ABSTRACT
For a wireless sensor network (WSN) with a random number of sensors, a decision fusion rule that uses the total number of detections reported by local sensors for hypothesis testing, is proposed. It is assumed that the number of sensors follows a Poisson distribution and the locations of sensors follow a uniform distribution within the region of interest (ROI). Both analytical and simulation results for the system level detection performance are provided. This fusion rule can achieve a very good system level detection performance even at very low signal to noise ratio (SNR), if the average number of sensors is sufficiently large. In addition, the problem of choosing an optimum local sensor level threshold is investigated for various system parameters.

1. INTRODUCTION
One of the most important tasks a WSN needs to perform is target detection, typically in a distributed manner. There are already numerous papers in the literature on the conventional distributed detection problem [1, 2, 3, 4].

However, most of these results are based on the assumption that the local sensors’ performances are known. For a dynamic target and passive sensors, it is very hard to estimate local sensors’ performances via experiments because these performances are time-varying as the target moves through the wireless sensor field. Even if the local sensors can somehow estimate their detection performances in real time, it will be very expensive to transmit them to the fusion center, especially for a WSN with very limited system resources. On the other hand, usually a WSN consists of a large number of low-cost and low-energy sensors, which are densely deployed in the environment. Taking advantage of these unique characteristics of WSNs, in our previous paper [5], we proposed a fusion rule that uses the total number of detections (“1”s) transmitted from local sensors as the statistic.

In [5], we assumed that the total number of sensors in the ROI is known. However, in many applications, the sensors are deployed randomly in and around the ROI, and oftentimes some of them are malfunctioning or out of battery. Therefore, in practice the total number of sensors that work properly in a ROI is a random variable. In this paper, the performance of the fusion rule proposed in [5] will be analyzed with this extra uncertainty about the total number of sensors.

2. MODELING AND DECISION FUSION RULE
2.1. Problem Formulation
As shown in Fig. 1, a total of $N$ sensors are randomly deployed in the ROI, which is a square with area $b^2$. $N$ is a random variable that follows a Poisson distribution:

$$p(N) = \frac{\lambda^N e^{-\lambda}}{N!} \quad \text{for} \quad N = 0, \ldots, \infty$$ (1)

The locations of sensors are i.i.d. and follow a uniform distribution in the ROI.

Noises at local sensors are i.i.d and follow the standard Gaussian distribution with zero mean and unit variance. For a local sensor $i$, the binary hypothesis testing problem is:

$$H_1: \quad s_i = a_i + n_i$$
$$H_0: \quad s_i = n_i$$ (2)
where $s_i$ is the received signal, and $a_i$ is the signal amplitude. We adopt the same isotropic signal power attenuation model as that presented in [5]:

$$a_i^2 = \frac{P_0}{1 + \alpha d_i^\alpha}$$

where $P_0$ is the signal power emitted by the target at distance zero, $d_i$ is the distance between the target and local sensor $i$. $n$ is the signal decay exponent and takes values between 2 and 3. $\alpha$ is an adjustable constant. Because the noise has unit variance, it is evident that the SNR at distance zero is

$$SNR_0 = 10 \log_{10} P_0$$

Assuming that all the local sensors use the same threshold $\tau$ to make a decision, we have the local sensor level false alarm rate and probability of detection:

$$p_{fa} = \frac{\tau}{\sqrt{2\pi}} e^{-\frac{\tau^2}{2}}$$

$$p_{dl} = Q(\tau - a_i)$$

where $Q(\cdot)$ is the complementary distribution function of the standard Gaussian.

We assume that the ROI is very large and the signal power decays very fast. Hence, only within a very small fraction of the ROI, which is the area surrounding the target, the received signal power is significantly larger than 0. By ignoring the border effect of the ROI, we assume the target is located at the center of the ROI, without any loss of generality.

### 2.2. Decision Fusion Rule

Since it is difficult to estimate $p_{dl}$s at local sensors, the fusion center is forced to treat every sensor equally. As proposed in [5], the system level decision is made by first counting the number of detections made by local sensors and then comparing it with a threshold $T$:

$$\Lambda = \sum_{i=1}^{N} \frac{H_1}{H_0} \quad T$$

where $I_i = \{0, 1\}$ is the local decision made by sensor $i$.

### 3. PERFORMANCE ANALYSIS

#### 3.1. System Level False Alarm Rate

At the fusion center level, the probability of false alarm $P_{fa}$ is

$$P_{fa} = \sum_{N=T}^{\infty} p(N)Pr\{\Lambda \geq T|N, H_0\}$$

Obviously, for a given $N$, under hypothesis $H_0$, $\Lambda$ follows a Binomial $(N, p_{fa})$ distribution, and

$$P_{fa} = \sum_{N=T}^{\infty} p(N) \sum_{i=1}^{N} \binom{N}{i} p_{fa}^i (1 - p_{fa})^{N-i}$$

It is well known that the Kurtosis of a Poisson distribution is $3 + \frac{1}{\lambda}$. As $\lambda$ increases, the Kurtosis of this Poisson distribution approaches that of a Gaussian distribution, and its distribution has a light tail. As a result, when $\lambda$ is large, the probability mass of $N$ will concentrate around the average value ($\lambda$), and

$$\sum_{N=N_1}^{N_2} e^{-\lambda} \frac{\lambda^N}{N!} \approx 1$$

where $N_1 = \left[ \lambda - 6\sqrt{\lambda} \right]$ and $N_3 = \left[ \lambda + 6\sqrt{\lambda} \right]$.

Hence, for a large $\lambda$, a "typical" $N$ is also a large number. The probability that $N$ takes a small value is negligible. For example, when $\lambda = 1000$, $Pr\{N < 810\} = 2.4 \times 10^{-10}$. Therefore, when $\lambda$ is large enough, (9) can be calculated by using Laplace-DeMoivre approximation [6]:

$$P_{fa} \approx \sum_{N=N_2}^{N_3} \frac{\lambda^N e^{-\lambda}}{N!} Q\left(\frac{T - \bar{N}p_{fa}}{\sqrt{Np_{fa}(1 - p_{fa})}}\right)$$

where $N_2 = \max(T, N_1)$.

### 3.2. System Level Probability of Detection

In [5], through approximation by using Central Limit Theorem, we derived the system level $P_d$ when the number of sensors $N$ is large:

$$Pr\{\Lambda \geq T|N, H_1\} \approx Q\left(\frac{T - \bar{N}p_{fa}}{\sqrt{N\bar{p}_{fa}(1 - p_{fa})}}\right)$$

where

$$\bar{p}_{fa} = \frac{2\pi}{b^2} \int_0^{\frac{1}{2}} C(r)rdr + \left(1 - \frac{\pi}{4}\right)p_{fa}$$

$$\bar{\sigma}^2 = \frac{2\pi}{b^2} \int_0^{\frac{1}{2}} \left[1 - C(r)\right]C(r)rdr$$

and

$$C(r) = Q\left(\frac{r - \sqrt{P_0/(1 + \alpha d^\alpha)}}{\sqrt{\sigma^2}}\right)$$

Similar to the derivation of (11), we have

$$P_d \approx \sum_{N=N_2}^{N_3} \frac{\lambda^N e^{-\lambda}}{N!} Q\left(\frac{T - \bar{N}p_{fa}}{\sqrt{N\bar{p}_{fa}(1 - p_{fa})}}\right)$$
Fig. 2. ROC curves obtained by calculation and simulations. $N = 1000$, $n = 2$, $b = 100$, $\alpha = 200$, and $\tau = 0.77$, 0.73, 0.67 for $P_0 = 1000$, 500, 100, respectively.

Fig. 3. ROC curves obtained by calculation and simulations. System parameters are the same as those listed in Fig. 2.

3.3. Simulation Results

In Figs. 2 and 3, the receiver operative characteristic (ROC) curve obtained by using approximations in Section 3.1 and 3.2 and that by simulations are plotted. The simulation results in Figs. 2 and 3 are based on $10^5$ and $10^6$ Monte Carlo runs, respectively. From Figs. 2 and 3, it is clear that the results by using approximations are very close to those obtained by simulations, even when the system level $P_{fa}$ is very low (Fig. 3).

3.4. Asymptotic Analysis

In (11), we know that

$$\max \left( T, \left[ \lambda - 6\sqrt{\lambda} \right] \right) \leq N \leq \left[ \lambda + 6\sqrt{\lambda} \right]$$

(17)

Hence, as $\lambda \to \infty$, we have $N \to \lambda$, if $T \leq \left[ \lambda + 6\sqrt{\lambda} \right]$. Assume that the system level threshold is in the form of

$$T = \beta \lambda,$$

we have

$$P_{fa} \approx \sum_{N=N_0}^{N_3} \frac{\lambda^N e^{-\lambda}}{N!} Q \left( \frac{(\beta - p_{fa})\sqrt{\lambda}}{\sqrt{p_{fa}(1-p_{fa})}} \right)$$

(18)

Similarly, from (16) we have

$$P_d \approx \sum_{N=N_0}^{N_3} \frac{\lambda^N e^{-\lambda}}{N!} Q \left( \frac{(\beta - p_d)\sqrt{\lambda}}{\sqrt{\sigma^2}} \right)$$

(19)

Therefore, when $\lambda \to \infty$, if $\beta < p_{fa}$, $P_{fa} = P_d = 1$; if $p_{fa} < \beta < \bar{p}_d$, $P_{fa} = 0$ and $P_d = 1$; if $\beta > \bar{p}_d$, $P_{fa} = P_d = 0$. As a result, as long as $\beta$ takes a value between $p_{fa}$ and $\bar{p}_d$, as $\lambda \to \infty$, the WSN’s detection performance will be perfect with $P_d = 1$ and $P_{fa} = 0$. In Fig. 4, $P_d$ and $P_{fa}$ as functions of $\lambda$ are plotted. It is clear that the $P_d$ converges to 1 as $\lambda$ increases and $P_{fa}$ converges to 0. In this example, we set $\beta$ such that $\beta = \frac{p_{fa} + p_{fa}}{2}$. Another conclusion is that when $\lambda$ is large enough, even for a small $SNR_0$, the system can achieve a very good detection performance.

4. THRESHOLD FOR LOCAL SENSORS

In this paper, we will find the optimum local sensor level threshold $\tau$ by maximizing the so-called deflection coefficient, which is defined as

$$D(\lambda) = \frac{[E(\Lambda|H_1) - E(\Lambda|H_0)]^2}{Var(\Lambda|H_0)}$$

(20)

In the case of $Var(\Lambda|H_1) = Var(\Lambda|H_0)$, this is in essence the SNR of the detection statistic.

Under hypothesis $H_0$, we have

$$E(\Lambda|N, H_0) = N p_{fa}$$

(21)
and
\[ \text{Var}(\Lambda | N, H_0) = Np_{fa}(1 - p_{fa}) \] (22)

With (21) and (22), it is easy to show that
\[ E(\Lambda | H_0) = \lambda p_{fa} \] (23)

and
\[ \text{Var}(\Lambda | H_0) = \lambda p_{fa} \] (24)

Similar to the derivation of (23), we have
\[ E(\Lambda | H_1) = \lambda \tilde{p}_d \] (25)

Therefore, the deflection coefficient is
\[ D(\tau) = \frac{\lambda[\tilde{p}_d(\tau) - p_{fa}(\tau)]^2}{p_{fa}(\tau)} \] (26)

The optimum \( \tau \) can be found by maximizing \( D(\tau) \) with respect to \( \tau \).

As we can see in Fig. 5, the ROC curve corresponding to the optimal threshold \( \tau_{opt} \) (0.77) is above those for other thresholds, meaning that \( \tau_{opt} \) provides the best system level performance. In Fig. 6, \( \tau_{opt} \) as functions of \( SNR_0 \) and \( \alpha \) are shown. It is clear that \( \tau_{opt} \) is a monotone increasing function of \( SNR_0 \) and a monotone decreasing function of \( \alpha \). This is because with a strong target signal (high \( SNR_0 \) and low \( \alpha \)), by adopting a higher threshold, local sensors lower their false alarm rate, while at the same time they can still maintain a relatively high probability of detection.

**5. CONCLUSIONS**

We have analyzed the performance of a decision fusion rule that is based on the total number of detections made by local sensors, for a WSN with a random number of sensors. The number of sensors in a ROI has been modeled as a Poisson random variable. We have shown that even at very low SNR, this fusion rule can achieve a very good system level detection performance given that there are, on an average, a sufficiently large number of sensors deployed in the ROI.

To achieve a better system performance, we also design an optimum threshold at the local sensors by maximizing the deflection coefficient. Guidelines on how to choose this optimal threshold are provided for various system parameters.

**6. REFERENCES**


