Adaptive Fuzzy Torque Control of Passive Torque Servo Systems Based on Small Gain Theorem and Input-to-state Stability

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Abstract

Passive torque servo system (PTSS) simulates aerodynamic load and exerts the load on actuation system, but PTSS endures position coupling disturbance from active motion of actuation system, and this inherent disturbance is called extra torque. The most important issue for PTSS controller design is how to eliminate the influence of extra torque. Using backstepping technique, adaptive fuzzy torque control (AFTC) algorithm is proposed for PTSS in this paper, which reflects the essential characteristics of PTSS and guarantees transient tracking performance as well as final tracking accuracy. Takagi-Sugeno (T-S) fuzzy logic system is utilized to compensate parametric uncertainties and unstructured uncertainties. The output velocity of actuator identified model is introduced into AFTC aiming to eliminate extra torque. The closed-loop stability is studied using small gain theorem and the control system is proved to be semi-globally uniformly ultimately bounded. The proposed AFTC algorithm is applied to an electric load simulator (ELS), and the comparative experimental results indicate that AFTC controller is effective for PTSS.

Keywords: flight simulation; adaptive control; fuzzy control; passive torque servo system; electric load simulator; extra torque; small gain theorem; input-to-state stability

1. Introduction

Passive torque servo system (PTSS) is often required in modern avionic area, which is called load simulator or loading system. It is an important equipment in hardware-in-loop (HIL) simulation for aircraft control system \cite{1}, which can exert aerodynamic load on aircraft actuation system to verify the performance of flight control system on ground \cite{2}.

Usually, in order to simulate aerodynamic load, PTSS connects actuation system directly with rigid shaft coupling. It is obvious that PTSS inevitably endures strong position coupling disturbance from active rotary motion of actuation system \cite{3}. This disturbance is called extra torque, and it is an inherent disturbance which may even be bigger than the desired torque output sometimes. Much literature focuses on designing appropriate mechanical structure or compensation controller to eliminate extra torque. The basic idea of mechanical structure optimization \cite{4} is to decouple torque servo and position tracking by introducing a synchronizing motor between system base and loading motor. The stator of synchronizing motor is connected to system base while the rotator is connected to loading motor \cite{5}. In this system, the stator of loading motor is used to realize position tracking while the rotator is used to realize torque tracking. But this structure makes system more complex and increases cost. For compensation controller, constant structure control and velocity synchronization are widely used in application of load simulator \cite{1}. Constant structure control was designed by Liu \cite{6} whose idea was feedforward actuator velocity to eliminate extra torque. This method...
depends on velocity measurement, so phase delay influence is inevitable. Velocity synchronizing compensation was presented by Jiao, et al. in Ref. [1], which foreknows the actuator motion by collecting its servo-valve signal and compensated extra torque in advance. Feedforward and compensation were introduced [7] to eliminate extra torque in electro-hydraulic load simulator. Plummer [8] presented a valve cross-compensation design approach to improve force tracking accuracy, which was also based on velocity synchronizing control essentially. Besides, there are many robustness improvement methods considering actuator movement as disturbance. Nam and Hong [9] proposed a robust force control system using quantitative feedback theory designed for loading system to enhance the robustness of force control system. Chantawatanathan and Peng [10] presented a modular adaptive robust control technique to improve the force control performance of vehicle active suspensions. A hybrid control scheme of dynamic loading system for ground testing of high-speed aerospace actuator was introduced in Ref. [10], which included velocity compensation, torque input feedforward and PID closed-loop control. Truong and Ahn [11] proposed a self-tuning grey predictor combined with a fuzzy PID controller to improve the robustness of force control of hydraulic load simulator.

Additionally, although many nonlinearities such as unknown system parameters, unstructured uncertainties and external disturbance, are neglected in system modeling, they really exist in practical system. Therefore, it is difficult to achieve high precision torque tracking and good robustness without effective control method. Adaptive backstepping design by Lyapunov function was proposed for parametric strict-feedback systems in Ref. [12]. Besides Lyapunov method, input-to-state stability (ISS) approach is also a useful tool to design controller and analyze system stability, which was firstly proposed by Sontag [13]. Furthermore, small gain theorem was proved [14] and the synthesis of adaptive fuzzy tracking controller was developed [15,16], where Takagi-Sugeno (T-S) fuzzy system is utilized to approximate nonlinear unstructured uncertainties.

In this paper, adaptive fuzzy torque control (AFTC) of PTSS is proposed to guarantee transient tracking performance as well as final tracking accuracy. T-S fuzzy logic system is used to compensate nonlinear unknown system function caused by parametric uncertainties and unstructured uncertainties. Only one parameter needs to be tuned online at each step and the proposed controller structure is simple and easy to be implemented in practical PTSS. Application on electric load simulator (ELS) shows that the method presented in this paper is effective.

2. Mathematical Preliminaries

In this section, the concepts of ISS and small gain theorem [13-14, 17] will be introduced, which have been widely used in nonlinear control problems for stability analysis [16]. The class $K$, $K_{+}$ and $KL$ functions [18] will be firstly reviewed. Then the structure of fuzzy logic system will be briefly described.

2.1. ISS and small gain theorem

**Definition 1** A function $\alpha : (0, \infty) \rightarrow (0, \infty)$ is said to belong to class $K$ if it is continuous, strictly increasing and $\alpha(0) = 0$. It is said to belong to class $K_{+}$ if additionally $\alpha$ is unbounded.

**Definition 2** A function $\beta : (0, \infty) \times (0, \infty) \rightarrow (0, \infty)$ is of class $KL$ if $\beta(\cdot, t)$ is said to belong to class $K$ for each fixed $t \geq 0$, and $\beta(\cdot, \cdot)$ is strictly decreasing for each fixed $r > 0$ and $\lim_{t \rightarrow \infty} \beta(r, t) = 0$.

**Definition 3** For a nonlinear system $\dot{x} = f(x,u)$, it is said to be input-to-state practically stable (ISpS) if there exist a class $KL$ function $\beta$ and a class $K$ function $\gamma$, such that for any initial condition $(0,u(0))$, each bounded control input $u(t)$ defined for all $t > 0$ and a constant $d > 0$, the associated solution $x(t) \in [0, \infty)$ satisfies
\[
\|x(t)\| \leq \beta(\|x(0)\|, t) + \gamma(\|u\|_t) + d
\]
where $\|u\|_t = \max_{0 \leq s \leq t} |u(s)|$ and $u(t)$ is a truncated function defined as
\[
u_\tau(t) = \begin{cases} u(t) & 0 \leq \tau \leq t \\ 0 & \tau < 0 \text{ or } \tau > t \end{cases}
\]

When $d = 0$ in Eq. (1), the ISpS property becomes the ISS property [13,19].

**Definition 4** A function $V$ is said to be an ISpS-Lyapunov function for the system $\dot{x} = f(x,u)$ if
1) there exist class $K_{+}$ functions $\alpha_{1}$, $\alpha_{2}$, such that
\[
\alpha_{1}(\|x\|) \leq V(x) \leq \alpha_{2}(\|x\|), \quad \forall x \in \mathbb{R}^n
\]
2) there exist class $K$ functions $\alpha_{3}$, $\alpha_{4}$ and a constant $d > 0$, such that
\[
\frac{\partial V(x)}{\partial x} f(x,u) \leq -\alpha_{3}(\|x\|) + \alpha_{4}(\|u\|) + d
\]

When $d = 0$ in Eq. (4), $V$ is said to be an ISS-Lyapunov function [19]. Then nonlinear $L_{\infty}$ gain $\gamma$ in Eq. (1) can be chosen as
\[
\gamma(s) = \alpha_{1}^{-1}(s) \alpha_{2}(s) \alpha_{3}^{-1}(s) \alpha_{4}(s), \quad \forall s > 0
\]

**Proposition 1** The nonlinear system $\dot{x} = f(x,u)$ is ISpS if and only if there exists an ISpS-Lyapunov
Theorem 1 (Small Gain Theorem) Consider a system in composite feedback form of two ISpS systems \[ \Sigma_{\mathbf{x}} : \dot{\mathbf{x}} = f(t, \mathbf{x}, \mathbf{w}) \]

\[ \dot{\mathbf{e}} = H(t, \mathbf{x}, \mathbf{w}) \]

\[ \Sigma_{\mathbf{e}} : \dot{\mathbf{y}} = g(t, \mathbf{y}, \mathbf{e}) \]

\[ \mathbf{w} = K(t, \mathbf{y}, \mathbf{e}) \]

where \( \mathbf{x} \) and \( \mathbf{e} \) are state vector and output vector of system \( \Sigma_{\mathbf{x}} \) while \( \mathbf{y} \) and \( \mathbf{w} \) are state vector and output vector of system \( \Sigma_{\mathbf{e}} \). In particular, if there exist class \( KL \) functions \( \beta_{\mathbf{x}}, \beta_{\mathbf{e}}, \) class \( K \) functions \( \gamma_{\mathbf{e}}, \gamma_{\mathbf{w}}, \) and two constants \( d_1 \geq 0, d_2 \geq 0, \) for all \( t \geq 0 \), the solutions \( X(\mathbf{x}, \omega, t) \) and \( Y(\mathbf{y}, \mathbf{e}, t) \) satisfy

\[ \|H(X(\mathbf{x}, \omega, t))\| \leq \beta_{\mathbf{x}}(\|X(0)\|, t) + \gamma_{\mathbf{e}}(\|\mathbf{w}\|) + d_1 \]

\[ \|K(Y(\mathbf{y}, \mathbf{e}, t))\| \leq \beta_{\mathbf{e}}(\|Y(0)\|, t) + \gamma_{\mathbf{w}}(\|\mathbf{e}\|) + d_2 \]

If

\[ \gamma_{\mathbf{e}}(\mathbf{y}(s)) < s \text{ (or } \gamma_{\mathbf{w}}(\mathbf{y}(s)) < s), \quad \forall s > 0 \]

Then the solution of the composite systems (6) and (7) is ISpS.

2.2. Fuzzy logic system

In the past years, fuzzy logic system has been widely researched. In this paper, it will be used as a practical function approximator \[ ^{16} \] in adaptive fuzzy torque controller design. T-S fuzzy system \[ ^{20} \] can be described as the following form which consists of \( q \) rules.

Rule \( R_i : \)

IF \( x_1 \) is \( M_i^1 \) and \( x_2 \) is \( M_i^2 \) and \( \cdots \) and \( x_n \) is \( M_i^n \)

THEN \( y_i = a_{i0} + a_{i1} x_1 + a_{i2} x_2 + \cdots + a_{in} x_n \)

where \( M_i^j (j = 1, 2, \cdots, n) \) is fuzzy subset, \( a_{i0}, a_{i1}, \cdots, a_{in} \) denote unknown constant parameters in fuzzy rule conclusion step, \( y_i \) is fuzzy output, and \( q \) the total number of fuzzy rules.

By using singleton fuzzifier production inference engine and center-average defuzzifier, the output of T-S fuzzy system can be given as

\[ y = \sum_{i=1}^{q} y_i \tilde{\xi}_i(\mathbf{x}) = \xi(\mathbf{x}) A_i \Xi \]

where

\[ \mathbf{x} = [x_1, x_2, \cdots, x_n]^T, \quad \Xi = [1, \mathbf{x}^T]^T \]

\[ A_i = \begin{bmatrix} a_{i0} & a_{i1} & \cdots & a_{in} \\ a_{i20} & a_{i21} & \cdots & a_{in} \\ \vdots & \vdots & \ddots & \vdots \\ a_{in0} & a_{in1} & \cdots & a_{in} \end{bmatrix} \]

\[ \xi_i(\mathbf{x}) \text{ and } \tilde{\xi}_i(\mathbf{x}) \text{ are fuzzy basis vector and fuzzy basis function respectively, and can be expressed as } \]

\[ \xi_i(\mathbf{x}) = \left[ \begin{array}{c} \xi_{i1}(\mathbf{x}) \\ \xi_{i2}(\mathbf{x}) \\ \cdots \\ \xi_{in}(\mathbf{x}) \end{array} \right] \]

\[ \tilde{\xi}_i(\mathbf{x}) = \frac{\prod_{j=1}^{n} \mu_{M_i^j}(x_j)}{\sum_{j=1}^{q} \prod_{j=1}^{n} \mu_{M_i^j}(x_j)} \]

where \( \mu_{M_i^j}(x_j) \) is the membership function of fuzzy set \( M_i^j \) and usually is given as

\[ \mu_{M_i^j}(x_j) = \exp \left[ -\frac{(x_j - b_{i}^j)^2}{2(b_{i}^j)^2} \right] \]

\( (i = 1, 2, \cdots, q; j = 1, 2, \cdots, n) \)

3. Dynamic Models and Basic Assumptions

3.1. Dynamic models of PTSS

PTSS is used to simulate aerodynamic load of aircraft rudder for ground testing. In HIL simulation process, the actuation system is controlled by onboard flight control computer while the relevant aerodynamic loading is calculated by simulation computer according to height, speed and posture of aircraft. PTSS is designed to exert this loading torque to actuation system.

Generally, PTSS connects to actuation system directly in order to simulate aerodynamic load as shown in Fig. 1, in which the right side is the actuation system (i.e., position servo system) and the left side is loading system (i.e., torque servo system).
It is obvious that PTSS and actuation system are two mutually coupling systems. For actuation system, this position servo system is required to follow desired angle trajectory in the presence of external disturbances, so loading torque can be treated as external disturbance acting on the position closed-loop system. On the other hand, torque servo system needs to exert desired loading torque on actuation system when actuator operates, and active motion of actuator can be considered as an external disturbance to the torque closed-loop system. A good PTSS design is that PTSS could catch up with actuator motion and put corresponding aerodynamic load to actuation system without extra torque.

For a kind of electric-torque-motor-driven PTSS, the dynamics of motor amplifier can be ignored when PTSS operates in normal condition. Thus, the relationship of driving torque $T_m$ versus input control voltage $u$ to motor amplifier can be represented by the following equation [22]:

$$T_m = K_m u$$  

where $K_m$ is proportional coefficient between input control voltage $u$ and driving torque $T_m$.

Considering friction torque and external disturbances, torque balance equation of loading system can be given as

$$m_d \ddot{\theta}_m + m_b \dot{\omega}_m + T_i + T_f = d_n$$  

where $J_m$ is total inertia of motor rotator and loading shaft, $B_m$ combined coefficient of the damping and viscous friction on the load, $\omega_m$ velocity of torque motor, $T_d$ the lumped effect of external disturbances, $T_i$ loading torque, $T_f$ the combined effect of stiction, Coulomb friction and Stribeck effect, and it will be formulated later.

In addition, torque sensor is a key component which connects actuation system and load simulator. Since its inertia is very small, it can be considered as an elastic model:

$$T_i = K_s (\theta_m - \theta_a) + d_n$$  

where $K_s$ is total stiffness coefficient of torque sensor and loading shaft, $d_n$ represents measurement noise, $\theta_m$ and $\theta_a$ are rotary displacements of torque motor and actuation system, respectively. Then differentiating Eq. (16) leads to the following function:

$$\dot{\theta}_m = K_s (\theta_m - \theta_a) + \dot{d}_n$$  

where $\dot{\theta}_m$ represents actuator velocity and it is the source of coupling disturbance of PTSS.

### 3.2. Model design

Define loading torque, velocity of PTSS as state variables, i.e. $x = [x_1 \ x_2]^T \triangleq [T_i \ \dot{\theta}_m]^T$, and noting Eq. (15) and Eq. (17), then the entire PTSS system can be expressed in state space form as:

$$
\begin{align*}
\dot{x}_1 &= K_s x_2 - K_a \omega_a + \dot{d}_n \\
\dot{x}_2 &= K_m u - B_m x_2 - \frac{1}{J_m} \dot{x}_1 - \frac{1}{J_m} T_f + \frac{1}{J_m} T_d
\end{align*}
$$  

From previous analysis, it is clear that active motion of actuator is the main disturbance to PTSS, so it is necessary to predict actuator motion. For this purpose, this paper utilizes the output velocity $\dot{\theta}_a$ of an identified actuation system model as the estimated value of actuator velocity. For the convenience of system analysis and controller design, the mathematical model of PTSS can be written in a general model of typical strict-feedback nonlinear system as

$$
\begin{align*}
\dot{x}_1 &= g_1 x_1 - f_1 + \Delta_1 \\
\dot{x}_2 &= g_2 x_2 + f_2 + \Delta_2
\end{align*}
$$  

where $g_1$ and $g_2$ are unknown control gains with lower bounds $g_1 \min$ and $g_2 \min$ respectively, $f_1$ and $f_2$ unknown smooth functions which represent the lumped effects of compensation errors, parametric uncertainties and unstructured uncertainties, and they are given as

$$
\begin{align*}
\Delta_1 &= \hat{K}_s \dot{\theta}_a - K_a \omega_a \\
\Delta_2 &= \frac{g_2}{g_2 \min} x_1 - \frac{B_m}{J_m} x_2 - \frac{1}{J_m} T_f
\end{align*}
$$

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\begin{align*}
\dot{x}_1 &= g_1 x_1 - f_1 + \Delta_1 \\
\dot{x}_2 &= g_2 x_2 + f_2 + \Delta_2
\end{align*}
$$

### 3.3. Basic assumptions

The following basic assumptions are introduced.

**Assumption 1** The uncertain control gains are...
bounded, such that

\[ 0 < g_{1_{\min}} < g_1 < g_{1_{\max}} \]  \hspace{1cm} (22)

\[ 0 < g_{2_{\min}} < g_2 < g_{2_{\max}} \]  \hspace{1cm} (23)

where \( g_{1_{\min}} \) and \( g_{1_{\max}} \) \((i=1, 2)\) are lower and upper bounds of \( g_j \).

**Assumption 2** In Eq. (19), the disturbance uncertainties \( A_i \) \((i=1, 2)\), have the property:

\[ |A_i| < p_i \phi_i(\mathbf{x}_i) \]  \hspace{1cm} (24)

where \( p_i \) are unknown positive constants, \( \phi_i(\mathbf{x}_i) \) are known positive smooth functions and \( \mathbf{x}_i = [x_1 \ x_2 \cdots x_i]^{T} \).

4. Controller Design and Stability Analysis

Considering a PTSS in the form of Eq. (19), the desired loading torque is given as \( x_i(t) = T_i(t) \), which is assumed to be known and bounded and its first and second derivatives \( \dot{x}_i, \ddot{x}_i \) are also continuous and bounded. The control objective is to design a control input \( u \) such that output \( y = x_1 \) tracks \( x_d(t) \) as closely as possible in spite of coupling disturbance from actuator.

**4.1. Adaptive fuzzy torque controller design**

In this subsection, AFTC laws will be designed by using backstepping technique based on small gain theorem and ISpS. The following error variables are defined:

\[ e_1 = x_1 - x_d \]  \hspace{1cm} (25)

\[ e_2 = x_2 - \mu \]  \hspace{1cm} (26)

**Step 1** Design a virtual control law \( \mu \) for the first subsystem of Eq. (19) as

\[ \mu = \mu_1 + \mu_2 \]  \hspace{1cm} (27)

where \( \mu_1 \) is adaptive fuzzy control law which will be designed in the following text, and \( \mu_2 \) is velocity synchronization compensation law which is given as

\[ \mu_2 = \frac{\lambda_i}{g_{1_{\min}}} \dot{x}_d \]  \hspace{1cm} (28)

Considering control law Eq. (27) and the first equation of Eq. (19), the time derivative of \( e_1 \) can be given as

\[ \dot{e}_1 = g_1(\varepsilon_2 + \mu) + f_1 + A_1 - \dot{x}_d \]  \hspace{1cm} (29)

Considering \( x_2 = e_2 + \mu \) and \( \mu = \mu_1 + \mu_2 \), Eq. (29) can be rewritten as

\[ \dot{e}_1 = g_1(\varepsilon_2 + \mu) + f_1 + A_1 - \dot{x}_d \]  \hspace{1cm} (30)

Since \( f_1 \) is an unknown continuous function, T-S fuzzy logic system can be used to approximate the uncertain term \( f_1 \) according to Lemma 1. Therefore, \( f_1 \) can be described as

\[ f_1 = \xi_i A_i [1 \ x_1]^{T} + \delta_i(x_i) \]

\[ \xi_i A_i^0 + \xi_i A_i e_1 + \xi_i A_i x_1 + \delta_i(x_i) \]  \hspace{1cm} (31)

where \( A_i = \begin{bmatrix} a_{i0} & a_{i1} \\ a_{i0} & a_{i1} \\ \vdots & \vdots \\ a_{i0} & a_{i1} \end{bmatrix} \), \( A_i^0 = \begin{bmatrix} a_{i0}^0 & a_{i1}^0 \\ a_{i0}^0 & a_{i1}^0 \\ \vdots & \vdots \\ a_{i0}^0 & a_{i1}^0 \end{bmatrix} \), \( \xi_i = [\xi_{i1} \xi_{i2} \cdots \xi_{i_d}] \). \( \delta_i(x_i) \) represents fuzzy approximating error, and for any \( x_i \in U_\delta \), \( |\delta_i| \leq \varepsilon_i \), where \( \varepsilon_i \) is unknown but bounded positive constant.

Let \( c_i = n\|A_i\| \), \( A_i^0 = c_i^0 A_i^0 \). It is clear that \( \|A_i\| \leq 1/n \). Then let \( \omega_i = \varepsilon_i^0 A_i^0 \). Eq. (31) can be rewritten as

\[ f_1 = \xi_i A_i^0 + c_i \xi_i \omega_i + \xi_i A_i x_1 + \delta_i(x_i) \]  \hspace{1cm} (32)

Substituting Eq. (32) into Eq. (30) leads to

\[ \dot{e}_1 = g_1(\varepsilon_2 + \mu) + c_i \xi_i \omega_i + v_i \]  \hspace{1cm} (33)

where

\[ v_i = \xi_i A_i^0 + \xi_i A_i x_1 + \delta_i(x_i) + A_1 - \dot{x}_d \]  \hspace{1cm} (34)

\[ |v_i| \leq \theta g_i \]  \hspace{1cm} (35)

where \( \theta_i = \max \{\|A_i\|, \|A_i^0\|, \varepsilon_i, p_i, 1\} \) is a positive constant and \( q_i = (1 + \|x_d\| \|\xi\| + 1 + \phi_i(\mathbf{x}_i) + |\dot{x}_d|) \).

Design the adaptive fuzzy control law \( \mu_i \) as

\[ \mu_i = -k_i \varepsilon_1 - \hat{\lambda}_i \psi_i e_i \]  \hspace{1cm} (36)

\[ \hat{\lambda}_i = \Gamma_i \left[ \psi_i e_i - \sigma_i (\hat{\lambda}_i - \lambda_{i0}) \right] \]  \hspace{1cm} (37)

where \( \lambda_{i0} = \max \{g_{1_{\min}}^2, \epsilon_{i0}^{-1}, \theta_i^2 \} \), \( \hat{\lambda}_i \) is an online estimate of unknown parameter \( \lambda_i \) with estimate error \( \hat{\lambda}_i = \lambda_i - \hat{\lambda}_i \), \( k_i > 0 \), \( \gamma_i > 0 \), \( \rho_i > 0 \), \( \Gamma_i > 0 \), \( \sigma_i > 0 \) and \( \lambda_{i0} > 0 \) are positive design parameters.

Design a Lyapunov function as

\[ V_i = \frac{1}{2} \varepsilon_i^2 + \frac{1}{2} g_{1_{\min}}^{-1} \hat{\lambda}_i^2 \]  \hspace{1cm} (38)
Considering $\dot{\lambda} = -\dot{\lambda}^o$, the time derivative of $V_1$ can be given as

$$V_1 = e_1\dot{e}_1 - g_{1\text{min}}F_1^{-1}\dot{\lambda}^o\dot{\lambda} = g_{1\text{min}}e_2 + g_{1\text{min}}\dot{\theta}q_1 + c_1e_3 + \xi_1\dot{e}_1 + v_1e_1 - g_{1\text{min}}F_1^{-1}\dot{\lambda}^o\dot{\lambda}$$

(39)

Noting that $|v_1| \leq \theta q_1$ and applying Young’s inequality, the following inequality can be obtained:

$$c_1e_1^2 + 2\theta q_1^2\epsilon_1^2 + 2\dot{\theta}q_1^2 = 2\theta^2 q_1^2$$

(40)

Consider that $q_1 = \|\xi_1\|^2 / (4\theta^2 q_1^2) + q_1^2 / (4\rho^2 q_1^2)$, and $\lambda^o = \max \{g_{1\text{min}}e_1^2, g_{1\text{min}}\theta q_1^2\}$, from Eq. (40), the following inequality holds:

$$c_1\epsilon_1^2 + \lambda^o\epsilon_1^2 + 2\theta q_1^2\epsilon_1^2 + 2\dot{\theta}q_1^2 = 2\theta^2 q_1^2$$

(41)

Substituting Eqs. (36)-(37) and Eq. (41) into Eq. (39) leads to

$$\dot{V}_1 \leq g_{1\text{min}}e_1^2 + 2\theta q_1^2\epsilon_1^2 + 2\dot{\theta}q_1^2 = 2\theta^2 q_1^2$$

(42)

where $d_1 = g_{1\text{min}}(\lambda^o - \lambda^o q_1^2 / 2 + \rho^2 q_1^2)$.

**Step 2** Design an actual control law $u$ for the second subsystem of Eq. (19) as

$$u = u_1 + u_2$$

(43)

where $u_1$ is adaptive fuzzy control law and $u_2$ model compensation law given as

$$u_2 = \frac{1}{g_{2\text{min}}f_m}x_1$$

(44)

Noting control law Eq. (43) and the second equation of Eq. (19), the time derivative of $e_2$ can be given as

$$\dot{e}_2 = g_2u - \phi_2 + f_2 + \lambda^o - \mu$$

(45)

The time derivative of $\mu$ can be given as

$$\dot{\mu} = \frac{\partial \mu}{\partial x_1}\dot{x}_1 + \frac{\partial \mu}{\partial \lambda^o}\dot{\lambda}^o + \frac{\partial \mu}{\partial \lambda^o}\dot{\lambda}^o + \frac{\partial \mu}{\partial x_3}\dot{x}_3 + \mu_2 = \frac{\partial \mu}{\partial x_1}\dot{x}_1 + w_2$$

(46)

where

$$w_2 = \frac{\partial \mu}{\partial \lambda} \dot{\lambda} + \frac{\partial \mu}{\partial x_3} \dot{x}_3 + \mu_2$$

(47)

is a computable intermediate variable. Substitute Eq. (43) and Eq. (46) into Eq. (45), then $\dot{e}_2$ can be rewritten as

$$\dot{e}_2 = g_2u_1 + \mu_2 + \dot{\lambda}^o - \lambda^o$$

(48)

where

$$\dot{\lambda}^o = \frac{\partial \mu}{\partial \lambda} \dot{\lambda}^o + \frac{\partial \mu}{\partial x_3} \dot{x}_3 + \mu_2$$

(49)

Then T-S fuzzy logic system can be used to approximate the uncertain nonlinear term $\dot{\lambda}^o$ according to Lemma 1, and $\dot{\lambda}^o$ can be described as

$$\dot{\lambda}^o = \xi_2 \bar{A}_2^* \bar{X}_2^* + \delta_2 \bar{x}_2$$

(50)

where $\bar{X}_2 = [e_1, e_2, \delta \mu / \delta x_3, x_3, w_2]^T$, $\bar{A}_2^* = \xi_2 \bar{A}_2^*$, $\bar{A}_2 = [a_{21}, a_{22}, \ldots, a_{25}]$, $\bar{A}_2^* = [a_{21}, a_{22}, \ldots, a_{25}]$.

$$\bar{A}_2 = \begin{bmatrix} a_{21} & a_{22} \\ a_{23} & a_{24} & a_{25} \\ & & & & \end{bmatrix}, \quad \bar{A}_2^* = \begin{bmatrix} a_{21} & a_{22} \\ a_{23} & a_{24} & a_{25} \\ & & & & \end{bmatrix}$$

represents fuzzy approximating error, and for any $x_2 \in U_{x_2}$, $|\delta_2 (x_2)| \leq 2$; here $x_2$ is unknown but bounded positive constant.

Let $c_2 = n\|A_2^*\|$, $A_2 = c_2 \bar{A}_2^*$. It is clear that

$$\|A_2^*\| \leq 1/n$$

Then let $\omega_2 = \bar{A}_2^*$, Eq. (50) can be rewritten as

$$\dot{\lambda}^o = \xi_2 \bar{A}_2^* \bar{X}_2^* + c_2\bar{A}_2\omega_2 + \xi_2 \bar{A}_2^* [0 \mu]^T + \delta_2 \bar{x}_2$$

(51)

Substituting Eq. (51) into Eq. (48) leads to

$$\dot{e}_2 = g_2u_1 + c_2\xi_2 \omega_2 + v_2$$

(52)

where

$$v_2 = \xi_2 \bar{A}_2^* \bar{X}_2^* + c_2\bar{A}_2\omega_2 + \xi_2 \bar{A}_2^* [0 \mu]^T + \delta_2 \bar{x}_2$$

(53)

and

$$|v_2| \leq \theta g_2$$

(54)
\[ \theta_2 = \max \left( \|\mathbf{A}_1\|, \|\mathbf{A}_2\|, \rho_1, \epsilon_1, 1 \right) \]

where \( \theta_2 \) is a positive constant and \( q_2 = \left( \|\mathbf{B}_2 \| / \| \partial \mathbf{x}_1 / \partial \mathbf{z}_1 \| + \| \mathbf{C}_2 \| \right)^{1/2} + 1 \).

The adaptive fuzzy control law \( u_1 \) is given by

\[ u_1 = -k_2 \psi_2 \varepsilon_2 \quad (55) \]

Design the Lyapunov function \( V_2 \) as

\[ V_2 = V_1 + \frac{1}{2} \epsilon_2^2 + \frac{1}{2} g_2 \gamma_2 \lambda_2 \lambda_2 \quad (57) \]

where \( \nu \gamma_2 = \epsilon_2 / \gamma_2 \), \( \lambda_2 = \max \left( g_{2 \min} \sigma_1, g_{2 \min} \sigma_2 \right) \).

4.2. Closed-loop system stability analysis

**Theorem 2** Consider the closed-loop system consisting of system (18), controller (43), adaption laws (37) and (56). For bounded initial conditions, if choosing the gain of subsystem \( \Sigma_{\omega} \) as \( 0 < \gamma < 1 \), and \( k_2 > 1 / g_{2 \min} \) and \( k_2 > 1 / g_{2 \min} \) satisfy Assumptions 1-2, then the closed-loop control system is semi-globally uniformly ultimately bounded in the sense that all signals in this system are bounded. If control parameters are chosen suitably, the tracking error can be smaller than a prescribed error bound, which means the tracking error asymptotically converges to zero.

**Proof** Rewrite the closed-loop system into two composited subsystems, viz. \( \Sigma_{\omega} \) and \( \Sigma_{\omega'} \):

\[ e_1 = g_1 (e_2 + \mu_1) + c_1 \xi_1 (x_1) \omega_2 + v_1 \]

\[ e_2 = g_2 u_1 + c_2 \xi_2 (x_2) \omega_2 + v_2 \]

\[ \hat{\lambda}_2 = -\gamma^2 \left[ \psi_2 \varepsilon_2 - \sigma_2 (\hat{\lambda}_2 - \lambda_2) \right] \]

\[ \hat{\lambda} = E (E) = E \]

where \( \omega = [\omega_1, \omega_2]^T \) is given as input of subsystem \( \Sigma_{\omega} \) and \( \hat{\lambda} \) is output, \( E = [e^T, \hat{\lambda}^T]^T \), \( e = [e_1, e_2]^T \), \( \hat{\lambda} = [\hat{\lambda}_1, \hat{\lambda}_2]^T \).

Design a Lyapunov function as

\[ V = V_2 \]

Choosing \( k_2 > 1 / g_{2 \min} \) and \( \gamma = (\gamma_1^2 + \gamma_2^2)^{1/2} \), and noting Eq. (60), the time derivative of \( V \) can be obtained:

\[ \dot{V} \leq -[\epsilon_2]^2 + \gamma_2 |\omega_2|^2 + d_2 \]

According to Definitions 3-4, the subsystem \( \Sigma_{\omega} \) satisfies ISpS; thus, there exist class \( K_\pi \) functions \( a_1(s) \), \( a_2(s) \), such that \( a_1(\|E\|) \leq V(E) \leq a_2(\|E\|) \), and \( a_3(s) = s \), \( a_4(s) = \gamma^2 s^2 \), then the gain of subsystem \( \Sigma_{\omega} \) can be obtained:

\[ a_3(s) = a_1^{-1}(s) a_2(s) a_3^{-1}(s) a_4(s), \ \forall s > 0 \]

Consider subsystem \( \Sigma_{\omega'} \):

\[ \omega = (A_2 e_2 - \bar{\omega} \bar{e}) \]

where \( \bar{\omega} = [\omega_1, \omega_2]^T \), \( \bar{e} = [\bar{e}_1, \bar{e}_2]^T \), \( \bar{\omega} = [\bar{e}_1, \bar{e}_2]^T \), \( A = [A_1, 0] \), \( A_2 = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \)

\[ \bar{\omega} = A \bar{e} \]

Considering \[\|A_i\| \leq 1/n \ (i = 1, 2)\], the
The following inequality can be obtained:

\[
\begin{bmatrix}
\mathbf{A} \\
\mathbf{A}^T
\end{bmatrix} = \begin{bmatrix}
\mathbf{A}_1 & 0 \\
\mathbf{A}_1^T & \mathbf{A}_2^T
\end{bmatrix} \leq \left( \mathbf{A}_1^T \mathbf{A}_1 + \mathbf{A}_2^T \mathbf{A}_2 \right)^{1/2} \leq 1
\]

(66)

Let \( \gamma' = \| \mathbf{A} \| \), then we have:

\[
\| \omega \| \leq \| \mathbf{A} \| \| \mathbf{E} \| = \gamma' \| \mathbf{E} \|
\]

(67)

Therefore \( \gamma' \leq 1 \), and the gain of subsystem \( \Sigma_{\omega} \) satisfies \( \gamma' \cdot \omega \leq 1 \).

According to small gain theorem, if \( \gamma' \cdot \omega < \sigma \), then the closed-loop system (61) is ISpS. That means \( \lim_{t \to \infty} e(t) = 0 \), i.e., the asymptotic output tracking is achieved.

5. Application Example

The proposed AFTC controller is implemented on an ELS which is a typical application of PTSS driven by electric torque motor. The goal is to further illustrate that AFTC algorithm can effectively cope with extra torque and achieve desired torque tracking in a practical PTSS. Then comparative experimental results are carried out as follows.

5.1. Experiment setup

As shown in Fig. 2, a direct drive rotary torque motor D143M by Danaher is used as drive component in this ELS system and it is driven by a Danaher digital servo amplifier S620. A torque sensor AKC-17 with measurement range of 300 N·m is used to measure the loading torque. A Heidenhain rotary encoder ECN113 with Heidenhain PC counter card IK220 is used to measure the rotary displacement of torque motor and the velocity signal is obtained by the difference of rotary displacement. A 16 bit AD/DA card PCI-1716 by Advantech is used to sample torque signal and to send out control voltage. Original designed real-time control program based on RTX real-time operating system and Labwindows/CVI is applied to the system and its sampling frequency is 2 kHz.

Parameter identification is performed to get the nominal values of system parameters and the results are shown in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_m (N \cdot m \cdot V^{-1}) )</td>
<td>37.7</td>
</tr>
<tr>
<td>( K_s (N \cdot m \cdot rad^{-1}) )</td>
<td>1964.7</td>
</tr>
<tr>
<td>( J_m (kg \cdot m^2) )</td>
<td>0.045</td>
</tr>
<tr>
<td>( B_m (N \cdot m \cdot (rad \cdot s^{-1})^{-1}) )</td>
<td>2.16</td>
</tr>
</tbody>
</table>

As shown in Fig. 3, the membership functions (NB, NM, NS, ZERO, PS, PM and PB) of fuzzy set is chosen as Eq. (12), in which \( b_j = -3, -2, \ldots, 3 \) and \( h_j = 0.424 \) 7.

According to ELS parameters in Table 1 and control performance of practical experiment, AFTC controller parameters are chosen as: \( k_1 = 1.1 \), \( k_2 = 0.5 \), \( \gamma_1 = 0.2 \), \( \gamma_2 = 0.6 \), \( \sigma_1 = 0.15 \), \( \sigma_2 = 0.25 \), \( \rho_1 = 0.5 \), \( \rho_2 = 0.5 \), \( \lambda_{10} = 8.5 \), \( \lambda_{20} = 0.4 \), and \( \lambda_{20} = 0.5 \). In the following text, three typical experiments for ELS are carried out.

5.2. Static loading experiment

As a basic performance test item for PTSS, static
loading experiment is carried out firstly. In this condition, actuation system is absent and output shaft of torque motor is fixed directly, thus estimated actuator velocity can be set to zero, i.e., $\dot{\omega}_a = 0$ and $\mu = 0$. This experiment is to investigate torque tracking performance without velocity disturbance.

Three algorithms, PID with $k_p=0.7$, $k_i=0.1$, $k_d=0.0001$, fuzzy control and AFTC, will be compared in this experiment. Here, fuzzy control uses the same membership functions as AFTC.

The desired loading torque is given by a 1.0 Hz sinusoidal signal with 80 N·m amplitude. Then tracking errors with three controllers are shown in Fig. 4.

Fig. 4 Tracking errors of low-frequency in static loading experiment.

It is obvious that the tracking error of AFTC is much smaller than that of PID and smaller than fuzzy due to well-designed and excellent performance of AFTC.

In order to verify AFTC performance under high-frequency desired loading torque, experiment is also conducted for high-frequency desired signal which is given by a 6.0 Hz sinusoidal signal with 40 N·m amplitude, then the tracking errors with the above three controllers are shown in Fig. 5.

Fig. 5 Tracking errors of high-frequency in static loading experiment.

As shown in Fig. 5, fuzzy cannot get a better performance than PID at high-frequency but AFTC still have a good performance at high-frequency.

5.3. Eliminating extra torque experiment

The ability to eliminate extra torque is a key issue for ELS, so it is important to measure exact extra torque. An aviation actuator is used in this experiment and its shaft is connected to ELS by a coupling. Assume ELS desired loading torque $d_0 = 0$ and the actuator operates with the desired sinusoidal angle command, then the measured torque output of torque motor is so-called extra torque.

Since extra torque is caused by actuation system, the estimated actuator velocity $\dot{\omega}_a$ will be used in AFTC controller in this experiment. To achieve this goal, off-line model identification by frequency sweep technique for actuation system is carried out and the identification results are shown in Fig. 6.

Fig. 6 Frequency sweep model and identified models of actuation system.

As shown in Fig. 6, the 5th order model fits the frequency sweep model well and it is given as

$$G(s) = \frac{3.4 \times 10^5 s^5 + 2.0 \times 10^3 s + 2.8 \times 10^6}{s^5 + 4.8 \times 10^3 s^4 + 1.2 \times 10^3 s^3 + 6.7 \times 10^3 s^2 + 5.8 \times 10^3 s + 2.4 \times 10^6}$$

But it is impossible to apply this model in practical control algorithm due to its high order. Therefore, a 2th order model is found instead of the 5th order model:

$$G(s) = \frac{7328}{s^2 + 108s + 7372} \quad (71)$$
Although the 2nd order model cannot fit frequency sweep model well in high frequency band, it can fit very well in low frequency band, especially under 80 rad/s (about 13 Hz). So it is reasonable to use the 2nd order model for compensation because both ELS and actuation system operate at 10 Hz.

Since the desired angle signal \( \theta_d \) is easy to obtain and without noises, it is input into identified model \( G_s(s) \), then the estimated actuator velocity \( \omega_e \) can be obtained by differentiating the angle output of \( G_s(s) \).

Set the desired loading torque of ELS as \( x_d = 0 \) and the desired angle trajectory of actuation system as \( \theta_d = 0.1 \sin(8.0\pi t) \) rad. First, ELS operates without any control algorithm, i.e. open-loop, then two control algorithms, fuzzy and AFTC, which are mentioned in Section 5.2, are used to eliminate extra torque, respectively. The extra torque comparison with three control strategies is shown in Fig. 7.

\[ e_m = \max \{ |e(t)| \} \]
\[ e_p = \max \{ |e(t)| \} \]

\[ t_p \] represents total running time and \( t_p \) is periodic time.

According to the final tracking error \( e_p \), fuzzy control can only eliminate about 45% of extra torque while AFTC control can eliminate about 82%. Clearly, AFTC controller has a good performance in eliminating the extra torque in practical PTSS.

\[ 5.4. \text{Gradient loading experiment} \]

Gradient loading is the most common use of PTSS, in which loading torque command is proportional to actuator angle command. For example, the desired command of actuator is \( \theta_d = 0.2 \sin(4.0\pi t) \) rad and torque gradient coefficient is \( K_{TG} = 100 \text{ N·m/rad} \), thus the desired loading torque can be calculated by \( x_d = K_{TG} \omega_d \) as shown in Fig. 8.

Fuzzy and AFTC are also compared in this experiment, actual loading torque outputs are also shown in Fig. 8, while torque tracking errors are shown in Fig. 9.

\[ 6. \text{Conclusions} \]

The adaptive fuzzy torque control is proposed in this paper and the closed-loop system is proved to be semi-globally uniformly ultimately bounded by small gain theorem. The AFTC is applied to ELS and experimental results show its good control performance. Parametric uncertainties and unstructured uncertainties in PTSS are effectively compensated by T-S fuzzy logic system. The AFTC is convenient to be implemented in practical PTSS because only one parameter needs to be turned online at each step. The output velocity of actuator identified model is utilized in AFTC algorithm to eliminate extra torque. This technique is easy to be realized, the compensation signal is smooth without noise, and the experiment of eliminating extra
torque also illustrates the effectiveness of this technique.

References


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