Dynamics of a Robotics-based Hardware-in-the-Loop Simulator for Verifying Microgravity Contact Dynamics

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ABSTRACT

This paper presents the concept of a cable-manipulator based 6-DOF hardware-in-the-loop (HIL) dynamics simulation system for testing and verification of microgravity contact-dynamics behavior of a space system. It then focuses on the inverse dynamics problem of the 6-DOF cable-driven manipulator which is designed for the simulation system. Accurate modeling and solution of the inverse dynamics is a key requirement for the control and high-fidelity performance of the complex simulation system. The inverse dynamics problem is solved completely under the basic operational conditions of a cable manipulator - all the cables must be always in tension for any possible end-effector motion of the manipulator. It is the first time that a systematic method of determining whether or not the inverse dynamics problem has a solution is proposed with full mathematical proof. Based upon this proven method, two numerical examples are presented to demonstrate the inverse dynamics solution of a 6-DOF cable manipulator. The study results support the feasibility of using such a manipulator for hardware-in-the-loop simulation of microgravity contact-dynamics.

Keywords: hardware-in-the-loop, HIL, dynamic simulation, contact-dynamics, cable manipulator, cable robot.

1. INTRODUCTION

With the increasing activities for planetary exploration and satellite on-orbit servicing, space missions requiring physical contact (including low-speed impact) become more common than ever. A critical step for satellite on-orbit servicing is to successfully dock to and capture the flying satellite to be serviced. Autonomous capture or docking is a very difficult and risky task and therefore, the docking/capture system of a servicing spacecraft has to be thoroughly tested and verified before launching a real mission. Ground-based test and verification of contact-dynamics responses of a spacecraft to arbitrary physical contact in space environment is extremely difficult. The existing test technologies have difficulties to test full 6-DOF microgravity contact-dynamics of large and complex space systems. For examples, the parabolic flight can only mimic 20~30 seconds of flight time inside a limited cargo space, which is insufficient for a complete rendezvous and docking test; the counterweight-balance technology suffers extra inertia effects which becomes significant during impact motion due to large accelerations; the air-bearing based floating test method is a 2D or pseudo 3D system and also suffers to extra inertial burden due to massive supporting frame/structure; the water-based neutral buoyancy technique alters dynamic characteristics because of the water drag on the tested device. Only a robotics-based active gravity compensation system has no limits on the complexity of the space system to be simulated or tested while still retaining full 6-DOF motion condition. Plus, it can use physical contact interfaces to generate contact forces and thus it is more accurate than any mathematical contact-dynamics model. The concept of such a robotics-based, contact-dynamics test facility is illustrated in Fig.1. It consists of three basic parts:

1) a real-time computer simulator to predict the dynamic response of the simulated space system such as a spacecraft based on its dynamics model;
2) a 6-DOF robotic manipulator to physically deliver the computer-generated dynamic motion of the simulated space system; and
3) a physical mockup of the contact interface or environment to allow real physical contact of the simulated system.

The relationship among the three parts is illustrated in Fig.2. In the concept of this HIL simulation system, the dynamics of a spacecraft including the microgravity condition is predicted by computer simulation, because it is very difficult to experimentally produce a full 6-DOF on-orbit dynamic motion on the ground but easy to model and simulate such dynamics on a computer. On the other hand, contact action is represented by real physical contact because such contact is very difficult to model and simulate on a computer.

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A critical requirement for the control of this HIL simulator is that the 6-DOF hardware manipulator in the loop has to exactly mimic the dynamic response of the space system to be simulated during a contact operation. In other words, the dynamics of the 6-DOF cable manipulator should not alter the dynamic response of the simulated space system as it is exhibited at contact interfaces [1]. This requirement will be met using the computed-torque control strategy (one of the nonlinear robotics control techniques). The overall control architecture of the simulation system is shown in Fig.3.

In the diagram, \( \mathbf{x}_s \) and \( \dot{\mathbf{x}}_s \) represent the pose and velocity of the space system (e.g., the pose and velocity of the docking interface of the spacecraft to be simulated); \( \mathbf{x}_r \) and \( \dot{\mathbf{x}}_r \) represent the pose and velocity of the 6-DOF manipulator (e.g.,...
the pose and velocity of the end-effector of the cable manipulator); $\tau_r$ represents the control forces (i.e., the cable forces in this case) of the manipulator; and $f$ represents the contact force and moment resulting from the contact operation (e.g., the docking operation). It is emphasized that $\dot{x}_r$ and $\ddot{x}_r$ are purely a symbolic representations of velocities as opposed to the time derivatives of the poses $x_r$ and $x_{r,s}$, respectively. The control architecture shown in Fig.3 indicates that an accurate dynamics model of the cable manipulator has to be known so that the accurate control forces can be calculated in order to fully compensate the dynamics of the manipulator.

There have been several examples of HIL dynamic simulators for simulating microgravity contact-dynamics. NASA developed an HIL simulator using a Stewart platform for simulating the space shuttle berthing to the International Space Station [2]. The Canadian Space Agency built a SPDM Task Verification Facility (STVF) using a serial-type hydraulic manipulator to simulate SPDM performing contact tasks [1]. German Space Agency (DLR) also developed a similar facility earlier [3]. All the three facilities experienced a common technical difficulty: the manipulator used to deliver dynamic motion of the space system is too complicated so that the manipulator’s own dynamics cannot be accurately predicted and compensated, which, in turn, reduces the fidelity of the facility. To overcome this technical difficulty, a new HIL system has been proposed by the principal author and his colleagues at the New Mexico State University. The manipulator used in this new system is a cable-driven parallel manipulator. Comparing with the hydraulic Stewart platform such as the one used in [2] and other rigid-link serial manipulators such as the one used in [1, 3], the cable-driven manipulator has much simpler dynamics model because: (1) its cable “links” or “legs” have negligible inertias compared to its end-effector/payload and thus, the inertia forces and moments of legs/links can be ignored; (2) the force along each cable is 1-dimensional only while a rigid leg or link must have a 3-dimensional force plus a 3-dimensional moment. As a result, the manipulator’s own dynamics can be accurately predicted and then compensated, which is the most critical requirement for an HIL simulator.

In order to investigate the feasibility and effectiveness of the proposed HIL simulation system, one has to first investigate several major technical problems associated with the cable-driven robotic system. These problems are in the fundamental areas of kinematics, dynamics and controls of cable-driven parallel manipulators. The particular problem addressed in this paper is the inverse dynamics problem of the cable-driven parallel manipulator. The inverse dynamics problem has to be solved under the basic conditions of a cable-driven manipulator, i.e., all the cables must always be in tension within the acceptable motion and force ranges of any contact operations of the space system to be simulated.

The remaining of the paper is organized as follows: the modeling of cable manipulators is presented in Section 2, which is followed by introducing the systematic method of checking force-closure condition in Section 3. Two numerical examples of solving the inverse dynamics problem using the proposed systematic method is addressed in Section 4. The paper is concluded in Section 5.

2. MODELING OF THE CABLE-DRIVEN MANIPULATOR

The six 6-DOF cable-driven manipulator under study along with its potential application in an HIL simulator is illustrated in Fig.1. The end-effector (moving platform) of the manipulator is controlled by seven active cables with their driving actuators mounted to the fixed base. The purpose of using seven cables as opposed to six is to keep the cables always in tension during operation [4]. In fact, the number of cables can be more than seven.

Cable-driven manipulators have several advantages over rigid-link manipulators. First, they can have larger workspaces because their joints can reel out a large amount of cables. Second, all of their actuators and transmission systems can be mounted on the fixed base and thus, they have a higher payload-to-weight ratio, which makes them attractive for high-load and high-speed applications. Third, their special designs make them potentially inexpensive, modular, and very easy to reconfigure. Finally, and also the most importantly (for HIL simulation), they have much simpler dynamics models than their rigid-link counterparts because the inertias of their links (i.e., cables) can be ignored. A number of research works in fundamental areas such as the kinematics, dynamics and controls of cable-driven manipulators have been conducted in the recent years. For example, Tsai first reviewed several basic problems of cable-driven manipulators [5]; Roberts et al. [6] did a comprehensive study of the inverse kinematics problem of cable manipulators; Verhoeven and Hiller [7] studied the tension distribution and workspace problems; Gosselin and Barrette [8] investigated the dynamic workspace of a planar cable-robot; Riechel and Ebert-Uphoff [9] and Fattah and Agrawal [10] also studied workspace and design problems of cable-driven manipulators. Dagkalakis et al. [11] investigated the stiffness issue of a cable-suspended crane used for ship-building. In spite of these reported accomplishments, many research activities for cable-
driven manipulators are still actively going on. Issues regarding different types of cable-driven manipulators including the one proposed in this paper are still to be addressed.

The design of the cable-driven manipulator investigated in this paper is shown in Fig. 1. Compared to the cable-driven manipulators studied in [6-11], this type of manipulator does not have the potential problem of cable interference (due to its special cable arrangement) and, therefore, it can fully take advantage of redundant drive (i.e., having more than necessary number of cables) for performance optimization. This type of cable manipulator has been studied for HIL simulation [12] and for hepatic human-machine interface [13]. Although the authors of [12,13] pointed out that the positive cables forces can be obtained if the columns of the Jacobian matrix satisfying the force-closure condition [14], they did not provide a mathematical proof regarding the relationship between the condition of force-closure and the specific design and end-effector pose of a cable manipulator. In other word, a systematic method of determining whether or not the force-closure condition is satisfied needs to be addressed, which is the topic of this paper. Detailed studies of some fundamental problems of this particular type of cable manipulators regarding the issues in workspace, singularities, dynamic characteristics, and design optimization still require further research.

Figure 4: Kinematics notation (left) and dynamics notation (right) of the 6-DOF cable manipulator

The kinematics notation of a 7-cable manipulator is defined in Fig. 4, where \( q_i, \) for \( i = 1, 2, \ldots, 7, \) is the vector along the \( i \)th cable and has the same length as the cable. The length of the \( i \)th cable is represented by the manipulator’s joint variable \( q_i. \) \( u_i \) is the unit vector along the \( i \)th cable. \( A_i \) and \( B_i \) are the two attaching points of the \( i \)th cable on the base and the end-effector, respectively. The positions of the two attaching points are represented by vectors \( a_i \) and \( b_i, \) respectively. Obviously, \( a_i \) is a constant vector in the base frame \( F_0, \) and \( b_i \) is a constant vector in the end-effector frame \( F_e. \) Based on the kinematics notation, the position of the end-effector (referred to by a reference point \( P \) on the end-effector) can be described as

\[
p = a_i + q_i - b_i \quad \text{for} \quad i = 1, 2, \ldots, 7
\]

from which one has

\[
q_i^2 = (p + b_i - a_i)^T[p + b_i - a_i] \quad \text{for} \quad i = 1, 2, \ldots, 7
\]

Differentiating (2) with respect to time, and then assembling the seven resulting equations into matrix form, one obtains

\[
\dot{q} = J_t \dot{t}
\]

where

\[
\dot{q} = \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \cdots & \dot{q}_7 \end{bmatrix}^T, \quad J_t = \begin{bmatrix} u_1 & u_2 & \cdots & u_7 \\ b_1 \times u_1 & b_2 \times u_2 & \cdots & b_7 \times u_7 \end{bmatrix}^T
\]

\[
\dot{t} = \begin{bmatrix} \dot{p} \\ \omega \end{bmatrix} = \begin{bmatrix} \dot{x} & \dot{y} & \dot{z} & \omega_x & \omega_y & \omega_z \end{bmatrix}^T
\]
where \( \dot{p} \) is the velocity vector of reference point \( P \) and \( \omega \) is the angular velocity vector of the end-effector; and \( t \) represents the twist vector in \( \mathbb{R}^6 \) which includes both the linear and angular velocities of the end-effector. Apparently, \( J \) is the \( 6 \times 6 \) Jacobian matrix corresponding to this cable manipulator.

For this type of manipulators, the inertia of each link is negligible compared to that of the end-effector because the so-called link is just a cable or wire. Therefore, one can ignore the dynamics of the links, which will significantly simplify the dynamics model of the manipulator. Based upon the dynamics notation in Fig.4, one can derive the Newton-Euler equations of motion of the manipulator with respect to the reference point \( P \) on the end-effector as follows

\[
J^T \ddot{f} = \mathbf{M} \ddot{t} + \mathbf{N} \dot{t} - \dot{w}_e - w_g
\]

(6)

The equation can be re-written into a compact form as

\[
A \ddot{f} = w
\]

(7)

where

\[
A = J^T, \quad W = \mathbf{M} \ddot{t} + \mathbf{N} \dot{t} - \dot{w}_e - w_g
\]

(8)

\[
\mathbf{M} = \begin{bmatrix}
m1 & mc^e \\
mc^e & I
\end{bmatrix}, \quad \mathbf{N} = \begin{bmatrix}
0 & -m(w \times e)^T \\
0 & -(mc \times \dot{p} + Iw)^T
\end{bmatrix}
\]

\[
f = [f_1, f_2, \ldots, f_7]^T, \quad w_e = [f_e]^T, \quad w_g = [mg, ec \times mg]
\]

and \( f \) is a 7-D vector consisting of all individual cable forces; \( w_e \) and \( w_g \) are a 6-D external wrench and the gravity wrench exerted on the reference point \( P \) of the end-effector; \( m \) is the mass of the end-effector including any attached payload; \( I \) is the 3×3 inertia tensor of the end-effector about reference point \( P \); \( g \) is the 3-D gravity acceleration vector; \( \dot{p} \) and \( \omega \) are the 3-D linear and angular acceleration vectors of the end-effector at point \( P \); \( f_i \) is the force value of the \( i \)th cable. When the cable is under tension, \( f_i \) is positive. In addition, \( \mathbf{0} \) and \( \mathbf{I} \) are the 3×3 zero matrix and the 3×3 identity matrix, respectively; \( \mathbf{c} \) is the 3-D position vector of the mass center of the end-effector with respect to the reference point \( P \); and \( (\cdot)^\times \) is the operator for vector cross product \((\cdot) \times\).

### 3. METHOD OF CHECKING FORCE-CLOSURE

What is intended in our project is to use the cable-driven manipulator as an output device to mimic the dynamics motion of a spacecraft or space robot. In the HIL simulation system shown in Fig.2, the end-effector of the cable-driven manipulator has to be controlled to follow the dynamic motion of the simulated spacecraft. The model-based nonlinear compensation technique [1] and the computed-torque control technique [15] may be used to improve the dynamic performance of the cable-driven robotic system. This requires determining the driving force in each cable by online solving the inverse dynamics problem from (7). Because the cables cannot provide any compression forces, the inverse dynamics problem should be stated as: solve

\[
A \ddot{f} = w \quad \text{subject to} \quad f > 0
\]

(9)

A few researchers have pointed out that the force feasibility problem of cable-driven manipulators is similar to the frictionless multi-finger rigid-body grasping problem. In the former case, all the cable forces must be in tension while in the latter situation all the finger forces must be in compression. The grasping problem has been extensively studied in 80’s and 90’s of the last century. The most cited approaches for solving the grasping problem is based on the vector-closure and convexity theories [14,16]. Some researchers proposed force/workspace analyses based on the analog between cable-driven manipulators and multi-finger hands [9,13,17]. To avoid the burden of rigorously proving such an analog, we introduce a systematic method of determining whether or not the inverse dynamics problem has a solution, without referring to any techniques specifically developed for the multi-finger grasping problem.

The set of cable forces satisfying dynamics equation (7) can always be partitioned into two parts, namely,

\[
f = \bar{f} + \bar{f}
\]

(10)

where \( \bar{f} \in \mathbb{R}^7 \) and \( \bar{f} \in \mathbb{R}^7 \). Moreover, the first part is assumed to satisfy...
\[ \mathbf{A} \vec{f} = 0 \]  

(11)

which means that \( \vec{f} \) lies in the null space of \( \mathbf{A} \). It follows that the second part must balance the total wrench, namely,

\[ \mathbf{A} \vec{f} = \mathbf{A} \vec{\tilde{f}} = \mathbf{w} \]  

(12)

In general, \( \vec{f} \) is nonzero because there are only 6 equations but 7 unknowns in (11). Physically, such a nonzero \( \vec{f} \) vector represents a set of self-balanced cable forces (i.e., preloading of the cables). If matrix \( \mathbf{A} \) is of full rank (nonsingular), then, the null space solution of (11) can be expressed as \( \vec{f} = \alpha \mathbf{v} \), where \( \mathbf{v} \in \mathbb{R}^7 \) is a basis vector of the null space of \( \mathbf{A} \) and \( \alpha \) is an arbitrary constant. It follows that

\[ \mathbf{f} = \alpha \mathbf{v} + \vec{\tilde{f}} \]  

(13)

Equation (13) indicates that, if all the components of the basis \( \mathbf{v} \) have the same sign, then, for any given \( \vec{\tilde{f}} \) satisfying (12), one can always find an \( \mathbf{f} \) with all positive components by selecting a constant \( \alpha \) with large enough absolute value such that the most negative component of \( \vec{\tilde{f}} \) is cancelled by the corresponding positive component of \( \alpha \mathbf{v} \).

With the above definition and discussion, the inverse dynamics problem can be converted into the problem of finding whether (11) has an all-positive solution or the null space of \( \mathbf{A} \) has an all-positive basis \( \mathbf{v} \). This is exactly the definition of vector-closure as described below [14, 16].

**Theorem 1 (Vector-Closure Theorem):** A set of \( n \) vectors \( \{ \mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_n \} \) in \( \mathbb{R}^{n-1} \) is vector-closure if and only if \( n - 1 \) of the \( n \) vectors are linearly independent and a strictly positive combination of the \( n \) vectors is zero, namely,

\[ \sum_{j=1}^{n} \mathbf{a}_j f_j = 0, \quad f_j > 0 \]  

(14)

This theorem only tells us that, if (11) has an all positive-component solution, the 7 column vectors of matrix \( \mathbf{A} \), denoted by \( \mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_7 \), must form a vector-closure. However, it does not offer a systematical method of checking whether a given matrix \( \mathbf{A} \) satisfies the vector-closure condition. As pointed out by [18], a computationally more appealing theorem for verifying the existence of a vector-closure is as follows.

**Theorem 2:** The necessary and sufficient condition for (11) to have all-positive solution is that the projections of all 7 column vectors of \( \mathbf{A} \) on every direction in \( \mathbb{R}^6 \) do not have the same sign. In other words, for every nonzero \( \mathbf{v} \in \mathbb{R}^6 \), the 7 scalars \( \mathbf{v}^T \mathbf{a}_1, \mathbf{v}^T \mathbf{a}_2, \cdots, \mathbf{v}^T \mathbf{a}_7 \) must include both positive and negative numbers.

Note that the wording of the above theorem has been modified from its original form in [18] for easy application. A proof of this theorem is given in [16]. The theorem tells us that if and only if the set of products \( \mathbf{v}^T \mathbf{a}_i \), for \( i = 1, 2, \cdots, 7 \), have both positive and negative elements for every nonzero \( \mathbf{v} \in \mathbb{R}^6 \), then the \( \mathbf{A} \) matrix satisfies the vector-closure condition. Therefore, the inverse dynamics problem defined by (9) has a solution. Based on this theorem, we propose a systematic method of determining whether a given matrix \( \mathbf{A} \) satisfies the vector-closure condition based on the fact that any vector \( \mathbf{v} \in \mathbb{R}^6 \) can be represented by a linear combination of a set of basis vectors in \( \mathbb{R}^6 \) choosing from the set \( \mathbf{a}_i \) (\( i = 1, 2, \cdots, 7 \)) as long as the \( \mathbf{A} \) matrix (or its transpose – the Jacobian matrix \( \mathbf{J} \)) is nonsingular. The method is described below:

1) Find a set of six linearly independent columns of matrix \( \mathbf{A} \), say, \( \mathbf{a}_1, \mathbf{a}_2, \cdots, \mathbf{a}_6 \).

2) Generate a set of six induced vectors \( \mathbf{v}_j \) (\( j = 1, 2, \cdots, 6 \)) where \( \mathbf{v}_j \) is a nonzero vector orthogonal to the five column vectors \( \mathbf{a}_j \) (\( j = 1, 2, \cdots, 6; \ j \neq i \)).

3) Compute the dot products \( \mathbf{v}_j^T \mathbf{a}_1, \mathbf{v}_j^T \mathbf{a}_2, \cdots, \mathbf{v}_j^T \mathbf{a}_7 \), among which five must be zero due to orthogonality. Put the rest two into a new vector defined as \( \mathbf{b}_j = [b_{j1}, b_{j2}]^T \). Repeat this step for all \( \mathbf{v}_j \) (\( i = 1, 2, \cdots, 6 \)), we get six \( \mathbf{b}_j \)'s.

4) Check the signs of the two elements of each \( \mathbf{b}_j \). If the elements of any \( \mathbf{b}_j \) have the same sign, the necessary and sufficient condition of Theorem 2 is not satisfied and thus, the inverse dynamics problem at the current pose has no solution. Otherwise, the inverse dynamics problem at the current pose has a solution.
The differences of the above-described method from that described in [17, 18] are: 1) only a total of \( n \) \( \mathbf{v} \) vectors in \( R^6 \) need to be formed and checked while the methods in [17, 18] require to check a combination of \( (n+1)n/2 \) vectors instead; 2) no proof is given in both references that by checking only \( (n+1)n/2 \) \( \mathbf{v} \) vectors will sufficiently satisfy Theorem 2, which requires all \( 6n \) \( \mathbf{v} \) vectors to satisfy the sign condition.

The following is a proof that only \( n \) \( \mathbf{v} \) vectors need to be considered where \( n = 6 \) in our case.

Proof of the Necessity:

By definition, the induced vectors \( \mathbf{v}_i (i = 1, 2, \ldots, 6) \) lie in \( R^6 \) and are nonzero. Thus from Theorem 2 we know that the dot products \( \mathbf{v}_i^T \mathbf{a}_i, \mathbf{v}_i^T \mathbf{a}_j, \ldots \) must have different signs. Since vector \( \mathbf{v}_i \), by definition, is orthogonal to the five vectors \( \mathbf{a}_j (j = 1, 2, \ldots, 6; j \neq i) \), we are left only two nonzero items: \( \mathbf{v}_i^T \mathbf{a}_i \) and \( \mathbf{v}_i^T \mathbf{a}_j \). Hence \( \mathbf{v}_i^T \mathbf{a}_i \) and \( \mathbf{v}_i^T \mathbf{a}_j \) must have different signs. Then it is obvious that the two elements of vector \( \mathbf{b}_i = [\mathbf{b}_{u}, \mathbf{b}_{o}]^T \), where \( \mathbf{b}_{u} = \mathbf{v}_i^T \mathbf{a}_i \) and \( \mathbf{b}_{o} = \mathbf{v}_i^T \mathbf{a}_j \) must have different signs. This concludes the proof of the necessary condition.

Proof of the Sufficiency:

Define \( \mathbf{b}_i = [\mathbf{b}_{u}, \mathbf{b}_{o}]^T \), where \( \mathbf{b}_{u} = \mathbf{v}_i^T \mathbf{a}_i \) and \( \mathbf{b}_{o} = \mathbf{v}_i^T \mathbf{a}_j \) for \( i = 1, 2, \ldots, 6 \). Since \( \mathbf{b}_{u} \) and \( \mathbf{b}_{o} \) have different signs, there always exists a positive number \( \beta \) such that \( \mathbf{b}_{o} = -\beta \mathbf{b}_{u} \). Since \( \mathbf{a}_i \) (\( i = 1, 2, \ldots, 6 \)) are linearly independent vectors in \( R^6 \), the induced vector \( \mathbf{v}_i \) can be expressed as:

\[
\mathbf{v}_i = \sum_{j=1}^{6} \alpha_j \mathbf{a}_j \tag{15}
\]

Also, \( \mathbf{v}_i \) is orthogonal to \( \mathbf{a}_j (j = 1, 2, \ldots, 6; j \neq i) \) by definition. Thus \( \mathbf{v}_i \) reduces to \( \mathbf{v}_i = \alpha_i \mathbf{a}_i \) (\( \alpha_i \neq 0 \) since \( \mathbf{v}_i \neq 0 \)). Also,

\[
\begin{bmatrix}
\mathbf{v}_i
\mathbf{v}_2
\mathbf{v}_3
\mathbf{v}_4
\mathbf{v}_5
\mathbf{v}_6
\end{bmatrix}^T = \sum_{i=1}^{6} \alpha_i \mathbf{a}_i \tag{16}
\]

which implies that the induced vectors \( \mathbf{v}_i (i = 1, 2, \ldots, 6) \) are linearly independent vectors in \( R^6 \). Hence, for any vector \( \mathbf{v} \in R^6, \mathbf{v} \neq 0 \), it can be expressed as:

\[
\mathbf{v} = \sum_{i=1}^{6} \gamma_i \mathbf{v}_i \tag{17}
\]

where \( \gamma_i (i = 1, 2, \ldots, 6) \) are arbitrary constants. It follows that

\[
\mathbf{v}^T \mathbf{A} = \begin{bmatrix}
\gamma_1 \beta_1 & \gamma_2 \beta_2 & \gamma_3 \beta_3 & \gamma_4 \beta_4 & \gamma_5 \beta_5 & \gamma_6 \beta_6 & -\sum_{i=1}^{6} \gamma_i \beta_i \mathbf{b}_o
\end{bmatrix} \tag{18}
\]

We select a vector \( \mathbf{d} = [\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6]^T \) whose elements, as defined, are all positive. Left-multiplying this vector by \( \mathbf{v}^T \mathbf{A} \) obviously yields \( \mathbf{v}^T \mathbf{A} \mathbf{d} = 0 \), which implies that the elements of \( \mathbf{v}^T \mathbf{A} \) must have different signs. This completes the proof of the sufficient condition.

4. NUMERICAL EXAMPLES

In this section, the foregoing discussed systematic method of checking force-closure condition is employed to solve the inverse dynamics problem using a 6-DOF 7-cable manipulator like the one shown in Fig.1. The dimensions of the exampled cable manipulator are defined by the positions of the seven fixed nodes \( A_i \) and seven moving nodes \( B_i \) (\( i = 1, 2, \ldots, 7 \)), as listed in Table 1. In the table the left-superscript \( ^0(\cdot) \) indicates that the vector is expressed in the global (fixed) frame \( F_0 \) and \( ^e(\cdot) \) indicates that it is in the end-effector (moving) frame \( F_e \). The mass properties of the end-effector are

\[
m = 10 \text{ kg}, \quad ^e \mathbf{I} = \text{diag}(2.270, 0.069, 2.270) \text{ kg} \cdot \text{m}^2. \tag{19}
\]
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<td>-0.0625</td>
<td>1</td>
<td>-0.1083</td>
</tr>
<tr>
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<td>0.3464</td>
<td>$^e_b_6$</td>
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<td>1</td>
<td>0.1083</td>
</tr>
<tr>
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<td>0.1250</td>
<td>1</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

### 4.1 Inverse Dynamics Example 1

In the first example, the inverse dynamics problem is solved assuming that the end-effector is moving at a constant speed of 0.0314 m/s along a circular trajectory without any rotations with respect to the base frame. Having a radius of 0.15 m, the circular trajectory, as shown in Fig. 5, is located at $(y,z) = (0.45, 0)$ with $x = -0.05$ m. The displacements, velocities and accelerations of the end-effector in y and z directions respectively are shown in Fig. 6. As expected, they are sinusoid functions because the trajectory is circular. The computed cable forces are shown in Figs. 7 and 8. The forces of Cables 1 & 4 changes similarly (will be exactly the same if $x = 0.0$ m) because they are located almost symmetrically with respect to the motion trajectory, so are the forces of cables 2 & 3. Obviously, all the cable forces are positive. The minimum force value of 5 N is an assumed preload of the cables which was pre-defined in the solution procedure. Matlab was used for numerical solutions.
Figure 6: Linear displacements, velocities and accelerations of the end-effector in y and z directions

Figure 7: Computed driving forces for the four lower cables
4.2 Inverse Dynamics Example 2

In the second example, the same manipulator is used but with a different end-effector trajectory. In this example the end-effector performs a pure rotation while the reference point $P$ is kept stationary at its home position, (0, -0.45 m, 0) in the base frame. The angular displacements, velocities and accelerations of the end-effector about the y and z axes are shown in Fig. 9. The computed cable forces are shown in Figs. 10 and 11. Again, the minimum force value of 5 N is an assumed preload of the cables which was pre-defined in the solution procedure. And all the cable forces are positive.

Figure 9: Angular displacements, velocities and accelerations of the end-effector about the y and z axes
5. CONCLUSIONS

This paper presented the concept of a cable-manipulator based 6-DOF hardware-in-the-loop dynamics simulation system for test and verification of microgravity contact-dynamics behavior of a space system. The system has advantage over the existing HIL dynamic simulators because it uses a cable manipulator and cable manipulators have much simpler dynamics models than rigid-link manipulators of similar capabilities. The paper then focused on the solution of inverse dynamics problem of the cable manipulator as it is a critical requirement for the control of such a HIL contact-dynamics
simulator. The inverse dynamics problem is solved completely under the basic conditions of a cable-driven manipulator, i.e., all the cables must be always in tension during the operation of the manipulator. A systematic method of determining whether or not the inverse dynamics problem has a solution is proposed with a full mathematical proof. Based on this proven method, two numerical examples were presented to demonstrate the inverse dynamics solution of a 6-DOF 7-cable manipulator. The study results support the concept of using this type of cable-driven manipulator for HIL contact-dynamics simulations.

ACKNOWLEDGEMENTS

This project was partially supported by the 21st Century Aerospace Cluster of the New Mexico State University.

REFERENCES