Model Order Reduction for Contact Dynamics Simulations of Manipulator Systems

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Abstract— Dynamic simulation of manipulators doing contact tasks with stiff and complex contact interfaces is time consuming because very small integration step sizes have to be used for numerical stability. Existing model order reduction techniques cannot be readily applied due to high nonlinear nature of contact dynamics. A new method of model order reduction is proposed to deal with this problem. The method identifies the “stiffness” and “damping” terms by linearizing the contact force model and add them to those of the manipulator. Then the existing modal analysis and truncation technology is applied. An example is presented to demonstrate the significant gain in computational efficiency and improved output results.

Keywords: Dynamics, impact dynamics, contact dynamics, simulation, model order reduction, model reduction.

I. INTRODUCTION

Contact dynamics simulation has become a primary means for engineering verification of space robotic systems such as the Canadarm2 and the Special Purpose Dextorous Manipulator (SPDM) because ground-based physical testing of such large and delicate space robots for general contact operations is extremely difficult [1]. Several different methods have been developed over the past two decades for contact dynamics modeling and simulations, as surveyed by Gilardi and Sharf [2]. An effective modeling approach for dealing with multiple-contact between complex contacting objects is the surface-compliance method which uses spring and damping elements to model local contacts [3-4], as illustrated in Fig.1. The generality and fidelity of this modeling approach have been experimentally verified and the method has been accepted for use in the International Space Station Program [4]. However, the simulation efficiency of the method remains a challenging problem. The inefficiency of contact dynamics simulation mainly comes from the bottlenecks problems:

1) contact objects have complicated geometries, which requires intensive computation to identify all the contact locations and estimate the geometry interferences; and
2) contact objects and forces equations are stiff, which requires a very small numerical integration step size.

The first problem has been studied by several researchers. For example, Gilbert et al first introduced an efficient distance computation algorithm between polyhedrons [5]; Ma and Nahon proposed an optimization-based distance method [6]; Attaway et al studied a method of parallelizing the contact detection process [7]; and Carretero et al. applied genetic algorithms for detecting contact between concave objects [8].

On the contrary, the second problem received much less attention although it is also critical. For example, a contact between two solid metal objects may have a linear stiffness of approximately $6 \times 10^9$ N/m (or a Yonge’s modulus in the order of $10^{11}$ N/m$^2$) in a contacting area of 1.0 cm$^2$. Such high stiffness causes very high-frequency oscillation modes in dynamic response resulting from contact, which consequently limits the numerical integration step size to microseconds level in order to achieve a stable simulation. Obviously, such a simulation is extremely inefficient. Moreover, high stiffness contact at the tip of a long flexible arm can also lead to stiff forward dynamic formulation which is sensitive and difficult to handle by usual numerical integration methods [9].

Understandably, variable-step integration methods could be used to deal with the high-stiffness impact problem. Such a solution, however, does not always work well with some independently coded control modules because of inconsistency in step sizes between the simulator’s integrator and those in the control code. It is not easy to make the control code to follow the time-varying integration step size which is likely
controlled by a black-box integrator like Simulink. In addition, variable-step integration methods may not be acceptable for real-time simulations because of its uncertain execution time for each integration step. Real-time simulation is very useful for aerospace applications for studying human factors and training vehicle crews to handle various remote robotic operations under extreme conditions.

Studies showed that oscillation modes beyond 150 Hz in a vehicle system contribute little to the dominant motion of the overall system [10]. Simulation prediction of structure dynamics at or over several-hundred Hz will unlikely be accurate even for a finite-element based dynamics model [11]. During the validation of the Canadarm2 and SPDM dynamics models with NAStRAN analysis, it was found that good matches were only within about the first ten flexible modes among a total of over a hundred modes. In most of the verification or mission support simulations of these two Space Station manipulators, flexible modes above 50Hz (excluding contact dynamics models) are normally truncated by model order reduction techniques. Model order reduction (MOR) can increase simulation speed by one to several orders and have been well developed and used across the aerospace industry for controls and simulations of aircraft/spacecraft flexible structures. Unfortunately, such mature technology is not readily applicable to contact dynamics simulations because contact dynamics models are intrinsically nonlinear and also time-varying in dimension (i.e., the number of state variables is time-varying). Engineers have been wondering whether high-frequency modes resulting from contact dynamics models can also be eliminated using modal analysis based truncation techniques. This problem did not receive enough attention in the past possibly due to the lack of motivation in the aerospace field where physical contact between flying vehicles was just out of question. However, with the increasing interest in satellite on-orbit servicing missions, simulation study of physical contact between flying spacecraft is becoming necessary. Indeed, contact dynamics simulation will play more critical role in aerospace industry in the future when satellite on-orbit servicing is near or becomes a reality. Therefore, there is strong motivation to have an investigation of this problem aimed at developing a practical method to extend the well-established modal analysis and truncation techniques to general multibody contact dynamics simulations.

In this paper a method of model order reduction for contact dynamics simulation is introduced. This method solves the problem by identifying the implicit “stiffness” contribution from the contact dynamics and adding it to the structure stiffness of the manipulator system, after which, any well-established modal analysis and truncation method can be applied. In such an approach, the contact dynamics simulation becomes much faster and more stable. It can be performed just like the simulations of non-contact flexible flight systems, which have been benefited from the successful modal analysis and truncation techniques. An example of a two degree-of-freedom planar manipulator contacting a fixed wall is presented to illustrate the application of the method. Simulation results demonstrated that the method significantly increased the simulation speed and reduced the non-dominant high-frequency oscillatory behavior in the simulation output.

II. METHODOLOGY

A. System Dynamics and Contact Dynamics

Assume that two multibody systems are in contact through impact and/or continuous contact between some of their member bodies, as shown in Fig.1. The number of contact points (or regions) and their contacting locations vary from time to time depending on the relative motion between the contacting bodies. The governing dynamics equation of a general, flexible robotic system can be expressed in terms of a set of generalized coordinates as follows

$$M(q)\ddot{q} + N(q, \dot{q})\dot{q} + K(q)q = \Sigma \tau + \tau_c$$  (1)

where $q$ is a vector of the generalized coordinates including both rigid and flexible coordinates of the manipulator. $M(q)$, $N(q, \dot{q})$, and $K(q)$ are the generalized inertia matrix, nonlinear inertia forces, and stiffness matrix, respectively? $\Sigma \tau$ is the sum of all the generalized forces except the generalized contact force. The term $\tau_c$ stands for the generalized contact force represented in the generalized space (or the joint space if the system is a rigid manipulator). Since physical contact happens only at the end-effector or payload in Cartesian space (task space), contact force is always first formulated as a function of the Cartesian state variables $x$ and $\dot{x}$, namely,

$$f_c = f_c(x, \dot{x})$$  (2)

This function is nonlinear and discontinuous because a contact may be on or off frequently during a robotic task. Besides, the total number of contact points and the contacting locations also vary from time to time in a task. The contact force model (2) also depends on a variety of physical parameters accounting for the geometry, stiffness, friction, and damping properties of the contacting objects or environment. Details of the model can be found in [3-4].

The contact force in the Cartesian space and that in the generalized coordinate space can be described as

$$\tau_c = J^T f_c(x, \dot{x})$$  (3)

where $J$ is the Jacobian matrix defined such that

$$\dot{x} = Jq \quad \text{or} \quad J = \frac{\partial x}{\partial q}$$  (4)

Notice that this is an extended Jacobian matrix because $q$ includes both rigid and flexible coordinates of the system.

B. Model Linearization

Simulation efficiency and numerical robustness can be significantly improved by truncating the highest frequency modes on the left hand side of eq.(1) if the equation is linear and the contact force is absent. This is a common practice for simulating the dynamics of large flexible manipulator systems.
like the Canadarm2. However, such a technique used as is
does not provide any benefits when the contact force $\tau_c$ is
present because there still exist high frequency components
(coming from the high contact stiffness) residing on the right-
hand side of the equation. Therefore, model order reduction
has to be applied to both sides of the dynamics equation (1).
To this end, the equation has to be linearized first.

Indeed, equation (1) can be linearized in terms of a set of
small perturbation coordinates $\Delta q$ measured from a known
reference state $(q, \dot{q})$, usually called operating point, namely,

$$M\Delta \ddot{q} + \overline{N}\Delta q + \overline{K}\Delta q = \Sigma \Delta \tau + \Delta \tau_c$$  \hspace{1cm} (5)

where the coefficient matrices $\overline{N}$ and $\overline{K}$ are defined as

$$\overline{N} = N(q, \dot{q}) - \partial N(q, \dot{q}) / \partial \dot{q}$$
$$\overline{K} = K(q) + \partial M(q) / \partial q + \partial N(q, \dot{q}) / \partial q + \partial K(q) / \partial q$$  \hspace{1cm} (6)

which are evaluated at the reference state $(q, \dot{q})$ and kept
unchanged until the next update of the reference state. The
incremental forces on the right-hand-side of eq.(5) are with
respect to the force values at the known reference state. The
reference state will be updated periodically at time intervals
normally much larger than the numerical integration step size.
The contact force increment may be approximated as

$$\Delta \tau_c = K_c(q, \dot{q})\Delta q + N_c(q, \dot{q})\Delta \dot{q}$$  \hspace{1cm} (7)

where

$$K_c(q, \dot{q}) = \partial \tau_c(q, \dot{q}) / \partial q, \quad N_c(q, \dot{q}) = \partial \tau_c(q, \dot{q}) / \partial \dot{q}$$  \hspace{1cm} (8)

In fact, $K_c$ and $N_c$ are the linear “stiffness” and “damping”
matsrices of the contact force model represented in the
generalized coordinate space. They can be viewed as the
coefficients of the contact interface to the system dynamics.
Thus, we obtain the linearized dynamics equation:

$$M(q)\Delta \ddot{q} + \overline{N}(q, \dot{q})\Delta q + \overline{K}(q, \dot{q})\Delta q = \Sigma \Delta \tau + K_c(q, \dot{q})\Delta q + N_c(q, \dot{q})\Delta \dot{q}$$  \hspace{1cm} (9)

This equation has constant coefficients between every two
neighboring updates of the reference state $(q, \dot{q})$ and is linear
in perturbation variables $\Delta q$ and its time derivatives.

C. Modal Transformation and Reduction

Upon the arrival of eq.(9), we can move the “stiffness” and
“damping” terms of the linearized contact force model from
the right-hand side of the equation to the left-hand-side as
follows (for compactness, the state variables showing
dependencies in the equations are omitted):

$$M\Delta \ddot{q} + (\overline{N} - N_c)\Delta q + (\overline{K} - K_c)\Delta q = \Sigma \Delta \tau$$  \hspace{1cm} (10)

Now, the contact force originally appearing on the right hand
side of the dynamics equation as an input force has become
part of the system dynamics on the left hand side of the
equation. This last step makes it possible to reduce the high-
frequency modes inside the contact force model. A similar
procedure can be applied to the sum of all the other
generalized forces, $\Sigma \Delta \tau_c$, consisting of control, gravity, and
friction forces, etc.) if one also wishes to reduce their
contributions to the system’s high-frequency modes.

Because all the coefficient matrices of equation (10) are
examined at the reference state, $(q, \dot{q})$, which is independent
of the perturbation variables $\Delta q$ and $\Delta \dot{q}$, the equation is not
only linear but also time-invariant in terms of the perturbation.
This makes it a perfect candidate for performing modal
analysis and reduction using well-developed model order
reduction techniques, as outlined next.

By introducing a set of independent modal coordinates $\eta$:

$$\Delta q = E\eta \quad \text{and} \quad \Delta \dot{q} = E\eta$$  \hspace{1cm} (11)

the linear dynamics equation (10) can be transformed into the
following form in modal space:

$$\ddot{\eta} + 2\zeta \Omega \eta + \Omega^2 \eta = E^T \Sigma E \eta$$  \hspace{1cm} (12)

where matrices $\Omega$ and $E$ are defined such that

$$E^T ME = 1, \quad E^T(\overline{K} - K_c)E = \Omega^2$$  \hspace{1cm} (13)

In the above equations, 1 is an identity matrix; $\Omega$ is a
diagonal matrix whose diagonal components are the modal
frequencies of the dynamic system (10); $E$ is the eigenmatrix
of the dynamic system; and $\zeta$ is a modal damping matrix
which is normally assumed to be diagonal and takes damping
ratios as its diagonal components. Except for the damping
terms, the individual scalar equations of eq.(12) are naturally
uncoupled in terms of the modal coordinates. Therefore, they
are ready for order reduction. Commonly used reduction
methods are to simply truncate or shift the modes associated
with the highest modal frequencies.

III. A SIMPLE EXAMPLE

In order to gain some insight into the feasibility and
performance of the proposed model order reduction method,
a simple flexible manipulator is employed as an application
example for a case study. The system under study is a 2
degree-of-freedom planar manipulator moving on a horizontal
plane, as shown in Fig.2. The joints of the manipulator are
assumed to be flexible with rotational stiffness $k_1$ and $k_2$. The
masses of the two links, $m_1$ and $m_2$, are assumed to be evenly
distributed along the links. A spherical contact object is rigidly
attached to the tip of the manipulator, which will be contacting
a fixed wall as shown in Fig. 3. In this case there can only be a
single contact point due to the assumed geometry of the two contact objects – a sphere and a wall.

For simplicity of the analysis, contact friction is ignored and the normal contact force is assumed to be linear in terms of the surface “deformation” $d$ which is approximated by the depth that the sphere penetrates into the wall (see Fig.3).

### A. Dynamics Model of the Manipulator

Based on the kinematics shown in Fig.2, the tip position and velocity of the manipulator can be expressed as

\[
x = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) \\ l_1 \sin(q_1) + l_2 \sin(q_1 + q_2) \end{bmatrix}
\]

\[
x = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \mathbf{J} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}
\]

where the Jacobian matrix $\mathbf{J}$ takes the following form

\[
\mathbf{J} = \begin{bmatrix}
- l_1 \sin(q_1) - l_2 \sin(q_1 + q_2) & - l_2 \sin(q_1 + q_2) \\
 l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) & l_2 \cos(q_1 + q_2)
\end{bmatrix}
\]

Applying the Lagrange formulation, we can derive the detailed dynamics analysis in the robotics field.

\[
\mathbf{M} = \begin{bmatrix}
\frac{1}{3} m_1 l_1^2 + m_2 \left( \frac{1}{2} l_1^2 + l_1 l_2 \cos(q_2) + \frac{1}{3} l_2^2 \right) \\
m_2 \left( \frac{1}{2} l_1 l_2 \cos(q_2) + \frac{1}{3} l_2^2 \right) \\
m_2 \left( \frac{1}{2} l_1 l_2 \cos(q_2) + \frac{1}{3} l_2^2 \right) \\
\frac{1}{3} m_2 l_2^2
\end{bmatrix}
\]

\[
\mathbf{N} = \begin{bmatrix}
- m_2 l_1 l_2 \sin(q_2) \dot{q}_2 + b_1 \\
\frac{1}{2} m_2 l_1 l_2 \sin(q_2) \dot{q}_2 \\
\frac{1}{2} m_2 l_1 l_2 \sin(q_2) \dot{q}_2 \\
b_2
\end{bmatrix}
\]

\[
\mathbf{K} = \begin{bmatrix}
k_1 & 0 \\
0 & k_2
\end{bmatrix}
\]

\[
\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \quad \text{← Generalized coordinates}
\]

\[
\Sigma \mathbf{\tau} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \quad \text{← Control torques}
\]

The generalized contact torques may be transformed from the local contact force in the following fashion (which is only a static relation but it is usually an acceptable approximation for dynamics analysis in the robotics field).

\[
\mathbf{\tau}_c = \mathbf{J}^T \mathbf{f}_c = \mathbf{J}^T \begin{bmatrix} 0 \\ f_c \end{bmatrix}
\]

where $f_c$ is the normal contact force applied at the contact location. The contact force is only along the $y$ direction because contact friction and contact share force are ignored in this analysis. Based on the assumption of linear contact force model, the normal contact force $f_c$ can be formulated as

\[
f_c = k_c d + b_c \dot{d} = \begin{cases} 0 & \text{if } y \geq r \\
k_c (r - y) - b_c \dot{y} & \text{if } y < r
\end{cases}
\]

where $k_c$ and $b_c$ are contact stiffness and contact damping coefficient, respectively. In addition to eq. (20), the contact force has to satisfy physical constraint $f_c \geq 0$.

Combining the foregoing formulation, we can derive the generalized contact torques in terms of the generalized coordinates and its time derivatives as follows

\[
\mathbf{\tau}_c = \begin{bmatrix}
k_c(l_1 c_1 + l_2 c_{12})(r - l_1 s_1 - l_2 s_{12}) \\
k_c[r - l_1 s_1 - l_2 s_{12}] l_2 c_{12} \\
b_c(l_1 c_1 + l_2 c_{12})^2 q_1 + b_c(l_1 c_1 + l_2 c_{12}) l_2 c_{12} q_2 \\
b_c(l_1 c_1 + l_2 c_{12}) l_2 c_{12} \dot{q}_1 + b_c(l_2 c_{12})^2 \dot{q}_2
\end{bmatrix}
\]

In the above equation, abbreviations $s_1$ and $c_1$ stand for $\sin(q_1)$ and $\cos(q_1)$, and $s_{12}$ and $c_{12}$ for $\sin(q_1 + q_2)$ and $\cos(q_1 + q_2)$, and so on. This final expression is ready for the linearization process, as described in Section II.B.

![Figure 2](image2.png) A 2-axis planar manipulator with a spherical contact object on its tip

![Figure 3](image3.png) Contact geometry modeling

### B. Linearization of the Dynamics Model

For linearization of the dynamics model, we can reasonably assume that the manipulator is operated only in a small neighborhood around the $x$ axis, which means

\[
q_1(t) \approx 0 \quad \text{and} \quad q_2(t) \approx 0
\]
From such an assumption, it follows that
\[ \sin(q_1) \approx q_1, \quad \cos(q_1) \approx 1, \quad \sin(q_2) \approx q_2, \quad \cos(q_2) \approx 1 \quad (23) \]
Thus, the coefficients of the linearized eq. (5) reduces to
\[
\Delta \mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}, \quad \Delta \mathbf{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}, \quad \Delta \ddot{\mathbf{q}} = \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}, \quad \Delta \mathbf{q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix},
\]
\[
\mathbf{M} = \begin{bmatrix}
\frac{1}{3} m_1 l_1^2 + m_2 \left( l_1^2 + l_1 l_2 + \frac{1}{3} l_2^2 \right) \\
\frac{1}{3} m_2 \left( l_1^2 + l_1 l_2 + \frac{1}{3} l_2^2 \right) \\
\end{bmatrix}
\]
\[
\mathbf{N} = \begin{bmatrix}
-m_2 l_1 l_2 q_2 \dot{q}_2 + b_1 - \frac{1}{2} m_2 l_1 l_2 q_2 \dot{q}_2 \\
\frac{1}{2} m_2 l_1 l_2 q_2 \dot{q}_1 \\
\end{bmatrix}, \quad \Delta \mathbf{\tau} = \mathbf{\tau} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix},
\]
Following eq.(21), the contact force model is correspondingly linearized as
\[
\mathbf{\tau}_c = \begin{bmatrix}
k_c \left( r - (l_1 + l_2) q_1 - l_2 q_2 \right) \\
k_c \left( r - (l_1 + l_2) q_1 - l_2 q_2 \right) l_2 \\
b_c (l_1 + l_2)^2 \dot{q}_1 + b_c (l_1 + l_2) l_2 \dot{q}_2 \\
b_c (l_1 + l_2) l_2 \dot{q}_1 + b_c l_2^2 \dot{q}_2 \\
\end{bmatrix},
\]
from which the “stiffness” and “damping” terms of the linearized contact force model (7) take the following forms:
\[
\mathbf{K}_c = \frac{\partial \mathbf{\tau}_c}{\partial \mathbf{q}} = \begin{bmatrix}
-k_c (l_1 + l_2)^2 - k_c (l_1 + l_2) l_2 \\
-k_c (l_1 + l_2) l_2 - k_c l_2^2 \\
-b_c (l_1 + l_2)^2 - b_c (l_1 + l_2) l_2 \\
-b_c (l_1 + l_2) l_2 - b_c l_2^2 \\
\end{bmatrix}, \quad \mathbf{N}_c = \frac{\partial \mathbf{\tau}_c}{\partial \mathbf{\dot{q}}} = \begin{bmatrix}
-k_c (l_1 + l_2)^2 - k_c (l_1 + l_2) l_2 \\
-k_c (l_1 + l_2) l_2 - k_c l_2^2 \\
-b_c (l_1 + l_2)^2 - b_c (l_1 + l_2) l_2 \\
-b_c (l_1 + l_2) l_2 - b_c l_2^2 \\
\end{bmatrix}
\]
As a result, the final linear equation ready for model order reduction becomes
\[
\mathbf{M} \ddot{\mathbf{q}} + (\mathbf{N} - \mathbf{N}_c) \dot{\mathbf{q}} + (\mathbf{K} - \mathbf{K}_c) \mathbf{q} = \mathbf{\tau} \quad (27)
\]
where the combined stiffness and damping matrices are
\[
\mathbf{N} - \mathbf{N}_c = \begin{bmatrix}
-m_2 l_1 l_2 q_2 \dot{q}_2 + b_1 + b_c (l_1 + l_2)^2 \\
-m_2 l_1 l_2 q_2 \dot{q}_2 + b_1 + b_c (l_1 + l_2)^2 l_2 \\
\frac{1}{2} m_2 l_1 l_2 q_2 \dot{q}_1 + b_c (l_1 + l_2) l_2 \\
\frac{1}{2} m_2 l_1 l_2 q_2 \dot{q}_1 + b_c (l_1 + l_2) l_2 + b_c l_2^2 \\
\end{bmatrix}, \quad \mathbf{K} - \mathbf{K}_c = \begin{bmatrix}
k_1 + k_c (l_1 + l_2)^2 \\
k_1 + k_c (l_1 + l_2) l_2 \\
k_c (l_1 + l_2) l_2 \\
k_2 + k_c l_2^2 \\
\end{bmatrix}
\]
Notice that the final stiffness matrix is symmetric. This satisfies the necessary condition for matrix digitalization, which is the next step of the process, namely, the modal transformation. However, in general this is not always true. If the resulting stiffness matrix is not symmetric, the equation will not be ready for model order reduction. This problem is being studied in on-going research.

C. Simulation Results

A numerical simulation of this example was implemented on Matlab. The assumed parameters of the manipulator in the simulation model are listed in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Link1-Joint1</th>
<th>Link2-Joint2</th>
<th>Contact</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass ( m_i )</td>
<td>3 kg</td>
<td>1 kg</td>
<td>n/a</td>
</tr>
<tr>
<td>Stiffness ( k_i )</td>
<td>10 Nm/rad</td>
<td>200 Nm/rad</td>
<td>( k_c = 10^5 ) N/m</td>
</tr>
<tr>
<td>Damping ( b_i )</td>
<td>0.38 Nm/s/rad</td>
<td>1.00 Nm/s/rad</td>
<td>63.2 Ns/m</td>
</tr>
<tr>
<td>(Ratio ( \zeta ))</td>
<td>(0.35%)</td>
<td>(0.35%)</td>
<td>(10%)</td>
</tr>
<tr>
<td>Link length ( l_i )</td>
<td>0.5 m</td>
<td>0.5 m</td>
<td>n/a</td>
</tr>
<tr>
<td>Radius ( r )</td>
<td>n/a</td>
<td>n/a</td>
<td>0 m</td>
</tr>
</tbody>
</table>

In the table “n/a” means not applicable.

The two joint stiffness values are purposely chosen to be very different in order to better demonstrate the effectiveness of the model order reduction method.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Without Contact</th>
<th>With Contact</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mode 1</td>
<td>Mode 2</td>
<td>Mode 1</td>
</tr>
<tr>
<td>Frequency (rad/s)</td>
<td>3.5</td>
<td>80</td>
</tr>
<tr>
<td>Step size (ms)</td>
<td>14.3</td>
<td>0.6</td>
</tr>
</tbody>
</table>

The frequencies of the two modes are listed in Table 2. Obviously, the highest frequency with contact is much higher than that without contact because of the contribution from the high contact stiffness. For simplicity and also without losing the generality, the modified Euler integration method is used in the simulation study. For a second-order integrator like the modified Euler method, the numerical integration time step size, as recommended by Woods and Lawrence [12], is \( \Delta t = 1/20 \omega_{\text{max}} \) where \( \omega_{\text{max}} \) is the highest frequency of the system. The integration step sizes for the cases with and without contact are also listed in Table 2. It is apparent that the integration step size can increase from 0.09 ms to 1.0 ms after cutting off the highest frequency mode for the contact simulation. This means that the resulting simulation speed is boosted by approximately 11 times. For the same reason, we can predict that the efficiency of a simulation can be improved by possibly 1~3 orders for the SPDM grasping simulation case where the equivalent contact stiffness is around \( 10^9 \) N/m.

The outputs of the simulation without model order reduction (in solid lines) and those with model order reduction (in dashed lines) are superimposed in the plots shown in Fig.4 for comparison. Initially the manipulator is at rest and the two
joint angles are 5 and 0 degrees. From the plots, we can see that the dominant part of the simulated motion with model order reduction matches that without model order reduction very well. Obviously, the high-frequency but non-dominant oscillatory response is no longer in the simulated motion (more visible in the velocity plots) after the model reduction. That is expected and also justified in many practical applications as discussed in Section I. Note that in the two comparison runs the damping ratio for the model reduction run was increased by about nine times in order to match the same amount of energy loss as in the non-reduction run.

A new model order reduction method for contact dynamics simulations of general robotic systems has been introduced. The method first identifies the stiffness and damping contributions from the contact force model to the robot’s system dynamics by linearizing the model along with the multibody system dynamics. The existing modal analysis and truncation techniques for linear elastic systems are then employed to reduce the model order of the linearized system. A case study of a two-degree-of-freedom flexible manipulator with a single-point contact was presented. The simulation example demonstrated that after applying the model order reduction the simulation speed can be increased by eleven times while the simulation output results still well matches those without the model reduction. This case study, although very simple, demonstrated the benefits of the proposed model reduction method.

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