is computed as the weighted local average of the magnitude of the image gradient on a neighbour of each pixel with the exclusion of a narrow strip oriented along the candidate edge. Then, a subsequent contour map extraction procedure that emphasizes object contours characterised by long connected stokes and discard short strokes related to textured areas and noise, is applied (see Section 3.3). This is the preliminary step in the iterative denoising chain. In fact, after object edge extraction via surround inhibition and contour map evaluation, the information resulting from these two blocks is used to estimate noise and the edge image statistical properties needed for Bayesian denoising. At this stage, the Bayesian estimate of the original signal is obtained from its noisy observation through the minimization of the corresponding absolute risk in a finite iteration chain. Parametric probabilistic models based on complex Gaussian mixtures are adopted for both noise and signal edge features. Based on them, the MMSE estimate, i.e. the conditional expectation with respect to the observed features, is calculated as a non-linear combination of optimum linear estimates individually matched to Gaussian sub-model pairs. Starting from the MMSE estimate of the Circular Harmonic Wavelet coefficients for each complex edge image at different resolution, the recovered image can be finally reconstructed from the restored features by inverse wavelet transform.

3.1 The Observation model

Introducing a shorthand notation, let us consider an image \( X = \{ x[n,m] \} \), and its observed version \( Y = \{ y[n,m] \} \) corrupted by an additive independent observation noise \( W = \{ w[n,m] \} \). After having performed the multiresolution analysis described in Section 2 and sketched in Fig. 1, for the generic \( k \)-th channel we have

\[
\begin{align*}
    z_k[n,m] &= \tilde{z}_k[n,m] + \Delta \tilde{z}_k[n,m],
\end{align*}
\]

being \( z_k[n,m] = y[n,m] \ast h_k[n,m] \) the complex edge image at resolution \( s_k \) of the observation image \( Y \), \( \tilde{z}_k[n,m] = x[n,m] \ast h_k[n,m] \) the complex edge image at resolution \( s_k \) of the original image \( X \), and \( \Delta \tilde{z}_k = w[n,m] \ast h_k[n,m] \) be the representation in the CHW domain of the observation noise \( W \).

Within this framework, denoting, for convenience, with \( \tilde{z} = [\tilde{z}_r \quad \tilde{z}_i]^T \) the 2D vector corresponding to 1D complex number \( \bar{z} = \bar{z}_r + j\bar{z}_i \), we choose to describe the marginal distribution of \( \tilde{z}_k[n,m] \) with a rather general model constituted by a complex Gaussian mixture with mixing parameters \( \lambda_i \), i.e. a weighted sum of complex Gaussian distributions:

\[
    p_{\tilde{z}}(\tilde{z}_k[n,m]) = \sum_{i=1}^{K} \lambda_i \mathcal{N}_2[\tilde{z}_k[n,m], \mu_{\tilde{z}_i}, \sigma^2_{\tilde{z}_i}, R_{\tilde{z}_i}]
\]

where \( \mathcal{N}_2[z; \mu, R] \) denotes the Gaussian probability density function of a random bivariate \( z = [z_r \quad z_i]^T \) with expectation \( \mu = [\mu_r \quad \mu_i]^T \) and covariance matrix \( R \):

\[
    \mathcal{N}_2[z; \mu, R] = \frac{1}{2\pi|\text{det}(R)|^{1/2}} e^{-\frac{1}{2}(z - \mu)^T R^{-1} (z - \mu)}.
\]

On the other hand, the distribution of the observation noise \( \Delta \tilde{z}_k[n,m] \) in the CHW transform domain is also modeled by means of a zero mean complex Gaussian mixture with mixing parameters \( \beta_i \), namely,

\[
    p_{\Delta \tilde{z}}(\Delta \tilde{z}_k[n,m]) = \sum_{i=1}^{M} \beta_i \mathcal{N}_2[\Delta \tilde{z}_k[n,m], 0, R_{\Delta \tilde{z}_i}]
\]

For each channel of the multiresolution decomposition shown in Fig.1 an iterative Bayesian denoising method, described in Section 3.4, is applied and finally the restored image is obtained as follows:

\[
    \hat{x}[n,m] = \sum_{i=1}^{J} \hat{\tilde{z}}_i[n,m] \ast g_i[n,m] + \hat{\tilde{y}}_i[n,m].
\]
3.2 Surround Inhibition

The surround inhibition operator takes into account the context influence of the surrounding of each point [17, 18] in the contour extraction problem. The inhibition term is evaluated as the integral of the gradient magnitude in the surroundings of a point; then it is subtracted from the gradient magnitude in the point of interest. The inhibition term is expected to be large in textured areas and low on object contours, thus allowing the suppression of texture while keeping meaningful edges. This operator is motivated by psychophysical and neurophysiologic findings (see [19] for arguments and further references).

Specifically, let \( M_s[n,m] \) be the gradient magnitude:

\[
M_s[n,m] = \sqrt{\left( \frac{\partial y}{\partial n} \right)^2 + \left( \frac{\partial y}{\partial m} \right)^2}.
\]

The inhibition term \( T_s[n,m] \) is defined as the weighted local average of \( M_s[n,m] \) on a special geometry support and it is computed as the convolution of \( M_s[n,m] \) and a weighting function \( w_s[n,m] \).

Such support is defined by two half rings around each pixel: a band region of width \( 2a \) oriented along the edge is excluded from the ring shaped surround of the considered point to avoid self-inhibition. We define the inhibition term \( T_s[n,m] \) as the minimum of the two weighted local averages of \( M_s[n,m] \) on the two resulting half-rings. More specifically, we define two weighting functions \( w^+[s]\phi[n,m] \) and \( w^-[s]\phi[n,m] \):

\[
w^+[s]\phi[n,m] = \frac{\text{DoG}^+[s]\phi[n,m]}{\sum_n \sum_m \text{DoG}^+[s]\phi[n,m]},
\]

where

\[
\text{DoG}^+[s]\phi[n,m] = \text{DoG}_s[n,m] \cdot U_{-1} \left[ \pm \left( n \cos \phi + m \sin \phi - a \right) \right].
\]

\( \phi \in [0, \pi) \) is a generic orientation, \( U_{-1} \) is the step function defined as follows:

![Fig. 1 - Restoration block diagram](image-url)
and, according to [20], \( \text{DoG}[n,m] \) is the difference of two concentric Gaussian functions:

\[
\text{DoG}_\sigma[\xi] = \left[ \mathcal{N}_2\left[ \xi, 0, \sigma^2 I \right] - \mathcal{N}_2\left[ \xi, 0, i^2 \sigma^2 I \right] \right]^+, 
\]

(17)

where \( |\cdot|^+ \) denotes half-wave rectification,

\[
|\xi|^+ = \begin{cases} 
\xi, & \xi \geq 0 \\
0, & \xi < 0 
\end{cases} 
\]

(18)

Then, we define and compute the modified inhibition term as follows:

\[
T_{\chi}[n,m] = \min \left\{ \left[ M_{\chi} * w^+_\eta \right][n,m], \left[ M_{\chi} * w^-_{\eta} \right][n,m] \right\}. 
\]

(19)

In practice, we compute the convolutions in (11) for a discrete set of four orientations \( \{\phi\}_{i=1}^{N_F} \), \( \phi = \frac{\pi i - 1}{N_F} \) and then, for each pixel, we use the result obtained for the angle that is closest to the gradient orientation \( \theta[n,m] \) for that pixel. The parameter \( a \) sets the width of the band region excluded by the support and it is set to be a fraction of the radius \( \rho_0 \). The edge strength \( c_{\chi}[n,m] \) is computed in the way that follows:

\[
c_{\chi}[n,m] = \left[ M_{\chi} [n,m] - \alpha T_{\chi}[n,m] \right]^+. 
\]

(20)

The exclusion of the central band region avoids the self-inhibition and is motivated by neurophysiologic studies according to which, inhibitory modulation originates from the regions flanking the receptive field of an orientation selective neuron on both sides of the optimal stimulus for that neuron. The parameter \( a \) controls the width of the excluded band region and we set it to be a fraction \( \eta \) of the radius \( \rho_0 \). Our experiments show that for values of \( \eta \) around 1 the choice of \( \eta \) is not critical for the performance of our algorithm. Therefore, we use \( \eta = 1 \) in the following.

The coefficient inhibition strength \( \alpha \) specifies the extent to which the inhibition term is taken into account. By varying \( \alpha \), the inhibition term can partially or completely suppress the sharpening of textured areas.

### 3.3 Binarization

After Surround Inhibition, objects contours are obtained by performing edge thinning by means of non-maxima suppression and binarization by thresholding. Object contours lead to long and wide connected components of nonzero pixels, while texture edges, especially after surround inhibition, lead to relatively short and thin components. Specifically, we apply non-maxima suppression to the signal \( c_{\chi}[n,m] \). Let \( u_{\chi}[n,m] \) be the unit vector parallel to the gradient \( \nabla c_{\chi}[n,m] \), i.e. \( \nabla c_{\chi}[n,m] = M_{\chi}[n,m]u_{\chi}[n,m] \); we consider the set \( S_{\chi} \) of all points which are local maxima of \( c_{\chi}[n,m] \) in the direction of \( u_{\chi}[n,m] \):
Let $S_{k_{i}}$, $i = 1, \ldots, N_{c}$, be the connected components of the set $S_{k_{i}}$.

$$S_{k_{i}} = \bigcup_{j} C_{j}^{(k_{i})}.$$

(22)

where $N_{c}$ is the number of such components. A morphological dilation is then applied to $C_{j}^{(k_{i})}$, with a $3 \times 3$ square $q_{3}$ as structuring element, and obtain dilated components $D_{j}^{(k_{i})}$:

$$D_{j}^{(k_{i})} = C_{j}^{(k_{i})} \oplus q_{3}.$$

(23)

Then, we define as global contour weight $G$ of $C_{j}^{(k_{i})}$, the sum of the values of $\hat{z}_{k_{i}}[n,m]$ over $D_{j}^{(k_{i})}$:

$$G[C_{j}^{(k_{i})}] = \sum_{[n,m] \in D_{j}^{(k_{i})}} |\hat{z}_{k_{i}}[n,m]|.$$

(24)

We finally compute a binary edge map $B_{k_{i}}$ by thresholding the global contour weight map, i.e.,

$$B_{k_{i}} = \bigcup_{G[C_{j}^{(k_{i})}] \geq \text{thres}} C_{j}^{(k_{i})}.$$

(25)

The binary map so obtained is used for the identification of the edge statistical model that is of the parameters $\lambda_{i}$ in (9).

3.4 Iterative Bayesian denoising

In this subsection we provide details about the iterative denoising scheme of Fig. 3. Our goal is to recover the transmitted image $X$ affected by additive white Gaussian noise $W$ from the noisy observation $Y$. Such observation is filtered with a bank of Laguerre-Gauss filters and decomposed at different scales. The single scale complex edge image resulting from the wavelet filter passes through the Surround Inhibition block where highly textured areas are suppressed, and it is further processed in the Binarization block, where a binary contour map is created, as described in Section 3.3. At this point, the real iterative denoising process takes place.

The Bayesian estimator, represented in Fig. 3 by the Bayesian Denoising block, will be introduced in Subsection 3.4.3. In order to recover the original signal from the noisy one, we want to clean the noisy high frequency contributions coming from the Surround Inhibition block. For this purpose, we will treat each single scale noisy observation separately, giving it as input to the Bayesian Denoising block. The goal of this process is to restore each high frequency contribution and to reassemble them all through inverse Laguerre Gauss filtering. For each single scale image, we need to provide additional parameters to the Denoising block, which are the signal and noise covariance matrices, $R_{Z_{i}}[n,m]$ and $R_{N_{i}}[n,m]$, the mixing parameters $\lambda_{i}$ and $\beta_{i}$, used in the signal and noise models (see (9) and (11)).

In the iterative approach we propose, we need to distinguish between the initialization stage and the steady state.

3.4.1 Initialization

As for the initialization, a preliminary denoising step is performed on the noisy image by means of the Surround Inhibition operator (see Fig. 3) which is detailed in Section 3.2. The output of the Surround Inhibition block is used to perform the estimation of the signal covariance matrix on a local basis.
At the first iteration, the covariance matrix needed to model the noise components are given, being the same value used for the additive white Gaussian noise. The mixing parameters $\lambda_i$ in (9) are evaluated in the Initialization step on a local base, using the border map information contained in the binary image generated by the Binarization operator (see Fig. 3) described in detail in Section 3.3. The same mixing parameters will be used for all the iterations.

With reference to Fig. 3, the output of the first Bayesian Denoising block feeds the following denoising step. The signal covariance matrix for the second Bayesian Denoising is the same calculated at the earlier step. At this stage, only the noise covariance matrix is updated, being calculated locally from the difference between the Laguerre-Gauss filtered image module and the module of the first Bayesian Denoising.

### 3.4.2 Generic iteration

After the algorithm is initialized, the generic iteration, which is represented in Fig. 3 by the switches changing their position, is performed. Specifically, for the odd denoising iteration, the signal covariance and the Bayesian Denoising input are taken from the previous denoising output, while the noise covariance is the one calculated at the former even step. For the even denoising iteration, the noise covariance matrices are updated from the previous ones, via a difference between the Laguerre-Gauss filtered image and the output of the odd Bayesian Denoising. The signal covariance matrix and the observation are taken from the previous denoising step. Specifically the estimation of the noise covariance matrix is performed as follows. It is straightforward to notice that $R_{N_i}[n,m]$ acts as a surround inhibition terms that takes into account the context influence of the surroundings of each point. $R_{N_i}[n,m]$ is supposed to be large in textured areas and small on flat regions thus leading to heaviest noise suppression inside texture while sharpening contours. Based on psychophysical and neurophysiologic findings (see [19], [20]), we compute $R_{N_i}[n,m]$ as the local average of the energy of the weighted complex edge image on a ring around each pixel with the exclusion of a narrow strip of width $2a$ oriented along the edge, as shown in Fig. 2.

Then, we define and compute the modified inhibition term as follows:

$$R_N[n,m] = \alpha \min \left\{ \left[ |\Delta z_{s_k}[n,m]|^2 \star w_{s_k}^+[n,m] \right], \left[ |\Delta z_{s_k}[n,m]|^2 \star w_{s_k}^-[n,m] \right] \right\} I$$

where $\theta[n,m]$ is the orientation of $\mathbf{\hat{z}}_{s_k}[n,m]$. 

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**Fig. 3 - Iterative scheme for the proposed denoising method (single scale)**
### 3.4.3 Bayesian denoising in the edge feature domain

A Bayesian estimate $\hat{X}$ of $X$, given $Y$, is obtained by the minimization of the associated absolute risk. Now, the calculation of such an estimate may be substantially simplified by removing stochastic interaction between samples. This operation can be approximated by spatial decorrelation. In our approach, we pursue this target through the CHW decomposition. As a matter of fact, spatial correlation of edges is generally quite lower than correlation of the original image. Based on (6), we want to recover $X$ from the MMSE estimates of its complex edge images at different resolutions, respectively constituted by the a posteriori expectation $\hat{Z}_s = \{\hat{z}_s[n,m]\}$ of $\hat{Z}_s = \{\hat{z}_s[n,m]\}$, given $Z = \{z_s[n,m]\}$. Neglecting residual spatial correlation, we propose a sub-optimum estimation procedure based only on the marginal a priori edge distribution. Thus, for each scale, we evaluate the conditional expectation $\hat{z}_s[n,m]$ of $\hat{z}_s[n,m]$ given $z_s[n,m]$ at site $[n,m]$ only.

By referring to the observation model given by (9)-(11), the derivation of the suboptimum Bayesian estimator based on multiscale complex edges requires the calculation of the $a$ posteriori probability density function of $\hat{z}_s[n,m]$ given $z_s[n,m]$. Applying the Bayes rule, the conditional expectation $\hat{z}_s[n,m]$ of $\hat{z}_s[n,m]$, given $z_s[n,m]$ can be written as:

$$\hat{z}_{s_k}[n,m] = E_{Z_{s_k}}[z_{s_k}[n,m] | z_{s_k}[n,m]] = CZNL(z_{s_k}[n,m]) =$$

$$= \sum_i \sum_j w_{ij} [z_{s_k}(n,m)] R_{Z_i}(n,m) \times [R_{X_j}[n,m] + R_{Z_i}[n,m]]^{-1} z_{s_k}[n,m]. \quad (27)$$

where $CZNL$ stands for Complex Zero-memory Non Linearity, and:

$$w_{ij}(z_{s_k}[n,m]) = \frac{\beta_j \lambda_i N_2[z_{s_k}[n,m], 0, R_{X_j}[n,m] + R_{Z_i}[n,m]]}{\sum_i \sum_j \beta_j \lambda_i N_2[z_{s_k}[n,m], 0, R_{X_j}[n,m] + R_{Z_i}[n,m]]}. \quad (28)$$

Eq. (27) says that in general, for signal and noise Gaussian mixtures, the MMSE estimator is a non-linear combination of conditionally optimal linear estimators, with gains $R_{Z_i}[n,m] [R_{X_j}[n,m] + R_{Z_i}[n,m]]^{-1}$ each matched to a pair $(i,j)$ of Gaussian submodels. The weights $w_{ij}(z_{s_k}[n,m])$ are just the posterior probabilities of each submodel pair. The CZNL acts as an anisotropic shrinking function enhancing or attenuating large/small edges more or less deeply, depending on their direction. Attenuation tends to zero for features orthogonal to the direction of the textured background.

Starting from the MMSE estimates of the CHW coefficients the restored image can be finally reconstructed as illustrated in the scheme of Fig. 1. In essence, the estimator (27) and the multiresolution representation (6) can be combined to yield the following restoration algorithm

$$\hat{X}[n,m] = \sum_{k=1}^4 CZNL(z_s[n,m]) * g_{s_k}[n,m] + \hat{y}_s[n,m]. \quad (29)$$

### 4. EXPERIMENTAL RESULTS AND CONCLUSIONS

Let us consider the problem of restoring images affected by an additive noise. The local character of our estimate is addressed using a 7x7 sliding window, centered on the considered pixel. As an illustrative application, we applied the restoration procedure to a set of images affected by additive white Gaussian noise with zero mean and $\sigma = 0.001$. The data set which was used to test the method is composed of 512x512 pixel gray level images.
Fig. 4 – Results of the proposed denoising method: in the left column, the original grayscale image; in the middle column, the AWGN corrupted image; in the right column, the restored image.
The Laguerre Gauss multiresolution decomposition is done through the choice of a dyadic set of scales. The multiresolution filters design implies an almost perfect coverage of the entire bandwidth \((-\pi,\pi]\). We know from the theory of the Circular Harmonic Functions that the peak of each single filter impulse response must be within this band. We choose filters with impulse responses with maxima in \(\{\pi/2,\pi,4\pi/8,\ldots\}\). In particular, our implementation employs four different scales, namely \(\{2\sqrt{2}/\pi,4\sqrt{2}/\pi,8\sqrt{2}/\pi,16\sqrt{2}/\pi\}\). The Surround Inhibition is applied to each single scale complex edge image in order to suppress texture due to noise, using an inhibition coefficient \(\alpha = 0.3\). In this implementation, a discrete set of \(N = 4\) directions is used, and connected components that contain less than \(G_{\text{min}} = 10\) pixels are considered too small to be retained as object contour. At each single scale, we calculate the mixing parameters \(\lambda_i\) locally, exploiting the contour activity highlighted by the binary border image generated after the Surround Inhibition. The noise mixing parameters \(\beta_i\) are fixed. As described in Section 3.4.1, the initialization is done via the first passage through the Bayesian Denoising. The initialization observation, signal and noise variances are identified in Fig. 3 by the dashed lines. Fig. 4 shows the results of the proposed method. From top to down, Lena, Barbara, Goldhill, and Peppers, images are shown, all from left to right in original, noisy and denoised version. It is easy to see how the proposed scheme is effective in removing all the undesired texture due to noise, while at the same time it is able to preserve all the significant high frequency details of the images. With respect to previous wavelet coefficient soft-thresholding techniques, the method described here presents augmented performance and flexibility due to directional discrimination capability and to the use of a priori probabilistic knowledge. In this regard, the technique can be operated in fixed or in adaptive mode. In fixed mode, the (estimated or a priori) probabilistic model is the same for the whole image or class of images, while in adaptive mode presented in this application mixture parameters can be locally estimated using moment matching. Besides, the method here presented shows additional features due to the use of the most perceptually relevant image features.

REFERENCES


