Continuous-Time Noise Modeling From Sampled Data

Rik Pintelon, Fellow, IEEE, Johan Schoukens, Fellow, IEEE, and Patrick Guillaume

Abstract—Most stochastic processes in engineering applications have an intrinsic continuous-time (CT) nature, for example, the noise generated by resistors and semiconductor devices. A few methods exist that identify CT noise models from sampled data. In this paper, a new method based on the concept of filtered band-limited white noise is introduced, an in-depth analysis of the basic assumptions made by the different approaches is given, and the pros and cons of each method are discussed. Finally, the new method is illustrated on a real measurement problem, i.e., the operational modal analysis of a bridge.

Index Terms—Continuous time (CT), frequency domain, noise models, system identification.

I. INTRODUCTION

Despite the fact that most noise-generating mechanisms have a continuous-time (CT) nature (e.g., thermal noise generated by resistors or noise generated by semiconductor devices), their impact in system identification has mostly been modeled as discrete-time (DT) filtered white noise [1]–[7]. The two main reasons for this are 1) the success of digital control and the related DT modeling and 2) the mathematical difficulty of handling stochastic differential equations [1], [2]. However, in various disciplines such as operational modal analysis [8], signal processing [9], astrophysics [10], and econometrics [11], CT noise modeling is of considerable importance. For example, in operational modal analysis of civil engineering structures (e.g., bridges and buildings), the system is excited by an unobserved input (e.g., turbulent wind flow and traffic), and the observed response (i.e., acceleration) carries information about the system poles (i.e., resonance frequencies and damping ratios). Knowledge of the latter is important for security/maintenance reasons.

Three methods are available to identify CT noise models from sampled data, namely 1) classical DT modeling with transformation of the results to the CT domain [6], 2) construction and identification of a difference equation that is parameterized in the original CT parameters [9], and 3) a new method, i.e., frequency-domain maximum-likelihood estimation [6] using the concept of band-limited white noise within a band-limited measurement setup. This paper presents an in-depth analysis of the basic assumptions made by the different approaches and discusses the pros and cons of each method. Especially, the new method is elaborated by 1) providing a theoretical justification for CT noise modeling, 2) showing that the nonidealities (deviations from the basic assumptions) of this method can be easily compensated for, 3) extending to the multivariable case, and 4) using a real-life example as an illustration, i.e., the operational modal analysis of an arc bridge.

II. CT NOISE MODELING

A. Classical Solutions

In the early days of system identification, CT noise was handled via stochastic differential equations [1], [2], where the noise-generating mechanism is a Wiener stochastic process (also called Brownian motion). Such stochastic processes have CT white Gaussian noise increments, and their variance increases linearly in time. Assuming that the noise-generating mechanism is a Wiener process and that the signals are sampled without antialias protection (see Fig. 1, top), it has been shown in [1] and [2] that an \( n \)th-order CT stochastic process can be described exactly at the sampling instances by an \( n \)th-order DT process. The poles of the DT process are related to those of the original CT process by the impulse invariant transformation, i.e.,

\[
z = \exp(sT_s)
\]

where \( T_s \) is the sampling period (see [1] and [2]).
In time-series analysis [3] and time-domain system identification [4]–[6], all these difficulties are avoided by assuming that the noise-generating mechanism is piecewise constant in-between white noise samples (also known as the zero-order-hold assumption). If in addition the observed signals are sampled without antialias protection (see Fig. 1, middle), then it has been shown in [5] and [6] that an nth-order CT stochastic process can be described exactly at the sampling instances by an nth-order DT process. The DT $H_d(z^{-1})$ and CT $H(s)$ noise models are related by the step invariant transformation, i.e.,

$$H_d(z^{-1}) = (1 - z^{-1})Z\left\{L^{-1}\{H(s)/s\}\right\} \quad (2)$$

where $L^{-1}\{\cdot\}$ is the inverse Laplace transform, and $Z\{\cdot\}$ is the $Z$-transform (see [7]). From a CT point of view, piecewise constant noise is a nonstationary stochastic process (see the Appendix).

The so-called direct approach starts from a CT white-noise-generating mechanism and replaces each derivative in the stochastic differential equation by a finite difference. The result is a difference equation that is parameterized in the original CT parameters. These parameters are identified from sampled data via a bias-correcting algorithm [9]. Since the method assumes that the signals are sampled without antialias protection, the sampling should be fast enough to avoid aliasing errors.

One can wonder whether Wiener processes, which have asymptotically (time → ∞) infinite variance, CT white noise, which has either infinite variance (power) or zero power spectral density, and piecewise constant noise, which is a nonstationary CT process, are realistic descriptions (approximations) of the true-noise-generating mechanism. Since in some applications it is impossible to lowpass-filter the signals before sampling (for example, sunspots data in astrophysics [10] or econometric data [11]), it makes sense to consider CT noise modeling without antialias protection [13]. However, one should realize that at low sampling rates, the noise power spectral density of the observed samples may then strongly depend on the true intersample behavior of the driving noise source.

B. Band-Limited White Noise in a Band-Limited Measurement Setup

A realistic description (approximation) of the true-noise-generating mechanism should at least have a finite variance (power). Since the variance of a stationary stochastic process $e_c(t)$ is related to its power spectral density $S_{e_c}(f)$ by

$$\text{var}(e_c(t)) = \int S_{e_c}(f)df \quad (3)$$

(see [1]), the condition $\text{var}(e_c(t)) < \infty$ imposes some limitation on the bandwidth of $S_{e_c}(f)$. The concept of CT band-limited white Gaussian noise introduced in [1] is a good candidate, i.e.,

$$S_{e_c}(f) = \begin{cases} \sigma_c^2/(2f_B), & |f| \leq f_B \\ 0, & |f| > f_B \end{cases} \quad (4)$$

where $\sigma_c^2 = \text{var}(e_c(t))$, and $f_B$ is the bandwidth of the power spectral density. We propose to combine this noise-generating mechanism with a band-limited measurement setup (see Fig. 1, bottom), where

$$AA(j2\pi f) = \begin{cases} 1, & |f| \leq f_s/2 \\ 0, & |f| > f_s/2 \end{cases} \quad (5)$$

in which $f_s$ is the sampling frequency and where the bandwidth of the noise is larger than or equal to the Nyquist frequency (i.e., $f_B \geq f_s/2$).

The power spectral density of the stochastic process $\eta(t)$ at the output of the antialias filter (see Fig. 1, bottom) can be written as

$$S_{\eta}(f) = |H(j2\pi f)|^2 |AA(j2\pi f)|^2 S_{e_c}(f) = |H(j2\pi f)|^2 S_{e_c}(f) \quad (6)$$

where $\varepsilon(t)$ is a CT band-limited white noise process with power spectral density defined by

$$S_{\varepsilon}(f) = \begin{cases} \sigma^2/f_B, & |f| \leq f_s/2 \\ 0, & |f| > f_s/2 \end{cases} \quad (7)$$

and $\sigma^2 = \text{var}(\varepsilon(t))$. Hence, $\eta(t)$ and $\varepsilon(t)$ are related by the following differential equation:

$$\eta(t) = H(p)\varepsilon(t) \quad (8)$$

where $p = d/dt$ is the derivative operator, and $H(s)$ is a rational form in the Laplace variable $s$. Since the autocorrelation function $R_{\varepsilon}(\tau)$ of $\varepsilon(t)$ is defined as

$$R_{\varepsilon}(\tau) = F^{-1}\{S_{\varepsilon}(f)\} = \sigma^2 \text{sinc}(\pi f_s \tau) \quad (9)$$

and is zero at the sampling instances where $F^{-1}\{\cdot\}$ is the inverse Fourier transform ($R_{\varepsilon}(nT_s) = 0$ for $n \neq 0$), it follows that $\varepsilon(t)$ is uncorrelated at the sampling instances $t = nT_s$. Since $e_c(t)$ is normally distributed, $e(n) = \varepsilon(nT_s)$ is a white Gaussian DT noise. Hence, the discrete Fourier transform (DFT) spectra $V(k)$ and $E(k)$ of the samples $v(n) = \eta(nT_s)$ and $e(n) = \varepsilon(nT_s)$, given

$$X(k) = N^{-1/2} \sum_{n=0}^{N-1} x(n)\bar{z}_k^n \quad (10)$$

are exactly related by

$$V(k) = H(s_k)E(k) + T_H(s_k) \quad (11)$$

where $E(k)$ is circular complex (i.e., $E(E^2(k)) = 0$) normally distributed, $T_H(s)$ is a rational form in $s$ with the same denominator as $H(s)$, and $s_k = j2\pi kT_s/N$ [14]. The noise transient term $T_H(s_k)$ takes into account the initial and final conditions of the experiment and some residual alias errors. It decreases as $O(N^{-1/2})$ with respect to the main term $H(s_k)E(k)$ [14].

From (10), it can be concluded that a CT model is the natural representation to be used when identifying noise models from sampled data collected with a band-limited measurement setup.
C. Discussion

- Model (11) is still valid when the condition $S_{e_c}(f) = 0$ for $|f| > f_B$ in (4) is relaxed to

$$S_{e_c}(f) = O\left(f^{-(1+\delta)}\right)$$

with $\delta > 0$ for $|f| > f_B$. (12)

Indeed, (12) is the weakest decay giving a finite variance (3), and $S_{e_c}(f) = |AA(j2\pi f)|^2S_{e_c}(f)$, with $AA(j2\pi f)$ defined in (5), still equals (7). An example of band-limited white noise satisfying (12) is the thermal noise generated by resistors in electrical circuits.

- Since the equivalent driving noise source $\varepsilon(t)$ in (8) is not observed [only $y(t)$ is measured], the noise model $H(s)$ is identified from an estimate of the noise power spectral density $S_n(f)$ (see Section III). Since $S_n(f)$ (7) is independent of the phase of $H(j\omega)$, the noise model $H(s)$ is only known within an all-pass section (no distinction can be made between minimum phase and nonminimum phase zeros).

- In practice, a perfect antialias filter (5) does not exist and, consequently, $\varepsilon(t)$ in (8) is only approximately band-limited white noise. However, the only requirement to be satisfied by the antialias filter is that the attenuation in the stopband ($f > f_s/2$) is sufficiently large (attenuations of more than a hundred decibels are easily realizable), even at the price of an increased passband ripple. Since the phase of the antialias filter does not influence the noise power spectrum $S_n(f)$ (6), the passband ripple of $AA(s)$ can easily be compensated for via an absolute amplitude calibration.

- Since the data acquisition channel itself produces some noise, model (11) should be extended as

$$V(k) = H(s_k)E(k) + T_H(s_k) + N_V(k)$$

(13)

where $N_V(k)$ is the measurement noise. Section III discusses the influence of $N_V(k)$ on the identified noise model and proposes a bias-correcting procedure.

- If the noise-generating mechanism $e_c(t)$ is not white within the frequency band of interest, then model (11) is still valid if the noise transfer function $H(s)$ also includes the coloring of $e_c(t)$. Unless some prior knowledge is available, no distinction can be made between the poles of the original $H(s)$ and those of the stochastic process $e_c(t)$. For example, in modal analysis applications (see Section IV), it is often known that the poles of the vibrating structure are lowly damped, whereas those of the noise-generating mechanism are highly damped, which allows distinguishing between them.

- Combining the results of Section II-B with those of [15], it follows that model (11), where $s_k$ is replaced by $\sqrt{s_k}$, is valid for diffusion phenomena. An example is the $1/f$ noise generated by semiconductor devices. Indeed, since $S_{e_c}(f)$ is proportional to $1/|f|$, it follows from (6) that $H(s)$ is proportional to $1/\sqrt{s}$.

- It is tempting to think that the results of Section II-A are valid, irrespective of the true intersample behavior of the noise-generating mechanism. The following reasoning shows that this is wrong. Consider a filtered CT band-limited white noise with an autocorrelation function $R(\tau)$. Its noise power spectral density given by

$$S(f) = \int_{-\infty}^{+\infty} R(\tau)e^{-j2\pi f\tau}d\tau$$

which is a rational function of $j\omega$ for $|f| \leq f_B$. Consider now a DT stochastic process with an autocorrelation function $R_d(n) = R(nT_s)$ (at the sampling instances, the DT process has the same first- and second-order moments as the CT process). Its noise power spectrum is given by

$$S_d(f) = \sum_{n=-\infty}^{+\infty} R_d(n)e^{-jn2\pi fT_s} = \frac{1}{T_s} \sum_{n=-\infty}^{+\infty} S(f - nf_s).$$

It clearly shows that, in general, $S_d(f)$ is not a rational spectrum in $\exp(-jn\omega T_s)$. Hence, a “high”-order rational function $H_d(z^{-1})$ is needed to model $S_d(f)$ accurately, and physical interpretation of $H_d(z^{-1})$ is dangerous, especially at low sampling rates.

III. IDENTIFICATION OF CT NOISE MODELS

A. Maximum-Likelihood Estimator

The noise model and the noise transient term in (11) are parameterized as

$$H(s, \theta) = \frac{C(s, \theta)}{D(s, \theta)} = \frac{\sum_{r=0}^{n_c} c_r s^r}{\sum_{r=0}^{n_d} d_r s^r}$$

$$T_H(s, \theta) = \frac{I_H(s, \theta)}{D(s, \theta)} = \frac{\sum_{r=0}^{n_h} i_r s^r}{\sum_{r=0}^{n_d} d_r s^r}. \quad (14)$$

Assuming that the DFT spectrum $V(k)$ is available at a set $\mathbb{K}$ of $F$ DFT frequencies $k \in \mathbb{K} \subseteq \{0, 1, \ldots, N/2\}$, the model parameters $\theta$ [coefficients $C(s, \theta)$, $D(s, \theta)$, and $I_H(s, \theta)$ polynomials] are found by minimizing the following nonlinear least squares cost function:

$$\frac{1}{F} \sum_{k \in \mathbb{K}} |g_F(\theta)|^2 \frac{S_V(k, \theta)}{|H(s_k, \theta)|^2}^2$$

subject to

$$\sum_{r=0}^{n_c} c_r^2 = \sum_{r=0}^{n_d} d_r^2 = 1 \quad (15)$$

with respect to $\theta$, where

$$g_F(\theta) = \exp\left(\frac{1}{F} \sum_{k \in \mathbb{K}} \log H(s_k, \theta)\right)$$

$$S_V(k, \theta) = |V(k) - T_H(s_k, \theta)|^2 \quad (16)$$

(see [12]). The minimizer $\hat{\theta}$ of (15) is calculated using a standard Newton–Gauss minimization algorithm [17]. To guarantee the numerical stability of the Newton–Gauss algorithm, (15) is expressed in the normalized model parameters obtained by scaling the angular frequencies by the median $\omega_{med}$ of the set.
of angular frequencies; for example, $d_r s^r = d_r \omega_{med}^r (s/\omega_{med})^r$ [18]. The variance $\lambda$ of the driving white noise source $E(k)$ is finally estimated as

$$\lambda(\hat{\theta}) = \frac{1}{F} \sum_{k \in K} \frac{S_V(k, \hat{\theta})}{|H(s_k, \hat{\theta})|^2}. \quad (17)$$

If $E(k)$ in (11) is independent (over $k$) circular process normally distributed, then (15) is the maximum-likelihood cost function. Under some standard assumptions concerning the true model, the model set, and the cost function, the minimizer $\hat{\theta}$ of (15) has the following asymptotic (i.e., $F \to \infty$) properties: strongly consistent (convergence to the true value); asymptotically normally distributed; and asymptotically efficient (minimal variance) [12]. Moreover, the consistency and asymptotic normality remain valid for non-Gaussian noise $E(k)$ that is correlated over the frequencies (mixing condition) [12].

B. Discussion

- The maximum-likelihood cost function (15) is the sum of the weighted ratio of the modeled $|H(s_k, \theta)|^2$ noise power spectrum. It is minimized by that value $\hat{\theta}$ of the model parameters for which $|H(s_k, \theta)|^2 \approx S_V(k, \theta)$ (see [12]).
- To reveal the influence of measurement noise $N_V(k)$ on the estimated noise model $H(s, \theta)$, we analyze the cost function (15) for $F \to \infty$. Applying the law of large numbers to (15) shows that the cost function converges for $F \to \infty$ to its expected value

$$\frac{1}{F} \sum_{k \in K} |g_F(\theta)|^2 \frac{E\{S_V(k, \theta)\}}{|H(s_k, \theta)|^2} \quad (18)$$

(see [12]). Using (13), $T_H(s_k, \theta) = O(F^{-1/2})$, and the fact that $E(k)$ and $N_V(k)$ are independent, we find

$$\lim_{F \to \infty} E\{S_V(k, \theta)\} = |H_0(s_k)|^2 \lambda_0 + S_NV(k) \quad (19)$$

where $H_0(s_k)$ is the true noise model, $\lambda_0 = \text{var}(E(k))$, and $S_NV(k)$ is the power spectrum of $N_V(k)$. From (18) and (19), it can be concluded that $|H(s_k, \theta)|^2$ models the sum of the desired noise power spectrum $|H_0(s_k)|^2 \lambda_0$ and the measurement noise power spectrum $S_NV(k)$. If $S_NV(k)$ is constant [i.e., $N_V(k)$ is white noise], then the poles of $H(s, \theta)$ are still correct (but not the zeros); otherwise, $H(s, \theta)$ contains the poles of both noise processes.

- The bias term $S_NV(k)$ introduced by the measurement noise [see (19)] can be compensated for via a consistent estimate $\hat{S}_N(k)$ of the noise power spectrum $S_N(k)$ of the data acquisition channel. Indeed, replacing $S_V(k, \theta)$ in (15) by $S_V(k, \theta) - \hat{S}_N(k)$ eliminates $S_NV(k)$ in (19). A consistent estimate $\hat{S}_N(k)$ is obtained by measuring the noise power spectrum of the acquisition channel without excitation.

- Cost function (15) can be extended to the multivariable case

$$\det\left(\text{Re}\left(\frac{1}{F} \sum_{k \in K} |g_F(\theta)|^2 H^{-1}(s_k, \theta) S_V(k, \theta) H^{-H}(s_k, \theta)\right)\right) \quad (20)$$

where

$$S_V(k, \theta) = (V(k) - T_H(s_k, \theta))(V(k) - T_H(s_k, \theta))^H$$

and where $V(k)$ is an $n_y \times 1$ vector, $H(s, \theta) = C(s, \theta)/D(s, \theta)$ is an $n_y \times n_y$ matrix, $C(s, \theta)$ is an $n_y \times n_y$ matrix polynomial, $I_H(s, \theta)$ is an $n_x \times n_y$ vector polynomial, and $D(s, \theta)$ is a polynomial [16]. The constraints (15) are replaced by

$$\sum_{r=0}^{n_d} \det(C_r) = 1 \sum_{r=0}^{n_d} C_r^T C_r = I_{n_y} \quad (22)$$

where $C_r$ is the $n_y \times n_y$ matrix coefficients. The covariance matrix of the driving white noise source $E(k)$ ($n_y \times 1$ vector) is finally estimated as

$$\Lambda(\hat{\theta}) = \frac{1}{F} \sum_{k \in K} H^{-1}(s_k, \hat{\theta}) S_V(k, \hat{\theta}) H^{-H}(s_k, \hat{\theta}). \quad (23)$$

Since all calculations are performed in the frequency domain, $H^{-1}(s, \theta)$ in (20) may be unstable. This is not true for the time-domain approaches, which require a stable inverse noise filter in each iteration of the nonlinear minimization algorithm. For multivariable systems, the stabilization of the inverse noise filter requires symbolic matrix polynomial calculus, which is the bottleneck of the time-domain methods.

IV. MEASUREMENT EXAMPLE: MODAL ANALYSIS OF A BRIDGE

The Villa Paso arch bridge shown in Fig. 2 is situated in the southeastern part of Italy. The bridge is excited by the traffic in the vertical direction and by the wind mainly in the horizontal direction. These excitations cannot be measured and are assumed to be white in the frequency band of interest (using operational modal analysis). Acceleration measurements have been performed on the deck of the bridge at 14 test points, both in the horizontal and vertical directions (see Fig. 2). All signals are measured simultaneously during about 14 min at the sampling rate $f_s = 400$ Hz, giving $N = 337,700$ data points per measurement channel. In the frequency band of interest [1.18 Hz, 4.14 Hz], the measurement noise $N_V(k)$ is white, and the passband ripple of the acquisition channel is negligibly small (i.e., < 0.01 dB). Only the measurements in the horizontal direction at test points 1, 6, and 7 are handled here. These data are modeled in the band [1.18 Hz, 4.14 Hz] (DFT lines $k = 999, 1000, \ldots, 3499 \Rightarrow F = 2501$) with a multivariable
Fig. 2. Villa Paso bridge. Top: picture bridge. Bottom: schematic top view with numbered test points.

Fig. 3. Measured (gray bullets) and modeled (solid line) noise power spectra and Cramér–Rao lower bound (dashed line) of test points 1, 7, and 8. Amplitude (in decibels) as a function of frequency (in hertz).


TABLE I

<p>| Estimated Resonance Frequencies $f_0$ and Damping Ratios $\zeta$ of the Villa Paso Bridge |
|----------------------------------|----------------------------------|</p>
<table>
<thead>
<tr>
<th>$f_0 \pm \text{std}(f_0)$ (Hz)</th>
<th>$\zeta \pm \text{std}(\zeta)$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5761 ± 0.0014</td>
<td>0.67 ± 0.09</td>
</tr>
<tr>
<td>2.6650 ± 0.0023</td>
<td>0.95 ± 0.09</td>
</tr>
<tr>
<td>3.5098 ± 0.0026</td>
<td>0.74 ± 0.07</td>
</tr>
<tr>
<td>3.7775 ± 0.0033</td>
<td>1.03 ± 0.08</td>
</tr>
</tbody>
</table>

Since the ratio of the 95% upper bound to the 95% lower bound of the measured noise power spectrum equals 22 dB (see [14, Table 2-1, p. 56]), it follows from Fig. 3 that the model explains the data very well. Table I gives the estimated resonance frequencies and damping ratios together with their standard deviation. Note that the relative uncertainty on the estimated resonance frequencies (less than 0.09%) is much smaller than that on the estimated damping ratios (less than 14%). The damping ratios themselves are about 1% and less.

V. CONCLUSION

We have shown that under the realistic conditions that the noise-generating mechanism has a noise power spectral density that is flat in a given frequency band ($|f| \leq f_B$ with $f_B \geq f_s/2$) and decreasing as $O(f^{-(1+\delta)})$ with $\delta > 0$ outside the band ($|f| > f_B$) and that the measurement setup is band limited with a sufficiently high stopband attenuation (passband ripple does not matter), CT noise modeling ($s$- or $\sqrt{s}$ domain) is the natural choice. Moreover, the bias errors introduced by the measurement setup can easily be compensated for through an absolute amplitude calibration of the acquisition channel (removal bias introduced by the passband ripple) or a calibration of the power spectral density of the noise generated by the acquisition channel (removal bias introduced by the measurement noise).

In those applications where it is impossible to lowpass-filter the data before sampling, the results may be (very) sensitive to the true intersample behavior of the noise-generating mechanism. This is not the case in a band-limited measurement setup.

APPENDIX

Calculating the autocorrelation of $e(t)$ using

$$E\{e(nT_s)e^*(mT_s)\} = \sigma^2 \delta(n-m)$$

where $\delta(n)$ is the Kronecker delta (i.e., $\delta(n) = 0$ for $n \neq 0$ and $\delta(0) = 1$), gives

$$R_{e}(\tau,t) = E\{e(t)\overline{e(t-\tau)}\}$$

$$= \begin{cases} \sigma^2, & (0 \leq \tau < T_s) \text{ and } (\tau + nT_s \leq t < (n+1)T_s) \\ 0, & \text{elsewhere} \end{cases}$$

(27)

where $n = \ldots, -1, 0, \ldots$. Since (27) depends on $t$, it can be concluded that $e(t)$ is a nonstationary CT stochastic process. It emphasizes that the zero-order-hold assumption is an approximation of the true stationary CT noise process.
REFERENCES


Rik Pintelon (M’90–SM’96–F’98) was born in Gent, Belgium, on December 4, 1959. He received the degree of electrical engineer (burgerlijk ingenieur) in July 1982, the degree of doctor in applied sciences in January 1988, and the qualification to teach at a university level (geaggregeerde voor het hoger onderwijs) in April 1994, all from the Vrije Universiteit Brussel (VUB), Brussels, Belgium.

From October 1982 to September 2000, he was a Researcher with the Fund for Scientific Research-Flanders at the VUB. Since October 2000, he has been a Professor with the Department of Fundamental Electricity and Instrumentation (ELEC), VUB. His main research interests are in the field of parameter estimation/system identification and signal processing.

Johan Schoukens (M’90–SM’92–F’97) was born in Belgium in 1957. He received the degree of engineer in 1980 and the degree of doctor in applied sciences in 1985 from the Vrije Universiteit Brussel, Brussels, Belgium.

He is currently a Professor with the Department of Fundamental Electricity and Instrumentation (ELEC), Vrije Universiteit Brussel. His main research interests are in the field of system identification for linear and nonlinear systems.

Patrick Guillaume was born in Anderlecht, Belgium, in 1963. He received the degree in mechanical–electrotechnical engineering in 1987 and the Ph.D. degree in 1992 from the Vrije Universiteit Brussel, Brussels, Belgium.

In 1987, he joined the Department of Fundamental Electricity and Instrumentation (ELEC), of the Vrije Universiteit Brussel. In 1996, he joined the Department of Mechanical Engineering, where he is currently a Lecturer. His main research interests are in the field of system identification, signal processing, and experimental modal analysis.