EFFICIENT ROBUST OPTIMIZATION TECHNIQUES FOR UNCERTAIN DENSE GAS FLOWS

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Abstract. Dense Gases (DG) are single-phase vapors of molecularly complex fluids operating close to saturation conditions which can be used to improve the efficiency of Organic Rankine Cycles (ORC) turbines. This work is devoted to the design of efficient procedures for optimizing a 2D dense gas turbine with multiple sources of uncertainty (thermodynamic model, operating conditions, geometry). Uncertainty Quantification (UQ) stochastic tools, dense gas CFD solvers and a multi-objective optimizer are coupled to produce a set of robust optimal shapes using mean efficiency and its standard deviation as objectives.
1 INTRODUCTION

Dense gases (DG) are defined as single phase vapors, characterized by complex molecules and moderate to large molecular weights. The use of dense gases as working media in turbomachinery, referred to as Organic Rankine Cycle (ORC) turbines, is proposed for the development of widely distributed, small yield thermal energy conversion devices, where the proposed heat sources typically includes variable energy sources such as solar thermal collectors or waste heat from industrial processes. To improve the feasibility of this technology, the resistance to variation in input conditions must be taken into account at an early stage of the development process.

Classic computational Fluid Dynamics (CFD) simulations are generally deterministic, where a set of constant input parameters result in a single solution. In order to evaluate the effects of input parameter variation in the CFD model, stochastic approaches aim to improve the performance of mechanical systems over a wide range of operating scenarios. Uncertainty quantification (UQ) consists of measuring the system response to random and unknown variations in input parameters using stochastic analysis. In the context of the numerical simulation of DG flows in ORC turbines, two main sources of physical uncertainty should be taken into account: fluid properties which are often difficult to measure accurately, and fluctuating turbine inlet conditions. In order to maximize the potential benefits derived from use of dense gases as a working medium, ORC turbine geometry must also be specifically optimized for the selected fluid, with geometric tolerances (hence uncertainty on the actual blade shape and position) taken into consideration. One approach to address this problem is the optimization procedure known as robust optimization, which takes into account the variability of one or more input parameters. The primary goal of robust optimization is to obtain a set of solutions which incorporate the variability of system performance, in order to improve the robustness of the mechanical system. In a multi-objective optimization, instead of a single optimal solution, the desired end result is a set of equally optimal solutions which form a Pareto front, representing different compromise solutions between potentially conflicting objective functions. Despite their relatively elevated computational cost, genetic algorithms have been widely adapted\textsuperscript{1,2} to a range of optimization problems, due to their strong ability to produce multiple Pareto-optimal solutions in a single optimization run. For CFD simulations in aeronautics, evolutionary algorithms have been successfully applied to develop optimized airfoil profiles in a range of configurations, including dense gas flows\textsuperscript{3-5}.

The present work investigates some efficient procedures for a 2D dense gas turbine cascade in the presence of multiple sources of uncertainty, primarily relating to the operating conditions, thermodynamic model, and geometric tolerances. Two different robust optimization strategies have been recently proposed\textsuperscript{4,5} for shape optimization of dense gas flows over symmetric airfoils. In the present work, the two UQ strategies featured in the previous studies are compared and analyzed for the basic test problem of a quasi-1D convergent-divergent nozzle. One of the UQ strategies will be selected and followed by
a robust optimization of the nozzle geometry using a multi-objective optimization based
on an evolutionary algorithm. Finally, the UQ techniques will also be analyzed in the
framework of a numerical simulation of a dense gas flow over a 2D turbine cascade, in
order to better reproduce the targeted ORC turbine design application. The primary goal
of this work is to analyze the different UQ methods, in order to evaluate the feasibility of
the overall robust optimization analysis in the context of complex numerical simulations
of fluid flow.

2 Governing equations and flow solver

In this section, we introduce the numerical tools used to carry out the numerical sim-
ulations, for both the perfect gas flow through the quasi-1D nozzle, and the dense gas
flow through the 2D turbine cascade. We consider the Euler equations, written in integral
form for a control volume $\Omega$ with boundary $\partial \Omega$:

$$\frac{d}{dt} \int_{\Omega} w \, d\Omega + \int_{\partial \Omega} f \cdot n \, dS = 0$$

In equation (1), $w$ is the conservative variable vector, where

$$w = (\rho, \rho \mathbf{v}, \rho E)^T$$

$n$ is the outer normal to $\partial \Omega$, and $f$, is the flux density:

$$f = \left( \rho \mathbf{v}, \rho I, \rho \mathbf{v} \mathbf{v}, \rho \mathbf{v} H \right)^T$$

where $\mathbf{v}$ is the velocity vector, $E$ the specific total energy, $H = E + p/\rho$ the specific
total enthalpy, $p$ is the pressure, $\rho$ is the density, and $I$ is the unit tensor. The preceding
equation sare completed by a thermal equation of state:

$$p = p(\rho(w), T(w))$$

where $T$ is the absolute temperature, and by a caloric equation of state for the specific
internal energy $e$, which must satisfy the compatibility relation:

$$e = e(\rho(w), T(w))$$

$$= e_r + \int_{T_r}^{T} c_{v,\infty}(T') \, dT' - \int_{\rho_r}^{\rho} \left[ T \left( \frac{\partial p}{\partial T} \right)_\rho - p \right] \frac{d\rho}{\rho^2}$$

In equation (3), $c_{v,\infty}$ is the ideal gas specific heat at constant volume, quantities with a
prime superscript are auxiliary integration variables, and subscript $r$ indicates a reference
state. The caloric equation of state is completely determined once a variation law for $c_{v,\infty}$
has been specified. To model the thermodynamic properties of an ideal polytropic gas flow, equation (2) is replaced with:

\[ p = \rho RT \] (4)

where \( R = 8.314472 \text{ J/(mol K)} \) is the universal gas constant. In the perfect gas case, we assume a constant specific heat, \( c_{v,\infty} \), so that:

\[ c_{v,\infty} = c_v = \frac{R}{\gamma - 1} \] (5)

where \( \gamma \) represents the ratio of specific heat capacities \( c_p/c_v \).

In the dense gas flow case, the Peng–Robinson–Strijek–Vera (PRSV) cubic equation of state (EoS) is used to describe the thermodynamic behavior of the fluid flow. The robustness of this equation with respect to more complex and potentially more accurate multi-parameter equations of state of the Span-Wagner type\(^7-9\) has previously been discussed.\(^4,5\) In the PRSV model, equation (2) is replaced by:

\[ p = \frac{RT}{v - b} - \frac{av}{v^2 + 2bv - b^2} \] (6)

where \( p \) and \( v \) denote respectively the fluid pressure and its specific volume, \( a \) and \( b \) are substance specific parameters related to the fluid critical-point properties \( p_c \) and \( T_c \). To achieve high accuracy for saturation pressure estimates of pure fluids, the temperature-dependent parameter \( a \) in equation (6) is expressed as:

\[ a = \left(0.457235R^2T_c^2/p_c^2\right) \cdot \alpha(T) \] (7)

while

\[ b = 0.077796RT_c/p_c. \] (8)

These properties are not completely independent, since the EoS should satisfy the conditions of zero curvature and zero slope at the critical point. Such conditions allow computing the critical compressibility factor \( Z_c = (p_cv_c)/(RT_c) \) as the solution of a cubic equation. Note that the correction factor \( \alpha \) is given by:

\[ \alpha(T_r) = \left[1 + K \left(1 - T_r^{0.5}\right)\right]^2 \] (9)

with

\[ K = 0.378893 + 1.4897153\omega - 0.1713848\omega^2 + 0.0196554\omega^3. \] (10)

The parameter \( \omega \) is the fluid acentric factor. The caloric behavior of the fluid is approximated through a power law for the isochoric specific heat in the ideal gas limit:

\[ c_{v,\infty}(T) = c_{v,\infty}(T_c) \left(\frac{T}{T_c}\right)^n \] (11)
with \( n \) a material-dependent parameter.

The governing equations are discretized using a cell-centered finite volume scheme for structured multi-block meshes of third-order accuracy, which allows the computation of flows governed by an arbitrary equation of state. A study of the accuracy of the numerical solver has been demonstrated in previous works,\(^{10,11} \) and is not discussed further. The two-dimensional flow domain for the baseline VKI-LS59 blade,\(^ {12} \) is discretized by a structured C-grid comprised of 192\( \times \)16 cells. The boundary conditions are imposed as follows: at the inlet and outlet boundaries, non-reflecting boundaries are applied using the method of characteristics; a slip condition is imposed at the wall, which uses multi-dimensional linear extrapolation from interior points to calculate the wall pressure; periodicity conditions are prescribed at the inter-blade passage boundaries.

3 Stochastic tools

Two non-intrusive uncertainty quantification methods are selected: Polynomial chaos\(^ 4 \) (NIPC) and the Probabilistic Collocation Method\(^ 5 \) (PCM). These methods are used to quantify the effects of uncertain input parameters on the flow of dense gases within a turbine, using concurrently these two strategies provides a high level of confidence in the accuracy of the computed stochastic solutions.

3.1 Non-intrusive polynomial chaos method

Let us consider a stochastic differential equation of the form:

\[
L (x, \theta, \phi) = f (x, \theta) \tag{12}
\]

where \( L \) is a non-linear spatial differential operator (for instance, \( L \) is the steady Navier-Stokes operator) depending on a random vector \( \theta \) (whose dimension depends on the number of uncertain parameters in the problem) and \( f(x, \theta) \) is a source term depending on the position vector \( x \) and on \( \theta \). The solution of the stochastic equation (12) is the unknown dependent variable \( \phi(x, \theta) \), and is a function of the space variable \( x \in \mathbb{R}^d \) and of \( \theta \). Under specific conditions, a stochastic process can be expressed as a spectral expansion based on suitable orthogonal polynomials, with weights associated with a particular probability density function.\(^ {13} \) The basic idea is to project the variables of the problem onto a stochastic space spanned by a complete set of orthogonal polynomials \( \Psi \) that are functions of random variables \( \xi (\theta) \), where \( \theta \) is a random event. For example, the unknown variable \( \phi \) has the following spectral representation:

\[
\phi (x, \theta) = \sum_{i=0}^{\infty} \phi_i (x) \Psi_i (\xi (\theta)) \tag{13}
\]
In practice, the series in equation (13) has to be truncated to a finite number of terms, here denoted with $N$. The total number of terms of the series is determined by:

$$N + 1 = \prod_{k=1}^{n} (p_k + 1)$$  \hspace{1cm} (14)

where $n$ is the dimensionality of the uncertainty vector $\theta$ and $p_k$ is the order of the expansion polynomial associated to the $k^{th}$ random variable. Substituting the polynomial chaos expansion (13), into the stochastic differential equation (12) we obtain:

$$L \left( x, \theta; \sum_{i=0}^{N} \phi_i (x) \Psi_i (\xi (\theta)) \right) = f (x, \theta)$$  \hspace{1cm} (15)

Equation (15) is solved through the weighted residual method. The collocation method is obtained by choosing Dirac delta weighting functions. The coefficients $\phi_i (x)$ are obtained using quadrature formulae based on tensor product of a 1D formula. When the number $d$ of variables is large, quadrature formulae based on tensor product of a 1D formula require too many numerical evaluations and Sparse Grids integration based on Smolyak’s construction are preferred.

Applying a collocation projection to equation (15), we obtain the solution of a deterministic problem for each collocation point.

In both cases, once the chaos polynomials and the associated $\phi_i$ coefficients have been determined, the expected value and the variance of the stochastic solution $\phi_i (x, \theta)$ are computed from:

$$E_{PC} = \phi_0 (x)$$  \hspace{1cm} (16)

$$Var_{PC} = \sum_{i=0}^{N} \phi_i^2 (x) \langle \Psi^2 \rangle$$  \hspace{1cm} (17)

Another interesting property of PC expansion is to facilitate sensitivity analysis based on the analysis of variance decomposition (ANOVA). With this method, we can evaluate the influence of the variance introduced by interactions of random variables, and the variance generated by a single random variable only. For further details see Congedo et al.4

### 3.2 Probabilistic collocation method

The stochastic analysis used to determine the system response to input parameter variation is the non-intrusive Probabilistic Collocation Method (PCM) with a subsequent Lagrange interpolation, as developed by Loeven et al.14. To illustrate the non-intrusive PCM implementation procedure, we consider the uncertain differential equation system (12). Let us decompose the solution $\phi$ into deterministic: $\phi_i (x, t)$, and stochastic: $h_i (\xi (\theta))$, components:

$$\phi (x, t, \xi (\theta)) = \sum_{i=1}^{P_{PCM}} \phi_i (x, t) h_i (\xi (\theta))$$  \hspace{1cm} (18)
where \( \phi_i(x,t) \) is the deterministic solution at the collocation point \( \xi_s(\theta_i) \). In the PCM, \( p \) is the order of the quadrature polynomial and the number of collocation points is given by \( P_{PCM} = p^n \), where \( n \) represents the total number of random input variables. The term \( h_i \) is the Lagrange interpolated chaos polynomial of order \( N_p = p - 1 \) that passes through the \( P_{PCM} \) collocation points:

\[
h_i(\xi(\theta)) = \prod_{s=1}^{n} \left[ \prod_{k=1}^{N_p} \frac{\xi_s(\theta) - \xi_s(\theta_k)}{\xi_s(\theta_i) - \xi_s(\theta_k)} \right]
\]  

(19)

The collocation points are chosen as the roots of a quadrature polynomial of the same type of chaos polynomials adapted for the solution expansion. These in turn are selected according to the Askey scheme. A decoupled deterministic CFD simulation is carried out at the \( p \) collocation points for each uncertain parameter, permuted by a full factorial design, resulting in \( P_{PCM} = p^n \) independent deterministic simulations. With all of the deterministic simulations completed, the output statistics can be calculated using a reconstructed Monte Carlo (RMC) method on the Lagrange interpolation. A large number \( (N_{RMC} \approx 10000) \) of values of \( \xi_s(\theta) \) are selected from the uniform distribution of each input parameter and substituted into equation (19). This equation rapidly reconstructs \( h_i, \) which is inserted into equation (18). Finally, the statistical moments of each output parameter distribution can finally be calculated using the classical definitions on the sample generated by the RMC method.

4 Genetic Algorithm based optimization strategy

In this section, we outline the procedure implemented to carry out the robust shape optimization of the single-stage turbine cascade, via the coupling of the PCM uncertainty quantification analysis to a multi-objective genetic algorithm (MOGA). In this study, the evolutionary algorithm selected for the robust optimization procedure is the NSGA-II non-dominated sorting algorithm proposed by Deb et. al. This multi-objective genetic algorithm is shown to provide a larger spread of solutions and better convergence towards the Pareto front than other similar evolutionary algorithms. Additionally, the NSGA-II has previously been coupled to a range of shape optimization problems involving dense gas flows over airfoils, including optimization under uncertain operating conditions. Despite the elevated computational cost associated with a multi-objective evolutionary algorithm, robust optimization studies of this type enable a compromise to be obtained between improved mean system performance and reduced system variability.

In this study several sources of uncertainty are considered, which relate to geometry (group G), thermodynamic gas properties (group T), or operating conditions (group O). Given that an increase in the number of uncertain parameters considered increases exponentially the number of deterministic simulations required in the UQ analysis, the following strategy is used to minimize the overall computational cost: 1) a preliminary
UQ analysis is performed with all the uncertainties active but without starting the optimization process, using a Sparse Grid strategy based on Smolyak’s construction to minimize costs; 2) an ANOVA strategy allows to identify the most influential parameters and reduce in accordance the number of uncertainties actually considered within the optimization loop; 3) the high-fidelity CFD solver may be replaced by substitute functions if its direct use at the optimization step is prevented by the ANOVA step yielding too many significant uncertain variables. Since a key point of an efficient robust optimization is to limit as much as possible the number of deterministic evaluations in the UQ step while keeping an accurate estimate of the statistics describing the stochastic solution, the NIPC and PCM strategies have both been used and the resulting solutions compared. Finally, to further reduce the total calculation time, both the NSGA-II optimization tool and the PCM stochastic analysis algorithm were fully parallelized for application to both local multiple core and cluster based systems.

The optimization strategy was tested on a simple case, i.e. a steady perfect gas flow through a supersonic nozzle, and then applied to a dense gas flow over a 2D turbine cascade. In the present work, an individual in the MOGA represented a given configuration of turbine blade VKI LS-59, defined in terms of the angle of attack $\beta$ and the stagger angle $\theta$. To ensure a wide spread of possible solutions, approximately 20 individuals were generated for each generation of the optimization. This value is approximate, as the MOGA does not calculate individuals which are very similar to a previously evaluated individual, thus reducing overall calculation time. In previous applications of the MOGA to the robust optimization of dense gas flows, convergence of the Pareto front was typically obtained after 25-30 generations. The objective functions, sources of uncertainty and parameters to optimize used in the robust optimization are presented in the following section.

5 Results

5.1 Steady perfect gas nozzle

Let us consider the steady flow of a perfect gas (with heat coefficient ratio $\gamma$) in the following convergent-divergent nozzle geometry:

$$A(x) = \begin{cases} 
1 + 0.75x^2 & \text{for } x < 0 \\
1 + \beta_d x^2 + (A_{esc} - 1.0 - \beta_d) x^3 & \text{for } x > 0 
\end{cases}$$

where $A_{esc}$ is the cross sectional exit area divided by the throat area and $\beta_d$ a geometric parameter. The exit pressure normalized by the reservoir pressure is denoted $p_e$ and fixed to yield a shock in the divergent. The uncertain parameters are therefore $A_{esc}$ and $\beta_d$ in group G, $\gamma$ in group T and $p_e$ in group O, which are assumed to vary within the ranges described in Table 1, following a uniform probability density function. The NIPC and PCM UQ methods were coupled to an exact nozzle flow solver.

Results of the 4th and 5th order ANOVA are displayed in Figure 1: the variance is decomposed computing the contribution of each source of uncertainty (T,G,O); the pressure
Table 1: Uncertain parameters for the nozzle, min/max values for the uniform probability density function

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>γ</td>
<td>1.39</td>
<td>1.41</td>
</tr>
<tr>
<td>p_e</td>
<td>0.8015</td>
<td>0.8515</td>
</tr>
<tr>
<td>A_{esc}</td>
<td>1.485</td>
<td>1.515</td>
</tr>
<tr>
<td>β_d</td>
<td>0.475</td>
<td>0.525</td>
</tr>
</tbody>
</table>

p_e contributes for nearly 98% to the variance so that it could be the sole uncertainty used for the UQ, allowing to reduce strongly computational cost. To confirm this analysis, a stochastic computation with NIPC and PCM taking into account the single uncertainty p_e is performed and its results compared in Figure 2 to the Monte Carlo (MC) solution obtained when considering all 4 uncertainties. The Mach variance along the nozzle computed with p_e uncertain only (using 21 deterministic evaluations) is nearly coincident with the full MC solution (obtained from 100000 deterministic evaluations). The global cost of the ANOVA and the stochastic evaluation of a 1-uncertainty problem requires 646 deterministic evaluations only, much less than MC.

Figure 1: ANOVA of the contribution of the uncertain parameters to the variance for the perfect gas nozzle.

This configuration was used also to validate the optimization loop, with the ratio of the exit total pressure to the inlet total pressure, p_{0ex}/p_{0in}, chosen as the fitness function. Then, A_{esc} was optimized within the range [1.4, 1.6] in order to minimize both the mean and the variance of p_{0ex}/p_{0in}. In order to validate the ANOVA, two optimization problems were considered: firstly, all uncertainties were taken into account at the same time, then only a uncertainty on p_e. Note that uncertainty on β_d is negligible in this case seeing that p_{0ex}/p_{0in} is not dependent on the nozzle shape. The two runs have been computed with
both NIPC and PCM (3rd stochastic order). The expected optimal value of $A_{esc} = 1.4$ was obtained for both runs and methods after nearly twenty generations. Note that the single uncertainty case was carried out with a computational cost reduced by a factor of sixteen times with respect to the all uncertainties case. Once again, this confirms the strong influence of the uncertain upstream pressure $p_e$ compared to the other uncertainties. With a better understanding of the computational requirements of the robust optimization procedure in the context of a simple test problem, the optimization loop could then be applied to more complex case of the dense gas flow simulation.

5.2 Dense gas turbine

5.2.1 Base configuration and sources of uncertainty

In the present work, the turbine blade under consideration was the two dimensional VKI LS-59 cascade, a configuration which has been widely studied.\textsuperscript{12,17} The chosen operating conditions, i.e. inlet total temperature $T_{in}/T_c$, inlet total pressure $p_{in}/p_c$, exit pressure $p_{out}/p_c$, angle of incidence $\beta$, stagger angle $\theta$, are defined in Table 3. The siloxane dodecamethylcyclohexasiloxane (C\textsubscript{12}H\textsubscript{36}Si\textsubscript{6}O\textsubscript{6}), commercially known as D\textsubscript{6} is the fluid used in this paper. The physical properties of D\textsubscript{6} are reported in Table 2). The PRSV parameters $c_{v\infty}$, $n$ and $\omega$ for for D\textsubscript{6} are defined in Table 4.
Table 2: Thermodynamic data for D₆₆, where M is the percentage molecular weight, and Tₜ is the boiling temperature at 1 atm. Properties are taken from Guardone et al.¹⁸

<table>
<thead>
<tr>
<th>M (g/mole)</th>
<th>Tₜ(K)</th>
<th>Pₜ(kPa)</th>
<th>Tₜb(K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>444.9</td>
<td>645.8</td>
<td>961</td>
<td>518.1</td>
</tr>
</tbody>
</table>

5.2.2 Convergence of Sensitivity indices via ANOVA

The stochastic performance of the turbine cascade under the influence of uncertain input parameters is evaluated by using three output criteria. The first is the turbine isentropic efficiency \( \Delta h / \Delta h_{\text{ideal}} \) where \( \Delta h \) represents the variation of the static enthalpy between turbine inlet and outlet boundaries, and \( \Delta h_{\text{ideal}} \) is the variation of enthalpy when an ideal isentropic transformation with the same initial conditions and pressure ratio as the real flow. The second criterion is represented by the relative temperature variation or Carnot factor \( \Delta T / T_{\text{inlet}} \), where \( \Delta T \) represents the variation of temperature between turbine inlet and outlet boundaries. Thirdly, the power output per unit depth (PO) expressed as \( \Delta h \cdot \dot{m} / w_{\text{mol}} [W] \), where \( \Delta h \) is the enthalpy variation through turbine stage, \( \dot{m} \) is the mass flow rate and \( w_{\text{mol}} \) is the molecular weight.

In order to compute the statistics of these output quantities, a full tensorization is used in order to compute a 2nd order polynomial for NIPC and PCM (6561 single computations). The results were compared in terms of mean and variance of the three output criteria. The differences in the mean calculated by the two different stochastic methods are approximately 2%, and approximately 3% for the variance. Then, ANOVA was applied to NIPC results in order to determine the hierarchy of the most influential uncertain input parameters. In order to validate the sensitivity indices, the ANOVA was applied also on sparse grids of 3rd order (701 single computations) and 4th order (5421) coupled with NIPC. In Figure 3, the contribution of each uncertainty to the variance for the three criteria in the 2nd order stochastic simulation is shown. It is clear that contribution of the

<table>
<thead>
<tr>
<th>n</th>
<th>c∞</th>
<th>( \omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.5729</td>
<td>105.86</td>
</tr>
<tr>
<td>Range</td>
<td>0.5385-0.6073</td>
<td>99.50-112.20</td>
</tr>
</tbody>
</table>

Table 4: Thermodynamic constants for D₆, PRSV equation of state, mean and min/max values for the uniform probability density function
three operating uncertainties, i.e. $T_{in}/T_c$, $p_{in}/p_c$ and $p_{out}/p_c$, are predominant with respect to the other thermodynamic and geometric parameters. It is therefore reasonable to retain only these three operating parameters for the optimization problem. Note that the results shown in Figure 3, were obtained with the NIPC but they are confirmed by Sparse Grid NIPC computations, where difference below 2% were computed for each uncertainty. This reduction of stochastic dimensional space makes it possible to lower the stochastic computational cost by a factor of more than 200 for the second order polynomial.

![Figure 3: Contribution of each uncertainty to the global variance for the three output criteria in the 2nd order stochastic dense gas turbine simulation.](image)

5.2.3 Robust design

Optimization problem is defined as follows: to find the optimal values for $\beta$ and $\theta$ in order to maximize mean and minimize variance of power output (PO). $\beta$ ($\theta$) vary in the intervals $25^\circ$-$30^\circ$ ($\pm 5^\circ$). Basing on ANOVA analysis reported in the previous section, three uncertainties are retained for the optimization problem. Two optimizations have been performed with both NIPC and PCM. Mean and variance of PO for the base configurations are equal to 1.0949 and 0.0137 respectively (values obtained with NIPC). Convergence is reached after nearly twenty generations (four hundreds of individuals, more than ten thousands of deterministic computations). Two similar Pareto fronts have been obtained, as shown in figure 4. Then results obtained by means of NIPC and PC are very similar. Some individuals of Pareto fronts dominate the base configuration. One of this individual, indicated BC in the following, has been obtained for $\beta = 27.1^\circ$ and $\theta = 0.13^\circ$. Highest mean indivial, indicated hereafter HM, is obtained for $\beta = 29.09^\circ$ and $\theta = -4.87^\circ$, while lowest variance individual (LV) is obtained for $\beta = 28.87^\circ$ and $\theta = 3.17^\circ$. A nearly linear dependence of PO mean (a) and variance (b) from $\theta$ can be observed, as shown in figure 5, for individuals belonging to Pareto fronts. In order to draw
differences among all set of individuals, BC, HM, HV and base configuration (where their representation on the objectives plane has been marked in figure 4) mean and variance mach contours have been reported in figures 6 and 7, respectively. As shown in figure 6, mean Mach solutions near turbine blade are very similar, even if $\beta$ and $\theta$ are different for each configuration. For the variance Mach solutions, BC and baseline profiles are very similar, while HM and LV solutions present a lower variance. Finally, a set of robust individuals has been derived with a reduced computational cost (twenty-seven times more than a deterministic optimization) but with a global analysis of all physical sources of uncertainties.

![Figure 4: Pareto fronts obtained with NIPC and PCM. Grey square represents the baseline configuration (obtained with NIPC).](image)

6 CONCLUSIONS

In this paper, we study some efficient procedure in order perform a robust stochastic optimization when a high number of uncertainties is taken into account at the same time. ANOVA analysis has been applied on the base configuration by considering all the sources of uncertainties. Then, when convergence is reached on the sensitivity indeces, optimization problem has been applied on the reduced problem where only the most important parameters are retained for the stochastic analysis. This allows a strong reduction of the
global computational cost. This strategy has been validated on the shape optimization of a perfect gas nozzle. Results given by full complete optimization when MC method is used for statistics have been reproduced with a gain of several orders of magnitude. Then the same strategy has been applied on the robust design of a dense gas turbine cascade. In this case, three source of uncertainties have been considered, related to the thermodynamic model, to the operating conditions and to geometrical parameters. Taking into account all the uncertainties at the same time makes the computation unfeasible seeing the computational cost of a single deterministic computation. Then, an ANOVA analysis has been applied, allowing to retain only three parameters for the optimization problem seeing their predominance with respect to other parameters. In this way, the cost for a single stochastic evaluation reduced of two hundred times (for a 2nd order polynomial). Moreover, the optimization problem produced a set of robust individuals allowing to improve the base configuration. A detailed study has been focalized on several robust individuals, the dominating one on the base configuration, the highest mean and lowest variance individuals, where comparison with respect to the base configuration has been drawn.

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REFERENCES

Figure 6: Mean mach contours for optimal robust individuals
Figure 7: Variance mach contours for optimal robust individuals


