Sparse Image Reconstruction in Diffuse Optical Tomography: An Application of Compressed Sensing

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Abstract: In this paper we study the application of Compressed Sensing (CS) framework for optical tomography based on the Rytov approximation to the heterogeneous photon diffusion equation. Simulations are performed on a sample system to validate and compare inverse image reconstructions with $l^1$-regularization (CS) and Singular Value Decomposition (SVD) respectively. Potential benefits and shortcomings of CS are discussed and are shown in the context of diffuse optical imaging.

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Tomographic imaging of tissues, i.e. turbid media, using low-energy near-infrared light is termed “diffuse optical tomography” (DOT) [1]. In practice, light is guided by optics to the surface of the media in consideration and measurements are done at other points on the tissue surface. The recovery of optical properties in the probed volume (or domain) from this boundary data requires complex image reconstruction algorithms [1]. Recent advances in signal recovery techniques from incomplete and inaccurate measurements has attracted considerable interest [2]. A framework called compressed sensing (CS) is based on the concept that one can recover randomly under-sampled sparse signals [3]. CS has been used in magnetic resonance imaging [4], recently applied to photo-acoustic tomography [5], fluorescent DOT [6] (without CS matrix analysis) and DOT [7], as a benchmark of a CS reconstruction algorithm. However, in this work, we seek a direct assessment of applying CS to DOT by checking CS properties of the weight matrix, comparing its results against conventional linear reconstructions and study the effect of the reduction of the number of measurements, which hasn’t been shown before [6, 7]. Overall, we investigate if CS can perform effectively in DOT, where the problem is ill-posed and often under-sampled.

The problem of recovery of optical properties in inhomogeneous media is mathematically expressed as an inhomogeneous photon diffusion equation (IPDE). It is often linearized and either relative measurements or calibration phantoms are employed as reference measurements. Rytov approximation (RA) is often employed to solve IPDE, where the photon fluence is approximated as $\Phi(r) = \Phi_0(r) \exp(\Phi_{sc}(r))$, where $\Phi_0(r)$ is the incident wave and $\Phi_{sc}(r)$ is the scattered wave due to an inhomogeneity. If the medium is discretized into volume elements (voxels) of volume $V$, then the RA can be cast into matrix form, $J \Delta \mu_a = \Phi_{sc}$. This is often called linear inverse problem, where $J$ is the Jacobian matrix. Following this RA approximation, the Jacobian is written as

$$J_{lk} = -vVG(r_i, r_k)\Phi_0(r_k, r_j)/(G(r_j, r_i)D_0)$$ (1)

where $v$ is the speed of light in the medium, $D_0$ is the diffusion coefficient, $i = 1, ..., ND$, $j = 1, ..., NS$, $k = 1, ..., NV$, $l = 1, ..., NS \times ND$, $r_i$ and $r_j$ are $ND$ detector and $NS$ source positions, respectively. Green’s functions must be provided according to given boundary conditions.

This linear inverse problem is ill-conditioned and often Tikhonov regularization is utilized to stabilize the solution for the vector $\Delta \mu_a$, after using SVD to invert the Jacobian,

$$\min \left( \|J \Delta \mu_a - \Phi_{sc}\|^2 + \lambda \|\Delta \mu_a\|^2 \right),$$ (2)

where $\lambda$ is the regularization parameter, and where $\|.\|^2$ is $l^2$ norm. We refer this method of image reconstruction simply as SVD.
The primary idea of CS is as follows: if one knows a priori that the resulting image is sparse or can be sparsified with a given orthogonal transformation $\mathcal{T}$, like Fourier or Wavelet (Haar), along with a randomly under-sampled measurement, the image could be recovered exactly [2]. Consider the DOT image (signal) $\Delta \mu_a$ that has a sparse representation in a given basis $\mathcal{T}$, such that $\Delta \mu_a = J \Delta \bar{\mu}_a$. Then, $\Delta \mu_a$ can be recovered by solving a problem of the form, in DOT notation,

$$\min \left( \| J^T \Delta \bar{\mu}_a - \Phi \|_2^2 + \lambda \| \mathcal{T}^T \Delta \bar{\mu}_a \|_1 \right).$$

(3)

This problem is called sparse $l_1$-regularization in CS [2, 3], due to the $l_1$ norm in the penalty term. We have implemented a conjugate gradient algorithm [4] to solve this system of equations. First, the CS-matrix ($J^T$) should have certain properties. The most important of these, the measurement matrix, Jacobian in the context of DOT, must be incoherent with respect to sparsifying transform matrix [3]. This means that the CS-matrix is computed with two uncorrelated bases. We utilize the cumulative coherence function (CCF) [8] of the Jacobian against transform bases to confirm this property. This is a more generalized approach [8] then the traditional ones [4, 5]. Basically, the CCF of order $n$ is computed by taking the maximum value of the inner product between the transform matrix vectors and all possible $n$ subsets in the Jacobian [8]. A slowly increasing CCF signifies incoherence [8].

We have simulated a 3D field of view (FOV) with the following ranges, $[-3.2, 3.2]$ on the $x$ and $y$ directions and $[0.0, -3.0]$ on the $z$ direction with voxel dimensions $0.2 \times 0.2 \times 0.3$ cm. A $5 \times 5$ rectangular array of sources (S) and detectors (D) were placed on the boundary ($z = 0.0$) resulting in 625 S/D pairs. A Jacobian with 1024 voxels and 625 measurements were constructed (Equation 1). A 2D heterogeneity was simulated by a linear approximation at $z = 1.5$ cm deep with $(x, y) = (1.0, 1.0)$ and a radius of 0.4 cm (Figure 2, leftmost image). The contrast ($\Delta \mu_a$) was varied between 0.1 to $1 \text{ cm}^{-1}$ to study extremes. We then reconstruct images.
with SVD and CS approaches with full data set (100%). Furthermore, we apply the same approach with randomly removing measurements (S/D pairs), 50% and 99% of the rows from the Jacobian.

We first verify that we have a CS-Matrix at hand by looking at CCF (Figure (1)) against Fourier and Wavelet (Haar) bases. $J^T$ is shown to be incoherent in both bases. Figure 2 shows the corresponding CS and SVD constructions with random removal of 99 percent of the S/D pairs for $\Delta \mu_a = 0.1 \text{cm}^{-1}$. We can observe that both methods perform in a similar manner, although, SVD image exhibits artifacts (rightmost).

To quantify these findings, we plot (Figure (3) the $\chi^2$ test between the reconstructed image and the simulated image (left) normalized to the full-data set. The CS reconstruction remains robust as the percentage of S/D removal increases while the SVD image is greatly affected. We also plot the linear signal strength (LSS), deducted as the sum of the volume weighted contrast over the full-width-half-maximum (FWHM) of the reconstructed heterogeneity. Interestingly, this indicates that if we ignore the shapes of the objects, the contrast is comparable between CS and SVD.

In conclusion, we have shown that DOT reconstruction with CS is possible and could be beneficial in situations where a large number of measurements are required. Even though both CS and SVD minimization parameters were not fully optimized against S/D positions or contrast location, we have observed that CS gives stable results with decreasing number of S/D pairs. Further studies on optimization on the CS parameters, the effect of random S/D pair removal, and signal-to-noise dependence of CS are needed, in order to apply CS on a real DOT measurement.

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References