Application of Robustified Model Predictive Control to a Production-Inventory System

C. Stoica, M.R. Arahal, D.E. Rivera, P. Rodríguez-Ayerbe and D. Dumur

Abstract—In this paper, a robustified control technique is applied to a production-inventory system. A linear model is considered for the demand/inventory system with a variable delay-time and a variable yield. The inventory is firstly controlled with a MPC (Model Predictive Control) law designed for some nominal values of the delay and yield. Using a Youla parameter-based procedure, this initial controller is robustified towards different types of uncertainties in order to manage the possible variations of the yield and of the delay-time. The robustification problem leads to a convex optimization, solved with LMI (Linear Matrix Inequality) tools. This robustified controller is further compared to another MPC law, which is slower, but remains stable for all the considered variations of the uncertain parameters. Therefore a trade-off between robust stability towards parametric uncertainties and nominal performances for the nominal system is highlighted.

I. INTRODUCTION

Supply chains for manufactured goods are an essential part of the modern economy [1]. The stochasticity and long throughput times associated with most manufacturing processes highlight the importance that robustness plays in Supply Chain Management (SCM). In recent years, a number of approaches inspired by chemical process control have been proposed to manage short-term tactical decision-making for inventory management problems associated with semiconductor manufacturing supply chains [2], [3], [4]. These approaches rely on Model Predictive Control (MPC) [5]. In these strategies, demand changes are treated as exogenous disturbance signals that must be properly "rejected" by a sensibly-designed control system.

Different policies (Economic Order Quantity policy) and control laws (feedback-feedforward internal model control [3], predictive control [6] etc.) have been already applied on the Supply Chain Management problem. For simplicity, a production inventory system (which is the basic unit of a supply chain) will be further considered into this paper. In order to assure robustness requirements, this paper proposes the application of a robustified predictive control technique to this production-inventory process (single node supply chain). The proposed procedure may be naturally extended for multi-node applications. The first part of the paper shows a model of the system derived from a fluid analogy [6], [7] producing a linear discrete-time system with a variable and uncertain time-delay and a variable and uncertain gain (the yield). The second part of the paper concerns the development of the control law. First, an initial stabilizing controller is designed for the nominal production-inventory model. Due to its performance [8], [9], [10] the predictive control has been chosen as an initial controller. Second, the robust stability (towards additive unstructured uncertainties) of this MPC law is improved via the Youla-Kučera parametrization. The third step considers the possible variations of the dead-time and of the yield. This means to consider structured uncertainties, leading to a polynomial representation (i.e. a segment variation of the yield) for each value of the dead-time. It has been noticed that improving the robust stability under multiplicative unstructured uncertainties [11] will increase the uncertain parametric stability domain. A Lyapunov-based technique (leading to a feasibility problem in a LMI form) is used in order to verify the stability for each segment of the yield variation and for each value of the delay. During simulations, two different cases are analyzed:

• an initial MPC controller that stabilizes the nominal system, offering very good responses in terms of performance, but which leads to instability for different values of the dead-time and the yield. This controller is robustified so that it remains stable for the considered mismatched systems;

• an initial MPC controller that offers a slower time-response but which guarantees the stability for the considered mismatched systems.

Therefore, the trade-off between robust stability over an uncertain domain and nominal performance specifications for a nominal model is highlighted using these controllers.

An advantage of the robustification procedure is that it allows to stabilize a system for a considered structured uncertain domain, even if the initial controller (which offers good performance) is unstable for some points of this domain.

This paper is organized as follows. Based on a fluid representation, Section II offers a specific way to model the production-inventory system. Section III deals with the theoretical aspects necessary to better understand the robustified MPC strategy. Section IV proposes a case study analysis. A summary of the work as well as some concluding remarks and future developments are presented in Section V.

II. SYSTEM MODEL

In most applications dynamic modeling is difficult whereas in manufacturing the most significant uncertainties are not in the process model structure but in some uncertain parameters and on the supply and demand signals. Using a fluid analogy
[3], [7] (Fig. 1), the factory is modeled as a pipe with a particular throughput time $\theta$ and yield $K$. The inventory is modeled as material (fluid) in a tank. Applying the principle of mass conservation leads to a differential equation relating net stock (tank level) at the end of the day $y(k+1)$ to net stock at the beginning of the day $y(k)$, factory starts $u(k)$ (pipe inflow) and customer demand $d(k)$ (tank outflow). This differential equation is represented by the following discrete time transfer, being the sampling time a whole day:

$$y(k+1) = y(k) + Ku(k - \theta) - d(k) \quad (1)$$

The demand $d(k)$ is composed of the forecasted customer demand $d_F(k - \theta_F)$ plus unforecasted customer demand $d_U(k)$, where $\theta_F$ is the forecast horizon and $d_F(k)$ represents an estimate of demand $\theta_F$ days into the future:

$$d(k) = d_F(k - \theta_F) + d_U(k). \quad (2)$$

Based on (1) and (2) it is possible to derive tactical decision policies that manipulate factory starts to maintain inventory level at a designated setpoint. If knowledge of future customer demand is available, it is advantageous to use feedforward compensation, a MPC formulation being a good choice. For ease of presentation, this paper focuses on robustification of MPC policies with respect to unforecasted demand changes, which enables to assume that the forecasted demand signal $d_F(k)$ will equal zero. The unforecasted demand $d_U(k)$ will be deviation from zero.

A state-space model is derived from equations (1) and (2), resulting in the following expressions:

$$x(k + 1) = Ax(k) + Bu(k) + Pd_U(k)$$

$$y(k) = Cx(k) \quad (3)$$

with $A \in \mathbb{R}^{(\theta+1)\times(\theta+1)}$, $B, P, x \in \mathbb{R}^{(\theta+1)\times1}$, $C \in \mathbb{R}^{1\times(\theta+1)}$, $u, y, d_U \in \mathbb{R}$ and $\theta \in \mathbb{N}^+$ and the following matrices:

$$A = \begin{bmatrix} 1 & 0_{1,\theta-1} & K \\ 0 & 0_{1,\theta-1} & 0 \\ 0_{\theta-1,1} & I_{\theta-1,1} & 0_{\theta-1,1} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 0_{\theta-1,1} \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0_{1,\theta} \end{bmatrix} \quad (4)$$

Note that here it is considered that all transactions are made within the working hours of a day and computed as their value at the end of these working hours. This means that goods either arrive one day or the next day, always leading to consider the delay $\theta$ as a multiple of a day. The value of parameters $\theta$ and $K$ are subject to changes. A nominal value can be estimated from historical data and used to develop an initial controller to be robustified at a later stage.

The model developed here might seem simple, however it does take into account the dynamics of the inventory. The main difficulty associated with the model is related to the uncertainties associated with the values of the delay and yield. In particular throughput times are in most cases dependent on load [12]. A reasonable nominal estimate value is not difficult to obtain [13] although variations around this estimate can amount to a whole day.

III. ROBUSTIFIED MODEL PREDICTIVE CONTROL

This Section states the theoretical aspects of the robustification procedure. The proposed method starts with the design of an initial stabilizing control and it may be applied for the general case of an initial stabilizing state-feedback controller coupled with an observer. Due to its performance and simplicity, a MPC is chosen here as an initial control law. The aim is to guarantee a certain level of robustness towards the possible variation of the delay $\theta$ and the possible variation of the yield $K$. In order to fulfill these necessities, this initial controller is further robustified via the Youla-Kučera parameter towards these uncertainties, using LMI techniques. Firstly, the MPC is robustified considering unstructured uncertainties. Secondly, the stability for the polytopic domains obtained for variations of the dead-time and of the yield is verified solving a feasibility (LMI) problem.

A. Youla-Kučera parametrization - Theoretical Background

The Youla parametrization is a well-known and powerful result. The major advantage is that it offers a way to parametrize the class of all stabilizing controllers through the set of stable systems, starting from an initial stabilizing controller [14], [15]. For an initial state-feedback controller and an observer, the modified controller paradigm [14] enables to modify this initial controller as it accepts an input $u'$ and an output $y'$ with a 0 transfer between them (Fig. 2).
Hence a stable $Q \in \mathbb{R} H_{\infty}$ parameter may be inserted from $y'$ to $u'$. As the transfer between $y'$ and $u'$ is zero ($T_{22} zw = 0$, Fig. 2), this stable system $Q$ does not destabilize the closed-loop. A bijection is obtained between stable systems and stabilizing controllers. Furthermore, conveniently choosing $y'$ and $u'$, the Input/Output transfer is not modified. This permits to act directly on the sensitivity function offering a way to increase the robustness of the initial control. In the general case, the transfer from an input $w$ to the output $z$ (Fig. 2) is represented as follows:

$$T_{zw} = T_{11z} + T_{12z} Q(I - QT_{22z})^{-1} T_{21z}$$  \hspace{1cm} (5)

Since the transfer $T_{22z}$ is zero, this is equivalent to the following expression which is affine in the $Q$ parameter:

$$T_{zw} = T_{11z} + T_{12z} QT_{21z}$$  \hspace{1cm} (6)

As $Q$ belongs to an infinite space, a sub-optimal solution is to use a polynomial or a FIR (Finite Impulse Response) filter [16] in the state-space form (7). Fixing the pair $(A_Q, B_Q)$ and $Q$ parameter $(C_Q, D_Q)$ must be found.

$$x_{Q}(k + 1) = A_Q x_Q(k) + B_Q y'(k)$$

$$u'(k) = C_Q x_Q(k) + D_Q y'(k)$$  \hspace{1cm} (7)

**B. Initial Model Predictive Controller**

The production inventory system was modeled as a linear system with a variable dead-time $\theta$ and a parameter $K \in [\bar{K}, \bar{K}]$. The basic idea of this paper is to consider a nominal model obtained for a fixed value of the dead-time and a given value of the yield $K$. An initial MPC law is designed for this model. Considering the linear time-invariant (LTI) discrete-time nominal representation (3), this subsection gives the main ingredients to obtain an initial stabilizing MPC control for this nominal model (obtained for $K_0$).

In order to cancel the steady state errors, an integral action on the control signal $u(k) = u(k - 1) + \Delta u(k)$ is added to (3), leading to the extended system (8). As the calculus of the extended matrices is trivial, it is omitted here.

$$\hat{x}_e(k + 1) = A_e \hat{x}_e(k) + B_e \Delta u(k) + P_e d_{L}(k)$$

$$y(k) = C_e \hat{x}_e(k)$$  \hspace{1cm} (8)

How MPC deals with the inventory management problems? The nominal discrete-time model (3) permits to predict the system output (inventory, net stock) on a fixed prediction horizon $p$. At each sampling period, minimizing a performance criterion (9), with $m$ the control horizon, $Q_e$, $Q_{\Delta u}$ appropriate weightings, $\Delta u(k + i) = 0$ for $i \geq m$:

$$\sum_{i=0}^{p} \| \hat{y}(k + i) - r(k + i) \|^2_{Q_e} + \sum_{i=0}^{m-1} \| \Delta u(k + i) \|^2_{Q_{\Delta u}}$$  \hspace{1cm} (9)

a sequence of control signals (factory starts) is obtained and only the first element (the starts level corresponding to the first entry) is implemented. At next sampling period, using the receding horizon principle, the same procedure is applied. The aim is to keep the inventories as close as possible to the inventory planning set-point $r(k + i)$ (supposed to be known over the prediction horizon). The predicted output $\hat{y}(k + i)$ is calculated as:

$$\hat{y}(k + 1) = CA \hat{x}(k) + \sum_{j=0}^{i-1} CA^{i-j} Bu(k + j)$$  \hspace{1cm} (10)

using the estimate of the extended state (11) obtained with an observer. The observer gain $K_{\text{obs}}$ is tuned by placing the eigenvalues of the matrix $A_e - K_{\text{obs}} C_e$ in a stable region, in order to obtain the desired observer dynamics.

$$\hat{x}_e(k + 1) = A_e \hat{x}_e(k) + B_e \Delta u(k) + K_{\text{obs}} [y(k) - C_e \hat{x}_e(k)]$$  \hspace{1cm} (11)

Minimizing (see [9] for details) the objective function (9) and using the receding horizon principle, the following expression is obtained for the control signal:

$$\Delta u(k) = F_r y_r(k) - L \hat{x}_e(k)$$  \hspace{1cm} (12)

where $F_r$ is a setpoint pre-filter and $L$ is the MPC gain (Fig. 3), calculated as detailed in [16].

**C. Robustified MPC - Theoretical Aspects**

After the development of an initial stabilizing MPC control for the nominal model, the next step is to increase its robustness via the $Q$ parametrization. Two aspects are further considered: improving the robustness under unstructured uncertainties and verifying the stability for a specified polytopic uncertain domain (based on a Lyapunov function). The two problems are solved with LMI techniques.

1) **Unstructured uncertainties**: This subsection offers a way to increase robust stability of an initial stabilizing controller toward unstructured uncertainties. This aims at reducing its influence by minimizing the $H_{\infty}$ norm of the following transfer (Fig. 3), where the control weighting $W_u$ should filter the specified frequency range:

$$\min_{Q \in \mathbb{R} H_{\infty}} \| T_{zb} \|_{\infty} = \min_{Q \in \mathbb{R} H_{\infty}} \| W_u T_{ub} \|_{\infty}$$  \hspace{1cm} (13)
The next step consists of rewriting (13) using the bounded-real lemma [17] for the discrete-time case.

**Bounded-real lemma** [18], [19]. A discrete-time system \((A_{cl}, B_{cl}, C_{cl}, D_{cl})\) is stable and its \(H_{\infty}\) norm is lower than a scalar \(\gamma\) if and only if:

\[
\exists X_1 = X_1^T > 0 / \begin{bmatrix}
-X_1^{-1} A_{cl, Q} B_{cl, Q} 0 \\
X_1^{-1} A_{cl, Q}^T -X_1 0 \\
0 & X_1^{-1} & 0 & C_{cl, Q}^T \\
B_{cl, Q} & 0 & -\gamma I \\
0 & C_{cl, Q} & D_{cl, Q} & -\gamma I
\end{bmatrix} \prec 0
\]  

(14)

where "> 0"("< 0") defines a strictly positive (negative) definite matrix. There exist techniques [18], [19] (based on congruence transformations and a bijective variable change) that allow to transform this matrix inequality into a LMI. The procedure which allows to transform (14) into a LMI is beyond the scope of this paper and it is omitted here. A complete description may be found in [20], [16]. Here the decision variables are the Lyapunov matrix \(X_1\), the scalar \(\gamma\) and the Youla parameter included in the closed-loop matrices. Denoting \(\text{LMI}_0\) \((\text{[16]}, \text{[20])}\) the LMI obtained from (14), the optimization problem is then rewritten as follows:

\[
\min_{\gamma \in \text{LMI}_0} \gamma
\]  

(15)

2) **Structured uncertainties:** The proposed production-inventory model (3) and (4) considers that the yield \(K\) and the time-delay \(\theta\) may vary around their nominal values. This may be seen as structured uncertainties. In fact considering only the variation of \(K \in [K, \overline{K}]\) is equivalent to a polytopic uncertainty: the model in this case may vary inside a polytope with 2 vertices (i.e. one segment). In addition, a possible variation of the throughput time \(\theta \in \{\theta_1; \ldots; \theta_k; \ldots; \overline{\theta}\}\) leads to different models for each value of the yield \(K\). In this case the uncertain domain is the union of all segments (Fig. 4).

\[
\begin{align*}
(A_{1,1}, B_{1,1}, C_{1,1}) \quad \bigcirc & \quad (A_{2,1}, B_{2,1}, C_{2,1}) \\
(A_{1,2}, B_{1,2}, C_{1,2}) \quad \bigcirc & \quad (A_{2,2}, B_{2,2}, C_{2,2}) \\
(A_{1,3}, B_{1,3}, C_{1,3}) \quad \bigcirc & \quad (A_{2,3}, B_{2,3}, C_{2,3}) \\
(A_{1,4}, B_{1,4}, C_{1,4}) \quad \bigcirc & \quad (A_{2,4}, B_{2,4}, C_{2,4}) \\
\end{align*}
\]

Fig. 4. General representation of the polytopic uncertainties for different variations of the yield and the time-delay

This subsection offers a tool to verify the stability robustness under the polytopic uncertainties. For time-invariant systems, due to the affine dependence, verifying the stability for each segment is equivalent to verifying the stability for its vertices [21]. A procedure based on a Lyapunov function is further used in order to verify the stability for each segment. This leads to solve the feasibility problem (16):

\[
\exists X_{2,\theta} = X_{2,\theta}^T > 0 / \begin{bmatrix}
-X_{2,\theta} & X_{2,\theta} A_{cl, \theta, K} \\
A_{cl, \theta, K}^T & X_{2,\theta}
\end{bmatrix} < 0
\]  

(16)

for each segment, with \(K \in [K, \overline{K}]\) and \(\theta \in \{\theta_1; \ldots; \overline{\theta}\}\). Note that this feasibility problem means finding \(X_{2,0}\) which verifies the LMI (16), with the Lyapunov matrix as decision variable.

Summarizing, for each value of \(\theta\), a feasibility problem containing two LMIs (with the same Lyapunov matrix) corresponding to the two vertices obtained for the bounds of \(K\) must be solved. The notation \(A_{cl, \theta, K}\) shows the dependence of the closed-loop evolution matrix on \(K\) and \(\theta\). When an initial MPC is used, \(A_{cl, \theta, K}\) has the form (17), where \(K_0\) and \(\theta_0\) are the values for the nominal system and the subscript "c" is used for the extended state-space representation (8).

\[
\begin{bmatrix}
A_1 & A_2 \\
A_3 & A_4
\end{bmatrix} = \begin{bmatrix}
A_{c, \theta, K} & -B_{c, \theta} L \\
K_{obs} C_{e, \theta} & A_{e, \theta} -B_{e, \theta_0} L -K_{obs} C_{e, \theta_0}
\end{bmatrix}
\]  

(17)

Considering the robustified controller, the matrix \(A_{cl, \theta, K}\) is given by (18), showing that the Youla parameter is included in the evolution matrix.

\[
\begin{bmatrix}
A_1 -B_{e, \theta_0} D_{Q} C_{e, \theta} & A_2 +B_{e, \theta} D_{Q} C_{e, \theta_0} -B_{e, \theta_0} C_{Q} \\
A_3 -B_{e, \theta_0} D_{Q} C_{e, \theta} & A_4 +B_{e, \theta} D_{Q} C_{e, \theta_0} -B_{e, \theta_0} C_{Q}
\end{bmatrix}
\]  

(18)

Please note that for time-variant systems, the same Lyapunov matrix must be used for all the vertices, after adding the corresponding zeros into the evolution matrix (4) (to enforce the same size of the corresponding zeros into the evolution matrix (4) (to enforce the same size of the evolution matrix).

A different approach consists into explicitly taking into account the polytopic uncertainties in the synthesis of the robustified controller. This means to find the \(Q\) parameter that guarantees (16), leading to a BMI (Bilinear Matrix Inequality) in the \(Q\) parameter and the Lyapunov variable. A sub-optimal solution of this BMI problem may be found with the 3-steps procedure from [11].

IV. **Simulations Results**

This part focuses on the simulation results obtained when applying the previous control techniques on the production-inventory system. Please note that the trajectories of variables have been selected to show differences between controllers and thus might not correspond to an actual production-inventory system as they might be too aggressive. There are three main steps:

- designing an initial stabilizing MPC controller (MPC0) which assures the stability and a certain level of performance for the nominal system. But this controller may not be stable for the mismatched systems (obtained for the variations of the dead-time and the yield);
- robustifying this first controller under unstructured uncertainties (leading to RMPC0) in order to stabilize also the mismatched systems [11];
- designing another initial stabilizing controller (MPC1) for the nominal system which has a slower time-response, but which offers stability properties for all the considered variations of \(\theta\) and \(K\).
A comparison between these controllers is finally realized. The following characteristics were used during the simulations. A step set-point with a magnitude of 1000 and a step disturbance (which may come from an unforecasted demand) of magnitude 10 applied at the time instant \( t = 100 \) days were used. The forecasted demand is taken as \( d_f = 0 \) for ease of presentation. The results are valid for other values of \( d_f \) as long as linearity is not violated. This is an assumption that can be made as long as the operation point does not enter the saturation regions that lie at the limits of the inventory capacity. The reference inventory level is kept constant. Note that a known-in-advance change in the reference is easily managed by the MPC whereas a sudden change in reference level is similar to an unforecasted demand.

In order to design the first controller (MPC0), the following MPC parameters are used: the output prediction horizon \( p = 20 \) days, the control horizon \( m = 10 \) days and the weightings \( Q_J = 1 \) and \( R_J = 5000 \). It may be noticed that applying MPC0 on the nominal model (\( \theta_0 = 5, K_0 = 1 \)) has a very good behavior in the time-domain (Fig. 5, left). The most significant remark is that this controller does not guarantee the stability for variations of the dead-time.

It has to be noted that for \( \theta_0 = 5, K_0 = 1 \) it is possible to find other MPC parameters \((p, m, Q_J, R_J)\) that provide stable closed-loops when applied to plants with some degree of mismatch in \( \theta \) and \( K \) at the cost of more sluggish responses. Also, for larger values of \( \theta_0 \), the set of MPC parameters that can cope with changes in \( \theta \) and \( K \) is greatly reduced. This is the main motivation of this paper: in order to stabilise the mismatched systems, this initial controller is further robustified leading to RMPC0.

By minimizing the \( H_\infty \) norm of the transfer \( T_{a,b} \) (Fig. 3), the controller MPC0 is robustified under additive uncertainties (15). It has been noticed that considering also the robustness toward multiplicative uncertainties (16) (which is equivalent to minimize the complementary sensitivity function) increases the stability domain toward structured uncertainties (11) (obtained from the variations of \( K \) and \( \theta \)). This adds a new LMI to the optimization problem (15):

\[
\min_{\gamma, \mu} \text{LMI}_{\gamma} \text{LMI}_{\mu} (c_1 \gamma + c_2 \gamma_{CS}), \tag{19}
\]

with the subscript "CS" related to the complementary sensitivity function. Solving this new LMI problem (with the parameters \( c_1 = 1, c_2 = 1 \), a degree 20 of the polynomial \( Q \) parameter and the weightings \( W_a, W_y \) as the state-space representation of \( W_a = W_y = (1-0.9z^{-1})/0.1 \) leads to the controller RMPC0. The stability of the uncertain domain is easily verified by solving the feasibility LMI problem (16) with the appropriate evolution matrices obtained for \( K \in \{0.8; 1\} \) and \( \theta \in \{4; 5; 6\} \). This leads to verify a number of six LMIs (two LMIs with the same decision variable for each considered value of \( \theta \)). In this case the problem is feasible and so RMPC0 offers the stability over the considered uncertain domain.

A trade-off has been realized: using RMPC0 the performances for the mismatched systems (Fig. 6, left) are deteriorated compared to the nominal case (Fig. 5, left), but the closed-loop system still remains stable for the considered variations of the parameters.

As a remark, the maximum value of \( K \) is limited to 1 because rarely the received items are more than the ordered goods. However many times the received items are less than the demand (\( K < 1 \)).

Another predictive controller (MPC1) which has a slower response for the nominal model (Fig. 5, left), but is stable on the entire parametric domain, is designed using the following tuning parameters: \( p = 20 \) days, \( m = 1 \) day and the weightings \( Q_J = 1 \) and \( R_J = 5000 \).
Figure 5 shows the output and the control signal obtained for the nominal model. The same I/O (Input/Output) behavior is obtained with MPC0 and RMPC0 without considering disturbances, as the $Q$ parameter does not modify it in the absence of disturbances. It can be noticed that MPC1 gives a much longer time-response than MPC0 and RMPC0, but it offers the stability over the considered polytopic domain. When disturbances (e.g. unforecasted demands) are acting, MPC0 offers the best response, the other controllers having a slower rejection. A trade-off between nominal performance for disturbances rejection and robust stability is thus realized. Figure 5, right shows the control signal for the nominal case. MPC1 offers smaller factory starts than MPC0 and RMPC0.

Figure 6 offers the results obtained for $\theta = 6$ and $K = 1$. Modifying the value of the delay, the closed-loop systems with MPC0 become unstable and this is the reason why they are not represented in Fig. 6. In fact, these results highlight the price to pay: better I/O performances for the nominal system with stability for the mismatched systems (RMPC0) or a slower I/O behavior for the nominal system (sometimes with better robust stability for the mismatched systems) which is propagated to the mismatched systems (MP1). It may be also noticed that MPC1 has small oscillations. Figure 6, right shows the control signal for $\theta = 6$ and $K = 1$.

Figure 7 offers a frequency analysis of the results with the nominal controller. The robustified controller RMPC0 has the smallest $H_\infty$ norm (Fig. 7, left) of the $T_{ab}$ transfer (Fig. 3) (and it may still be improved by choosing a higher order of the Youla parameter). The best $H_\infty$ norm (which is the maximum singular value) for the complementary sensitivity function (Fig. 7, right) is given by MPC1. These two graphics highlight the trade-off between RMPC0 and MPC1.

An interesting idea is to analyze the controlled system for the case of higher variations of the delay $\theta$. This seems to be very difficult (sometimes impossible) to be stabilized using a simple MPC. In fact, the controller MPC1 is not anymore stable considering $\theta = 8$ and $K = 0.8$ (Fig. 8). In this case the controlled system with the controller RMPC1 still remains stable, even if it is very oscillating.

V. CONCLUSIONS AND FUTURE RESEARCH

This paper proposes an application of an advanced technique (robustified Model Predictive Control) to a production-inventory system. The first step consists into modeling the system as a LTI system with a variable input delay $\theta$ and a variable parameter $K$ (the yield). As the bounds of variations of these parameters are known, a polytopic uncertain description may be associated to this system. For the control process, an initial MPC which stabilizes the nominal system with respect to some performance specifications is used. As this initial controller may be unstable for variations of the delay and the yield, it is robustified (via the Youla parametrization) under unstructured uncertainties in order to verify the stability for the considered variations. This robustified controller is compared to another initial MPC controller, which is slower, but remains stable for all the considered variations of $\theta$ and $K$.

The most important advantage is that the robustification procedure offers enough degrees of freedom (especially increasing the degree of the Youla parameter) in order to stabilize the mismatched systems. To implement the robustified controller, it should be approximated using common available techniques of balanced truncation.

In order to explicitly limit the effect of the unforecasted demand, a future development is to consider time-domain templates for this disturbance rejection [16]. Future work may also consider applying an on-line robust MPC [22] on the production-inventory system and compare the results.

REFERENCES


