Thermal network model of supercapacitors stack

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Abstract—In the field of urban transport, supercapacitors are submitted to relative high charge and discharge currents and therefore significant heat generation occurs. It has been shown that the temperature has a great influence on the aging mechanism of supercapacitors and, by consequence, on the lifetime of the system. That is why the thermal management becomes a key issue in the study of the performance of a supercapacitors stack. In this paper, thermal modeling of a supercapacitors stack is presented and a matrix representation of the model is deduced. Validation of the model has been carried out using measurement on a developed test bench. Sensitivity of the model according to input uncertainties is finally discussed.

Index Terms—Supercapacitors, thermal modeling, heat transfer, sensitivity analysis

I. INTRODUCTION

Electrical double layer capacitors, also called supercapacitors, are energy storage devices that operate intermittently between batteries and capacitors with respect to energy storage and power performance [1]. They are attractive for many low or high power applications such as power sources in order to reduce the constraints on the battery or as temporary energy storage devices in electric vehicles [1-7].

Although the internal serial resistance of supercapacitors (SC) is very low, power losses cannot be neglected due to the relative high current charge and discharge cycles (hundreds of amperes). Consequently, power dissipations involve an increase of the temperature.

Because of the relative low maximal operating voltage of a single SC, a large number of cells must be connected in series in order to obtain the required voltage. In a supercapacitors pack, the location of each cell and the cooling strategy has a great influence on the temperature of each component inside the pack. However, it has been shown by many authors that the temperature has a strong influence on performance, reliability and lifetime of supercapacitors [8-13].

Therefore, in order to properly qualify the lifetime and the performance of supercapacitors stack, thermal study becomes a key point. In this paper, we present the thermal modeling of a supercapacitors storage stack dedicated to supply electrical buses in case of electrical microcuts. The thermal model is based on a lumped parameter approach and an intuitive discretization of space. It takes into account the conduction, convection and transfer of mass phenomena.

The stack consists of 120 supercapacitors (Maxwell BCAP 3000F 2.7V) connected in series. Supercapacitors are distributed on both sides of the stack and each side is composed of 10 columns of 6 SC. Because of the symmetry of the stack, the problem can be reduced to the study of the half stack that is 60 SC. The supercapacitors are arranged in a staggered manner, which gives higher heat transfer rates than in line arrangement [14, 15]. The proposed thermal model is based on a lumped parameter approach and the discretization of space in finite volumes. However, in order to decrease computation time, the adopted methods are based on coarse volume elements, which allow making simulation in reasonable times especially for electro-thermal models simulations. In order to validate the model, thermal test has been carried out on the industrial supercapacitors stack. In section II, the thermal model of the stack is presented. Comparison between simulated and experimental results is illustrated in section III. In section IV, the sensitivity analysis of the model is discussed.

II. THERMAL MODEL OF THE SUPERCAPACITORS BANK

A. Introduction

The problem of thermal modeling of supercapacitors pack has been studied in [16-18]. In [16] and [17], the finite element model of small number of supercapacitors has been investigated. However, if the number of elements increases, the simulation time becomes larger and this approach becomes inadequate for a fast computation of the supercapacitors temperature dispersion inside the stack, which is required if the coupling between electrical (fast dynamics) and thermal (low dynamic) behavior has to be studied. In [18], nodal thermal model of a SC stack has been proposed in the case of ventilation in axial direction. However, this cooling solution becomes expensive and complicated, when the size of the bank becomes large, especially in the presence of balancing circuit (figure 1).

Figure 1: Supercapacitors bank.
In the system studied in this paper, transversal ventilation is adopted. As it is shown in figure 1 the air flow propagates from the inlet to the outlet of the stack while ventilating components on the transversal axis. This makes the modeling procedure more complex since the temperature of the air around a supercapacitors depends on its location inside the bank.

The adopted method to simulate thermal behavior of our supercapacitors stack is based on a lumped parameter modeling approach using a thermal-electrical analogy. This allows us to utilize the widely known Kirchhoff’s laws in order to model the thermal phenomena. First of all, it is necessary to divide the system into a finite number of volumes called thermal nodes [19]. The thermal properties of each node are supposed to be homogeneous in each elementary volume (figure 2) and are represented by a thermal capacitance associated to a temperature. Exchanges between contiguous nodes are represented by thermal resistances (conduction, convection) and also by heat transportation sources (heat transfer by mass transport), which reflect the different mechanisms of heat exchanges.

The chosen space discretisation is shown in figure 2. Due to the staggered arrangement of supercapacitors, the air around a supercapacitor has been divided into six volumes (figure 2). Moreover, two thermal nodes are considered for each SC: the first one represents temperature of the terminal and the second one represents the temperature of the case of the component. The boundary conditions correspond to a constant temperature for both the inlet air flow and the metal plates, which are placed on both sides of the assembly (figure 2). The thermal model can be implemented into electrical software using the thermal-electrical analogy. figure 3 represents the equivalent electrical circuit of the thermal model of an elementary part of the stack.

**Figure 2:** Discretisation of the supercapacitors stack.

As it is shown, each node corresponds to a thermal capacity \( C_{TH,SC1}, C_{TH,air}, ... \). Thermal resistances \( R_{cond}(1,2), R_{conv}(2,7), ... \) represent the heat transport phenomena respectively by conduction and by convection. The heat transfer by mass transport, in a node \( i \), depends on the temperature of the inlet flow into the node \( i \) and the temperature of the node itself. So, the heat transfer by mass transport is modeled via a heat flux source \( \phi_{air} \) modulated by the temperature gradient between the node itself and the upstream nodes. The heat flux sources \( (\Phi_1, \Phi_2, ...) \) represent the heat production into supercapacitors. Using Kirchhoff’s laws, the equations that represents the thermal model can be elaborated so that the model can be represented by a matrix form as it is represented in the next section.

**B. Matrix representation of thermal model**

Matrix representation of the thermal model can be build referring to Kirchhoff’s law. Introducing the thermal capacity vector \( C_{th} \) which comprises the values of the thermal capacity of each volume, the thermal model is given by:

\[
\begin{align*}
\frac{dT}{dt} &= [A_{cond} + A_{conv} + A_{trans}]T + q_{gen} \\
&= GT + q_{gen} 
\end{align*}
\]  

(1)

Where \( T \) is the vector of the temperature of each node, \( A_{cond} \) and \( A_{conv} \) is respectively the conduction and convection matrix which contains conduction and convection resistances between all nodes, \( A_{trans} \) is the fluid transport matrix that contains mass flow rate from one node to another one. Summing these different contributions leads to the conductance matrix \( G \). \( q_{gen} \) represents the heat production in each SC. The conductance matrix can be determined based on the following properties [20]:

- All the diagonal terms are negative, others ones are positive or null.
- Apart the terms, which are related to the air transportation phenomena, the matrix is symmetrical.
For large systems, the matrix G is sparse since the temperature of a node depends mainly on the temperature of its neighbors.

In the next section, the identification of the values of the conduction and convection resistances is presented before the modeling of the mass flow rate from one node to another one.

C. Thermal model of the supercapacitor

The development of thermal model for the supercapacitors requires to properly studying the two following points: the first one refers to the modeling of heat generation inside the supercapacitors; the second one concerns the model of the heat transfer phenomena. However, heat generation sources in supercapacitors are of two kinds:

1. Irreversible losses due to ionic transport of charge in the electrolyte, and electronic transport of charge in electrode and metallic collector \[16\], \[21\].

2. Reversible losses due to thermodynamic phenomena taking place inside the supercapacitor \[22\].

To simplify the thermal model and as the thermal constant time is larger than the period of charge and discharge, the reversible sources of heat can be neglected \[23\]. Based on electro-thermal analogy, a simplified thermal model is adopted in order to identify the thermal parameters of the SC (figure 4). Inside the supercapacitor, the conduction phenomena is taken into account via the conduction resistance (\(R_{\text{cond}}\)), but the convection and radiation phenomena are neglected in this case, as the studied SC is isolated in a quiet atmosphere. However, the free convection phenomenon between the supercapacitor and the ambient air is considered via the convection resistance (\(R_{\text{conv}}\)). Its value depends on the free convection coefficient and the heat exchange surface of supercapacitor.

To characterize this simple thermal model, cycling experiments were performed. The experiment test consists of a series of charge-discharge cycles followed by rest periods (figure 5). During these rest periods, the temperature of the supercapacitor decreases to ambient temperature. As the experimental conditions have a significant influence on the identification of the model parameters, the tests are realized in a climatized room where the air temperature can be considered as fixed throughout the test duration. The supercapacitor was charged and discharged with a constant current to obtain a nearly constant heat generation. The supercapacitor current and voltage are shown in figure 5 where the voltage of supercapacitor is swept between 2.5V and the half of this voltage. The behavior of the voltage around min and max values is related to the voltage drop across the serial resistance of the supercapacitor.

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![Figure 4: Thermal model of the supercapacitor.](image1)

![Figure 5: Voltage and current versus time during cycling.](image2)

![Figure 6: Supercapacitor under test.](image3)

Different temperatures were measured with thermocouples, one located on the terminal and another one sticks on the case of the supercapacitor (figure 6). Thermal grease has been added to attach the thermocouple which relatively isolates it from the air.

The heat generation \(\Phi_1\) is calculated in relation with the energy difference between the charge and discharge cycle, which is measured directly at the terminal of supercapacitor. Looking to figure 7, the energy loss for one charge-discharge cycle is equal to 504J. We can assume that the energy loss is fully converted into heat then, dividing the energy loss by the duration of one cycle (= 79 s) results on the average lost power during a cycle, which leads to a heat generation \(\Phi_1\) estimated to 6.4 W. The advantage of this method is that we can determine accurately the losses inside the supercapacitor without needing to precisely estimate the serial resistance of the component which depends on the frequency of the cycle and on the component temperature.
The temperature of the terminal has been assumed to be equal to the temperature inside the supercapacitor. This hypothesis is confirmed by the experimental data obtained by Maxwell Technologies [21]. From the steady state temperatures at terminal and case of the supercapacitor (figure 8), the values of \( R_{\text{cond}} \) and \( R_{\text{conv}} \) can be deduced:

\[
R_{\text{cond}} = \frac{T_{\text{terminal}} - T_{\text{case}}}{\phi_i} \quad (2)
\]

\[
R_{\text{conv}} = \frac{T_{\text{case}} - T_a}{\phi_i} \quad (3)
\]

Where \( T_a \) is the ambient temperature, \( T_{\text{terminal}} \) is the temperature of the terminal (°C), \( T_{\text{case}} \) is the temperature of the case (°C).

The thermal capacity of the supercapacitor is determined by calculating the thermal time constant of supercapacitor, so that:

\[
C_{TH-SC} = \frac{\tau}{R_{\text{cond}} + R_{\text{conv}}} \quad (4)
\]

Where \( C_{TH-SC} \) is the thermal capacity of supercapacitor (J/K).

The thermal time constant \( \tau(s) \) of supercapacitor is determined from the behavior of the temperature during the rest period after the cycling period.

For the Maxwell BCAP 3000F 2.7V supercapacitor, the thermal convection resistance \( (R_{\text{conv}}) \), thermal conduction resistance \( (R_{\text{cond}}) \) and thermal capacitor \( (C_{TH-SC}) \) obtained are respectively 0.627 K/W, 1.7 K/W and 700 J/K.

In our works, the thermal model is used for sizing and ageing study of industrial energy storage bank dedicated to transportation applications. In this type of applications, the stack is solicited with repetitive charge/discharge cycles for duration relatively bigger than the thermal constant time of the stack. So, for these two problems (sizing and ageing), the key value is the hottest steady state temperature inside the energy storage device. That is why in this paper only the steady state temperatures of the supercapacitors are of interest and it is not necessary to focus on the SC thermal capacity, which does not play here a crucial role. In other cases where transient temperatures are studied, calorimeter method is more adequate to determine the thermal capacity.

With the identified values of the conduction resistances we can establish the conduction matrix \( A_{\text{cond}} \). The heat transfer by conduction \( \frac{dQ_{\text{cond}}}{dt} \) can now be written in the following way:

\[
\frac{dQ_{\text{cond}}}{dt} = A_{\text{cond}} \cdot T
\]

Defining \( R_{\text{cond}}(i,j) \) as the conduction resistance between nodes \( i \) and \( j \), the terms \( A_{\text{cond}}(i,j) \) are equal to:

\[
A_{\text{cond}}(i,j) = \begin{cases} 
\frac{1}{R_{\text{cond}}(i,j)} & \text{if } j \in a_i \\
0 & \text{elsewhere}
\end{cases}
\]

Where \( a_i \) is the group of adjacent nodes that exchanged heat energy by conduction with nodes \( i \). The diagonal terms \( A_{\text{cond}}(i,i) \) are calculated using this equation:

\[
A_{\text{cond}}(i,i) = \sum_{j \in a_i} \frac{-1}{R_{\text{cond}}(i,j)}
\]

Since conduction occurs only between the terminal and the case of supercapacitor; only the terms that are related to the terminal and the case of supercapacitors are non zero, all the other ones are null. As an example and looking to figure 3, the term \( A_{\text{cond}}(1,2) \) is equal to \( \frac{1}{R_{\text{cond}}(1,2)} \), the term \( A_{\text{cond}}(1,1) \) is equal to \( \frac{-1}{R_{\text{cond}}(1,2)} \) and all other terms of the first lines of the matrix \( A_{\text{cond}} \) are null.

Note that the identified value of convection resistance from the previous represents natural convection around the supercapacitor in absence of ventilation and in horizontal position, this value cannot be used for simulating the stack during forced ventilation. In next section we quantify the value of this resistance in the presence of external ventilation and taken into account the geometrical configuration of the stack.

D. Convection heat modeling

The convection is a heat transfer between a fluid and a solid due to the motion of a fluid such as air or water [14]. In our study, we are concerned with the heat convection across a
stack of cylinders. In such system, the convection coefficient depends on the position of supercapacitors in the bank and the air velocity. However it's difficult to estimate exactly these coefficients for every row in the bank. On the other hand, several literature works have approached this problem by estimating the mean convection coefficient via experimental results and empirical expressions covering a range of tube configurations and flow conditions [14, 15, 24, 25]. In our work, we will refer to these empirical expressions in order to calculate heat convection coefficient. These works estimate the Nusselt number \((Nu)\) which is related to heat convection coefficient \((h)\) via this equation:

\[
Nu = \frac{hD}{\lambda}
\]  

(8)

Where \(h\) is the convection heat transfer coefficient \((W m^{-2} K^{-1})\), \(D\) is the diameter of supercapacitor \((m)\), \(\lambda\) is the thermal conductivity of air \((W m^{-1} K^{-1})\).

The Nusselt number is calculated from other dimensionless numbers such as Reynolds number \(Re\) and Prandlt number \(Pr\) which characterize the heat exchange according to turbulence, temperature gradient, geometrical characteristics and fluid properties.

According to heat transfer literature, the general form of the Nusselt number is:

\[
Nu = \alpha Re^{\beta_1} Pr^{\beta_2}
\]  

(9)

Where \(\alpha\), \(\beta_1\) and \(\beta_2\) are experimental coefficients. \(Pr\) is the Prandlt number which depends on the air characteristic, \(Re\) is the Reynold number.

The Nusselt number for supercapacitors placed in column 3 to 10 has been considered identical for all supercapacitors since the air flow can be considered as stable after a few columns. Given in literature, the Nusselt number for these columns is obtained from the following equation:

\[
Nu = 0.34 Re^{0.6} Pr^{0.36}
\]  

(10)

For the first column where the flow is similar to a single tube in front of the flow, and for second column the Nusselt number is deduced referring to equation (9):

\[
Nu = 0.9 Re^{0.6} Pr^{0.36}
\]  

(11)

The convection resistance \(R_{conv}\) is calculated based on this formula:

\[
R_{conv} = \frac{1}{hS}
\]  

(12)

Where \(S\) is the surface of supercapacitors in front of air \((S = \pi D l_{sc}, l_{sc}\) is the length of supercapacitor).

Finally the heat convection coefficients and the convection resistances are summarized in table 1 for two values of air velocity \((V_{air})\).

<table>
<thead>
<tr>
<th>(V_{air}=0.23) m/s</th>
<th>28</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>(h) ((W m^{-2} K^{-1}))</td>
<td>1.38</td>
<td>2.4</td>
</tr>
<tr>
<td>(R_{conv}(K/W))</td>
<td>1.5</td>
<td>2.6</td>
</tr>
</tbody>
</table>

Using these convection resistances, the heat transfer by convection \(\frac{dQ_{conv}}{dt}\) can be written as:

\[
\frac{dQ_{conv}}{dt} = A_{conv} \cdot T
\]  

(13)

Similarly to the study of conduction terms, the same rules are applied in order to deduce the terms \(A_{conv}(i,i)\) and \(A_{conv}(i,j)\).

### E. Heat transfer by mass transfer

The mass transfer plays a main role when the system is ventilated via forced external sources. Therefore, it is essential to be able to represent this phenomenon in order to obtain an accurate modeling of the stack. Being given three consecutive thermal nodes \((i-1, i, i+1)\) and supposing that the inlet mass flow from node \((i-1)\) to \(i\) is the same as the outlet mass flow from \(i\) to \((i+1)\), so we can write:

\[
\dot{m}_{air(i-1,i)} = \dot{m}_{in(i-1,i)} = \dot{m}_{out(i,i+1)}
\]

where \(T_i\) is the temperature of the node \((i)\) \((\degree C)\), \(\dot{m}_{in(i)}\) is the mass flow rate from node \(i\) to node \(j\) \((kg.s^{-1})\), \(C_p\) is the specific heat capacity \((J kg^{-1} K^{-1})\).

As it is shown in eq.14, in order to represent the mass transfer phenomena, the mass flow rate between nodes must be determined.

In in-line arrangement, the mass flow rate from one node to another one can be taken the same for all consecutives nodes as the direction of flow is essentially perpendicular to the cross section of the channel. In the case of staggered arrangement, we cannot keep this hypothesis as the displacement of air occurs in all directions (figure 9). In our case, we chose to propagate the mass flow throughout the supercapacitors stack as it is shown in (figure 10). We assume here that the pressure drop between nodes can be neglected. This allows the mass flow rate to be calculated considering the air velocity nearly constant in the stack. The mass flow rate is equally distributed between nodes in the case of nodes with equal cross sections. The propagation of air flow for nodes at the extremities has been calculated taking into consideration the ratio between the cross section of the nodes.

**Table 1. Convection parameters**
Figure 9: Air distribution for aligned and staggered configuration.

Figure 10: Mass flow transport for internal and external nodes

Taking as an example three thermal nodes where the air comes from node 1 and propagates towards nodes 2 and 3 (figure 10). Let $S_1$ and $S_2$ be the cross sections respectively between nodes (1,2) and nodes (1,3). The air mass flow rate from node 1 to node 2 (respectively 3) becomes:

$$m_2 = m_1 \frac{S_1}{S_1 + S_2} \quad m_3 = m_1 \frac{S_2}{S_1 + S_2} \quad (15)$$

Where $m_1$ is the total entering mass flow rate to node 1 ($Kg/s^{-1}$), $m_2$ is the mass flow rate between node 1 and 2 ($Kg/s^{-1}$), $m_3$ is the mass flow rate between node 1 and 3 ($Kg/s^{-1}$).

According to equation (14), the heat transfer by mass transport is linear and it can be represented in matrix form:

$$\frac{dQ_{\text{trans}}}{dt} = A_{\text{trans}} T \quad (16)$$

The computation of the coefficients of the matrix $A_{\text{trans}}$ is a slightly different from the calculation of matrix $A_{\text{cond}}$ and $A_{\text{conv}}$ since the heat transfer by mass flow in a node i do not depend on all adjacent nodes, but only on the upstream ones. So the terms $A_{\text{trans}}(i,j)$ are equal to:

$$A_{\text{trans}}(i,j) = \begin{cases} m_{(j,i)} C_p \quad \text{if } j \in u_i \\ 0 \quad \text{elsewhere} \end{cases} \quad (17)$$

Where $u_i$ is the group of nodes that are upstream with respect to i and exchange heat by mass transfer with i.

In other side, the diagonal terms of the matrix are equal to:

$$A_{\text{trans}}(i,i) = - \sum_{j \in u_i} (m_{(j,i)} C_p) \quad (18)$$

For the computation of the thermal capacitance of each control volume ($V$) surrounding the supercapacitors, the following formula is used:

$$C_{TH-air} = \rho V C_p \quad (19)$$

With $\rho$ is the density of air ($Kg.m^{-3}$), $V$ the volume of air ($m^3$).

### III. Model Validation

To validate the thermal model, a test bench has been developed. It consists of two stacks of supercapacitors that alternatively discharge in each other. An external supply compensates the losses taking place during phases of charge and discharge. As a result, the whole system consumes only its losses. One stack is equipped with thermocouples placed on different supercapacitors at different locations. Supercapacitors, which are marked with cross in figure 11, are those for which terminal temperature is available.

An acquisition board terminal with input-output of different nature (current, voltage, temperature) was developed and the data processing is done via Labview software. The cycling test is composed of charging and discharging periods at constant power (50 kW) with rest periods (figure 12). The duration of rest periods is adjustable so that different cycles with different RMS current can be applied. Six tests has been done by varying the rest periods ($t_r$) and the air velocity ($V_{air}$). The air flow is generated by a ventilator, and the air flow velocity is measured using an anemometer at the stack inlet. Different air velocities are obtained by varying the voltage supplying the ventilator.

As a result, figure 13 shows the dispersion of the temperature inside the stack obtained for ($V_{air} = 0.23 \ m/s$, $t_r = 90 \ s$). The steady state thermal model ($GT + q_{gen} = 0$) (equation 1) is implemented using Matlab@Mathworks software. Heat losses are calculated based on the value of the resistance of each supercapacitor and the RMS current of the cycle. Erreur ! Source du renvoi introuvable. details the comparison between experimental and simulated results. It should be noted that the used cycle (figure 12) is a stressed cycle and it is far from real operating conditions. This explains the large difference of temperature between components.

Concerning the location of the maximum temperature inside the stack, we should note that the SC6 exchanges heat by convection with the air volume located on the left side, which is shared with other components, and also with the air volume.
on its right side that is the air at the stack output. The air at the stack output is colder than on the middle because it is mixed with colder air flowing through the gap between supercapacitor and the metallic plate. Moreover, at output, the air exchanges energy by convection with the metallic plates (figure 2), which contributes to decrease the air temperature. This is the reason why the temperature for SC5 is warmer than for SC6.

We can observe in table 2 a good correlation between experimental and simulated temperatures especially for the first four supercapacitors. The absolute error between simulated and experimental temperature is less than 1 degree for supercapacitors SC1 to SC4, with a maximum relative error less than 4.5%. However, higher differences between computation and measurement are obtained for the two last supercapacitors, but it remains within acceptable limits. In fact, as pressure drop phenomena are not taken into account, it may contribute to the observed error for the inner supercapacitors, where the effect of this phenomenon is more substantial than for supercapacitors in the first columns. However, it would be difficult to consider these phenomena with a simple model without solving the Navier-Stokes equation.

On the other hand, the model input uncertainties can be considered as another source of the observed error. That is why in the next section, we are concerned by sensitivity analysis of the model with respect to uncertainties on input values.

Figure 11: Position of instrumented supercapacitors inside the stack.

![Figure 12: Charge-discharge cycle of the stack.](image)

<table>
<thead>
<tr>
<th>$V_{air}$ = 0.23 m/s</th>
<th>$t_r$ = 90 s</th>
<th>$T_{exp}$</th>
<th>$T_{sim}$</th>
<th>$diff$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.23 m/s</td>
<td>22</td>
<td>27</td>
<td>30</td>
<td>37</td>
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<td></td>
<td>22</td>
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<td></td>
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<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$V_{air}$ = 0.23 m/s</td>
<td>$t_r$ = 110 s</td>
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<td>36</td>
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</tr>
<tr>
<td></td>
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<td>0</td>
<td>0</td>
<td>0</td>
</tr>
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<td>0</td>
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<td>$t_r$ = 90 s</td>
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<td>28</td>
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<td>-1</td>
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<tr>
<td>$V_{air}$ = 0.2 m/s</td>
<td>$t_r$ = 110 s</td>
<td>21</td>
<td>26</td>
<td>29</td>
</tr>
<tr>
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</tr>
</tbody>
</table>

Table 2: Comparaison between experimental and computed overheating (temperature in °C, $T_{exp}$ is the experimental temperature, $T_{sim}$ is the simulated temperature, $diff = T_{exp} - T_{sim}$)

IV. UNCERTAINTY AND SENSIBILITY ANALYSIS

Uncertainty analysis is concerned with understanding how changes in the input parameters $x$ influence the model outputs $y$ but also the contribution of each input on the output uncertainty. The input factor such as supercapacitor’s serial resistance ($ESR$), the air velocity ($V_{air}$), the conduction and convection resistances is no more a constant value, but instead it is supposed to be uncertain and described by a Gaussian distribution. The probability density of all inputs parameters is 5 %. Being given the probabilistic distribution of each variable, a set of random numbers for each variable is generated. The computation of the output of the thermal model for all of these different values for the input variables will result in a probabilistic distribution of the response attributes.

In order to quantify which input parameter has the greatest influence on the output of the model, we introduce the sensitivity index $S_i$ which is equal to:


\[ S_i = \frac{V(Y/X_{-i})}{V(Y)} \]  

(20)

Where \( V(Y/X_{-i}) \) is the variance of the output \( Y \) when all the inputs are known apart the input \( X \) that is described by its probability distribution. \( V(Y) \) is the variance of the output when all the inputs are uncertain.

We deduce the values of \( V(Y/X_{-i}) \) by Monte Carlo analysis (10000 runs). The results are summarized in table 3.

<table>
<thead>
<tr>
<th>Input</th>
<th>mean</th>
<th>( V(Y/X_{-i}) )</th>
<th>( S_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{air} )</td>
<td>44</td>
<td>0.847</td>
<td>0.81</td>
</tr>
<tr>
<td>( ESR )</td>
<td>44</td>
<td>0.103</td>
<td>0.012</td>
</tr>
<tr>
<td>( R_{cond} )</td>
<td>44</td>
<td>0.05</td>
<td>0.00288</td>
</tr>
<tr>
<td>( R_{conv} )</td>
<td>44</td>
<td>0.312</td>
<td>0.11</td>
</tr>
</tbody>
</table>

As it is shown in table 3, the speed of air and the convection resistance have the greatest values of sensitivity index, so it affect the most the output of the model.

So any improvement of the modeling results or the thermal performance of the stack must focus on a more accurate measure of the air speed and also more precise calculation of the coefficient of convection.

V. CONCLUSION

In this paper, a thermal model of a supercapacitor stack has been developed based on a lumped parameter approach. This model is used to determine the temperature distribution inside the stack. Thermal test has been carried on an industrial supercapacitor stack for different conditions. The small differences between computed and experimental temperatures show the validity of the developed model. Finally, the sensitivity of the model with respect to input variables is studied. This work helps to know which parameter has more impact on the output of the model (highest SC temperature); we have shown that the uncertainty about the air velocity and convection coefficient has the greatest effect on the model output. Further research will now focuses on electrothermal model of the stack.

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