Playing with Models and Optimization to Overcome the Tragedy of the Commons in Groundwater

Groundwater is the natural resource most extracted in the world. It supplies 50% of the total potable water requirements, 40% of the industry take, and 20% of agriculture groundwater is a strategic resource for every country. That common-pool resources are highly susceptible to lead to a tragedy of the commons is a well-known fact. We claim that a combination of groundwater modeling, optimization, and a game theoretical analysis may in fact avoid the tragedy. A groundwater model in MODFLOW from Zamora aquifer in Mexico was used as input of a basic but instructive, optimization problem: extract the greatest possible volume of water, but at the same time minimizing the drawdown and drawdown velocity. The solutions of the optimization problem were used to construct the payoffs of a hypothetical game among the aquifer users, the resource's administrator, and a resource protector entity. We show that the success of the optimal management program depends heavily on the information that the users have about the resource. Therefore, better decision-making processes are a consequence of sustainability literacy. Particularly, water literacy could lead to the usage of water considering it as a part of an ecosystem and not only as a natural resource. Additionally, a new non-classical equation for underground flow was derived, that may be specially important to understand and predict the groundwater flow in highly heterogeneous conditions as in karstic aquifers or fractured media. © 2013 Wiley Periodicals, Inc. Complexity 19:9–21, 2013

Key Words: tragedy of the commons; groundwater; optimal management; genetic algorithms; game theory; water literacy

1. INTRODUCTION

1.1. The Nature of the Groundwater Management Problem

Although water resources are considered as renewable, because of their great spatiotemporal variability in terms of anthropogenic exploitation, this is not always true.

Currently, 884 million people do not have access to drinkable water and 2.6 billion people have no access to basic health care. Each year, 1.5 million children...
under the age of 5 die due to illnesses related to water and sanitation. Only nine countries hold 60% of fresh water available for human use. Approximately 8% of water is consumed domestically, 22% is destined to industrial use, and 70% is used in agriculture [1].

As groundwater is the cornerstone of the Green Revolution in Asia, providing about 70% of drinking water in the European Union and maintains almost all rural life of sub-Saharan Africa, its sustainable management has become a crucial topic in securing water availability.

What can we conclude from the previously mentioned data? What is the meaning of these statistics? The answer is indeed simpler than the apparent difficulty it presents. The bottom-line is a question of life. Without water, life is not possible in any aspect or in any possible manifestation. The metabolic processes that are fundamental to life, in its biological dimension, establish the pressing need for water. Taking into consideration the relentless eventual draining and depletion of upper aquifers, the extraction of groundwater is presented as a short and medium-term palliative measure.

It has been argued the importance of groundwater both locally and globally, not only for direct exploitation purposes and in securing food supply, but also from the perspective of social and economic development [2]. Nevertheless, the scarcity problem is not related to the shortage in the amount of water, because due to the global hydrological cycle, the total volume of it on the planet is constant. The real issues are how to overcome natural variability and how to reduce the difficulties of predicting water flow patterns and extractable water volumes, to name but a few [3].

Evidently, any management scenario should be consistent with the physical laws governing the system, which from a mathematical point of view, result in an optimization problem with constraints. These constraints may be imposed by the physical characteristics of the system, the users requirements, or by policies implemented by the authorities who run it [4–7]. In practice, it has been proved that many optimization problems of this type are better resolved by evolutionary computation techniques such as genetic algorithms (GA) that with traditionally or even by Monte Carlo techniques [8–12]. A fundamental advantage of GA versus traditional methods is that GA’s solve discrete, nonconvex, discontinuous, and non-smooth problems successfully [13,14]. Thus, they have become one of the most widely used techniques in water optimization problems. [15–32].

But even if one could resolve all the scientific and technical difficulties related with this kind of optimization problem, it has been recognized that the success of sustainability ultimately rests on cultural and socioeconomic elements [33].

In that sense, although water conflict resolution is highly suitable for a game-theoretic formulation and both GA and game theory have been used in groundwater, almost no application deals directly with management problems [34].

In fact, most decision making in water conflicts are strategic situations in which an individual’s success depends on the choices of others competing agents, the same kind of problem that game theory describes in a precise mathematical formulation [35–37].

1.2. The Tragedy of the Commons in Groundwater

Water in general and groundwater in particular are recognized as common pool resources susceptible of evolving to the tragedy of the commons. Common pool resources (CPRs) include natural and human-constructed resources in which (i) exclusion of beneficiaries through physical and institutional means is especially costly, and (ii) exploitation by one user reduces resource availability for others [38]. The central idea of the tragedy of the commons as introduced by Hardin [39] was that in any situation where a CPR is under unsupervised exploitation by a group of users, every user has the incentive to exploit the resource at a level that is collectively inefficient. This outcome is based on the assumption that each user is playing a prisoner’s dilemma game, a game in which Nash equilibrium is not Pareto optimal.

Prominent scholars have criticized Hardins among them the Nobel laureate Elinor Ostrom. She argues that there are many empirical examples of successful community management. This is supported by a reasonable number of laboratory experiments in which people are not only rational, narrowly self-interested maximizers as Hardin proposed but instead there are many people that actually do contribute significantly to the common well-being. Thus, there are not only self-interested rational maximizers, but also conditional cooperators and willing punisher. In fact, several strategies have been developed [40–42]. In this work, we propose that the use of Quantum Game Theory may overcome the tragedy as in this wider framework, the classically noncooperative game of the prisoner’s dilemma (prototype of Hardin’s Tragedy of the Commons) exhibits a new Nash equilibrium that is at the same time a Pareto optimal solution.

Traditionally prisoner’s dilemma is presented by a payoffs matrix as in Table 1. Each prisoner must choose to defect (D) the other or to cooperate (C). How should the prisoners act?

In this game, as in most game theory, the only concern of each individual player (prisoner) is maximizing his own payoff, without any concern for the other player’s payoff.

In the classic form of this game, cooperating is strictly dominated by
defecting, so the only possible Nash equilibrium for the game is defect for all players. No matter what the other player does, one player will always gain a greater payoff by defecting. Because in any situation defecting is more beneficial than cooperating, all rational players will play with this strategy.

A prisoner’s dilemma groundwater conflict structure is presented in [34] where two farmers tap a shared aquifer over a long period of time. The payoffs for each farmer are computed as revenues from crop sales minus pumping costs. Each player must choose between the cooperative and noncooperative pumping rates. If both farmers pump at the lower rate, the groundwater level will not drop and the farmers can enjoy long-term low pumping costs. However, both farmers pumping at the higher rate reduces groundwater levels, increases pumping costs, and reduces profit, eventually making pumping economically infeasible, and ending irrigation and profit. Cooperative pumping increases profit for both farmers. Getting “free ride” (D,C or C,D), letting others contribute and benefit from their contributions without paying oneself, would be the best outcome for each farmer. In that case, one farmer pumps at the higher rate while the other one has committed to pump at a lower rate. The free rider gains the highest payoff in this situation due to pumping costs lower than the case in which both farmers pump at the noncooperative rate and higher crop sale revenues than the cases in which he decides to cooperate. On the other hand, choosing a cooperative strategy while the other farmer is willing to cooperate results in the lowest payoff due to high pumping costs and low crop revenues.

As pointed out, with the prisoner’s dilemma structure, Nash equilibrium is a defective strategy for all players. This situation would lead to the eventual tragedy of the commons [39].

In classical game theory, a game \( G=(N, S, E) \) can be defined as set of \( N \) players, a set of strategies \( s=\{s_1, \ldots, s_N\} \), and a set of payoff functions \( E=\{E_1, \ldots, E_N\} \). The \( s_j \) is the set of strategies available to the \( j \)-th player, meanwhile \( E_j \) is the payoff function for the \( j \)-th player. A payoff function \( E_j \) for a player is a mapping from the cross-product of player’s strategy spaces to the player’s set of payoffs. \( E \) assigns a real number to the pair \((s_i, s_j)\). This number \( E(s_i, s_j) \) is the payoff obtained by a player who plays the strategy \( s_i \) against an opponent who plays the strategy \( s_j \). An action or a move is a choice available to a player during some moment in a game, while a strategy is a complete plan of actions for every stage of the game. A strategy space for a player is the set of all strategies available to the player. Strategies are considered as pure if they specify a unique move in a given game position. If strategies are given as a probability distribution over \( S \) which corresponds to how frequently each move is chosen, then they are considered as mixed. In classical game theory, strategies can be either pure or mixed, but can they be entangled?

Quantum entanglement, also called the quantum nonlocal connection, is the property of certain states of a quantum system, of being distributed in more than a single object. The information describing this objects is inextricably linked such that performing a measurement on one immediately alters properties of the other, even when separated at arbitrary distances.

In 1935, responding to Niels Bohr’s advocacy that quantum mechanics as a theory was complete, Einstein, Podolsky, and Rosen formulated the EPR paradox [43]. This paradox considers entangled pairs of particles which go off in different directions, separated by light-years. Nevertheless, a measurement of one particle will instantly affect the state of the other particle.

To clarify the EPR paradox, Bell [44] formulated a set of inequalities that would distinguish experimentally whether quantum mechanics was incomplete, or whether physics is nonlocal. Recent experiments seem to confirm the existence of entangled particles [45] and even more, there are several examples of physical phenomena that exhibit probabilistic entanglement instead of quantum physics entanglement [46–52]. In fact, one may understand quantum mechanics mathematical formulation as a nonclassic probabilistic theory that may be applied to other system beside quantum physics [53,54]. Then, entanglement and other phenomena considered as an exclusive property of quantum system, could be reformulated in probabilistic terms only.

Interestingly enough one of the mayor authors of game theory development, von Neumann, was also one of the pioneers in quantum mechanics mathematical foundations. In this work, we will give some feeling as to the nature of quantum mechanics and some of the mathematical formalisms needed to work with quantum game theory [55,56].

The quantum version of the prisoner’s dilemma [57] considers that the player may use quantum strategies represented by operators as Hadamard and Pauli’s, which may be used only when entanglement is present. The new payoffs for the game are presented in Table 2 [55,56].

The outcome \((1, 1)\) is no longer a Nash equilibrium, but we have a new one at \((3, 3)\) corresponding to quantum
\((\sigma_x, \sigma_z)\) strategies. This new Nash equilibrium takes the prisoner's dilemma to a happy end as it is also a Pareto optimal.

Of course, the possible existence of this quantum strategy does not imply they in fact exist. Fortunately for quantum game theoreticians, they have been physical experimental realization of the prisoner’s dilemma using a two qubit NMR quantum computer [58,59]. Additionally, recent work [60] shows experimental evidence that people, without training in quantum physics, plays a simple quantum game effectively. Furthermore, their observations are consistent with the game theory predictions that the quantum version of the prisoner’s dilemma is more efficient, in a Pareto sense, than the classical version. When extended to larger groups, the level of cooperation in the quantum game increased as predicted. This is evidence that subjects were responding to the strategic considerations of the quantum game.

### 1.3. Entanglement in Groundwater

The underground flow process may be conceptualized as the movement of agents (water particles) in heterogeneous media [61–63]. It has been showed [64] that when the media is very heterogeneous, an agent movement presents a 1/f power spectrum, which may be generated, an agent movement presents a 1/f power spectrum, which may be generated, and in a Pareto sense, than the classical version. When extended to larger groups, the level of cooperation in the quantum game increased as predicted.

**TABLE 2**

<table>
<thead>
<tr>
<th></th>
<th>Bob 1</th>
<th>Bob (\sigma_x)</th>
<th>Bob (H)</th>
<th>Bob (\sigma_z)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alice (1)</td>
<td>(3,3)</td>
<td>(0,5)</td>
<td>(1,3)</td>
<td>(1,1)</td>
</tr>
<tr>
<td>Alice (\sigma_x)</td>
<td>(5,0)</td>
<td>(1,1)</td>
<td>(1,3)</td>
<td>(0,5)</td>
</tr>
<tr>
<td>Alice (H)</td>
<td>(3,5)</td>
<td>(1,1)</td>
<td>(2,4, 2,4)</td>
<td>(1,1, 4)</td>
</tr>
<tr>
<td>Alice (\sigma_z)</td>
<td>(1,1)</td>
<td>(5,0)</td>
<td>(4,1, 2)</td>
<td>(3,3)</td>
</tr>
</tbody>
</table>

The probability of finding an agent in an arbitrary node \((\Delta x, \Delta y)\) at the next time \(k\Delta t = (k + 1)\Delta t\) is

\[
P(\Delta x, \Delta y, k\Delta t) = P(\Delta x, \Delta y, k_0\Delta t)
\]

\[
= e^{\|\nabla\| (\Delta y)_{j+1}}
\]

\[
e^{\|\nabla\| (\Delta y)_{j-1}}
\]

\[
e^{\|\nabla\| (\Delta y)_{j+1}}
\]

\[
e^{\|\nabla\| (\Delta y)_{j-1}}
\]

If we define \(\delta P_{ij}\) = \(P(\Delta x, (j+1)\Delta y, k\Delta t) - P(\Delta x, j\Delta y, k\Delta t)\), and so for the rest of directions, then Eq. (2) became

\[
P(\Delta x, \Delta y, k\Delta t) = P(\Delta x, j\Delta y, k_0\Delta t) + e^{\|\nabla\| (\Delta y)_{j+1}} P_{ij} \delta P_{ij} + e^{\|\nabla\| (\Delta y)_{j-1}} P_{ij} \delta P_{ij} + e^{\|\nabla\| (\Delta y)_{j+1}} P_{ij} \delta P_{ij} + e^{\|\nabla\| (\Delta y)_{j-1}} P_{ij} \delta P_{ij}.
\]

Which is the discrete form of the anisotropic diffusion equation [67]

\[
\frac{\partial P(x, y, t)}{\partial t} = -\nabla^2 \|\nabla\| P(x, y, t)
\]

The discrete anisotropic diffusion Eq. (4) could be rewritten as [68]

\[
P_{a+1} = P_a + \frac{1}{\eta_a} \sum_{b \neq \eta_a} e^{\partial P_{ab} \delta P_{ab}}
\]

with \(a\) the actual position, \(\eta_a\) the neighboring position of \(a, \eta_a\) the number of first neighbors of the agent in \(a, \lambda \in R^+\) a constant that defines the diffusion rate.

Of course, the functional form of \(\delta P_{ij}\) could be a more generic one as

\[
\delta P_{ij} = e^{\|\nabla\| (\Delta y)_{j} x, y, t}
\]

in which case the general anisotropic diffusion equation is obtained

\[
\frac{\partial P(x, y, t)}{\partial t} = \text{div} [e(x, y, t) \nabla P(x, y, t)] = \nabla e
\]

Even more, one could extend this analysis to a \(n\) players game and use the evolutionary theory of Nowak. In that context, if we relate \(e\) with a particular game with payoff matrix \(A\), and then \(\delta P_{ij}\) is the fraction of agents in the \((i, j)\) position that adopt strategy \(h\) at time \(t\). The corresponding replication equation is then

\[
\delta P_{ij} = e^{\|\nabla\| (\Delta y)_{j} x, y, t, \delta h}
\]

where \(f_h\), \(\phi_h\) are the fitness and mean fitness of strategy \(h\).

As an example, let be the matrix \(A\) with elements \(a_{hbc}\) the payoff of a Prisoner’s dilemma game given in Table 3, then the corresponding fitness are
\[ h = C \Rightarrow f_C = (R)_{ij}^k \chi_{ij}^k + (S)_{ij}^k \chi_{ij}^k, \quad (10) \]

\[ h = D \Rightarrow f_D = (T)_{ij}^k \chi_{ij}^k + (P)_{ij}^k \chi_{ij}^k. \]

And the mean fitness is

\[ \phi = \sum_k f_{ij}^k \chi_{ij}^k \chi_{ij}^k + \ldots + \sum_k f_{ij}^k \chi_{ij}^k \chi_{ij}^k + \ldots + \sum_k f_{ij}^k \chi_{ij}^k \chi_{ij}^k + \ldots + \sum_k f_{ij}^k \chi_{ij}^k \chi_{ij}^k. \]

Taking into account Eqs. (10) and (11) and that \( e_{ij}^k - e_{ij}^k \) is the discrete form of the time derivative, the replicator equation can be rewritten [69] in matrix form as

\[ \frac{dE}{dt} = [\Lambda(t), E(t)]. \]

Where \( E, \Lambda \) are two matrix with elements

\[ E_{hh} = (e_{hh}^k)^{1/2} \]

and

\[ \Lambda_{hh} = \frac{1}{2} \left[ \left( \sum_{k=1}^n a_{hh}^k e_{hh}^k \right) (e_{hh}^k)^{1/2} - (e_{hh}^k)^{1/2} \left( \sum_{k=1}^n a_{hh}^k e_{hh}^k \right) \right], \]

and the square brackets [ ] in Eq. (12), stands for commutation operation. It has been demonstrated that quantum game theory is a generalization of game theory an that the replicator Eq. (12) is equivalent to von Neumann equation

\[ i\hbar \frac{dp}{dt} = [H, p] \]

Where \( p \) is the density matrix, a self-adjoint (or Hermitian) positive-semidefinite matrix of trace one, that describes the statistical state of a quantum system. The \( H \) operator is the Hamiltonian of the system. The equivalence between the replicator and von Neumann equations are given by [69]

\[ E \leftrightarrow \rho, \Lambda \leftrightarrow -\frac{i}{\hbar} H. \]

Via the master equation, it can be demonstrated [70–72] that the von Neumann equation leads to a Fokker–Planck equation of the form

\[ \frac{\partial e(x, y, t)}{\partial t} = -\text{div}[D_1(x, y, t)e(x, y, t)] + \nabla^2[D_2(x, y, t)e(x, y, t)] \]

where \( D_1 \) and \( D_2 \) are traditionally associated with drift and diffusion.

In this game theory, context \( D_1(x, y, t) \) is associated with the fitness \( f(x, y, t) \) [Eq. (10)] and \( D_2(x, y, t) \) with the mean fitness \( \phi(x, y, t) \) [Eq. (11)].

The master equation is a first-order differential equation that describe the time evolution of the probability of the system to be in a particular set of states. Typically, the master equation is given by

\[ \frac{d\hat{P}}{dt} = \Lambda(t)\hat{P} \]

where \( \hat{P} \) is a column vector of the states \( i \), and \( \Lambda(t) \) is the matrix of connections. Many physical problems in classical, quantum mechanics, and other sciences can be expressed in terms of a master equation. Examples of these are the Lindblad equation in quantum mechanics and as we aforementioned, the Fokker–Planck equation which describes the time evolution of a continuous probability distribution. For more hydrological applications of the master equation, the reader may refer to [73].

Then, we can finally enunciate the discrete spatially extended game in continuum terms. The probability of finding an agent (water particle) in the position \((x, y)\) at the time \( t \) is given by

\[ \frac{\partial P(x, y, t)}{\partial t} = \text{div}[e(x, y, t)\nabla P]. \]

Where \( e(x, y, t) \) is the strategy (microphysics of the flow process) that the player in \((x, y)\) plays at the time \( t \) and that obeys the equation

\[ \frac{\partial e(x, y, t)}{\partial t} = -\text{div}[D_1(x, y, t)e(x, y, t)] + \nabla^2[D_2(x, y, t)e(x, y, t)] \]

This new groundwater flow equations resembles to classical one

\[ \frac{\partial h}{\partial t} = \text{div}[K \nabla h] \]

with the fundamental difference that our equations allows mixed stated, probabilistic entanglement, and there for quantum strategies.

From the hydrogeological empirical perspective, one of the sources of
groundwater entanglement is well interference. Whenever a well is pumping water from a groundwater system, a cone of depression occurs. In general, this is caused by a reduction in the head of the water well, which may lead to an actual depression of the water levels (drawdown) for unconfined aquifers. Because water flows from high to low water levels or pressure, a gradient appears producing a flow from the surrounding aquifer into the well.

The drawdown $s(r, t)$ at a distance $r$ from a fully penetrating well at time $t$ after the beginning of pumping at a constant rate $Q$ from a confined aquifer with transmissivity $T$ and storage constant $S$ is given by the Thies’s equation [74]:

$$ s(r, t) = \frac{Q}{4\pi T} \int_0^\infty \frac{e^{-z}}{z} dz - \frac{Q}{4\pi T} W(u) $$

$$ W(u) = \gamma - \ln u + u - \frac{u^2}{2(2!)} + \frac{u^3}{2(3!)} + \ldots + \frac{(-1)^{n+1} u^n}{n(n!)} $$

$$ u = \frac{r^2 S}{4Tt} $$

From a direct inspection of Thies’s equation, one can see that drawdown provoked by well pumping is a monotone function so the effects of the drawdown cone will, in principle, spread throughout the aquifer.

Furthermore, as in general groundwater, flow goes in the direction of the hydraulic head gradient (Darcy’s Law) and this changes by the presence of a well, then one may formulate a probabilistic uncertainty principle. Let be two physical observables $a,b$ represented by $A,B$ operators which are noncommutable $[A,B] \neq 0$. Then, it will not be possible to prepare an ensemble of all measurement systems in a state $\langle \Psi \rangle$ where the standard deviations of $a,b$ measures do not satisfy the condition:

$$ \Delta \Psi A \Delta \Psi B \geq \frac{1}{2} \langle \Psi | [A,B] | \Psi \rangle \quad (23) $$

where $\Delta \Psi X = \sqrt{\langle X^2 \rangle - \langle X \rangle^2}$ is the uncertainty taken as the standard deviation of the measurement of $|\Psi \rangle$ in the ensemble; and $[A,B] = AB - BA$ is the commutation operator. From this, it should be intuitive that hydraulic head and water velocity may satisfy the probabilistic uncertainty principle of Eq 23.

2. HOW TO OVERCAME THE TRAGEDY: A CASE STUDY

We have shown heuristically that groundwater is a probabilistic entangled system. In entangled systems, quantum strategies are available for players. Recent studies show that people do play quantum games [60] and if quantum strategies are available, then even in traditional noncooperative games as the prisoner’s dilemma, ultimate cooperation may emerge.

But even when groundwater is entangled, a water management game is made of groundwater system coupled to different players, the users of the resource. We think that when a player makes an observation of the groundwater system to make a decision about its exploitation, a decoherence operator enter in action that may or may not maintain the original entanglement. We pose that this operator that works over the coupled groundwater-user system during an observation should be related with the player perception of the resource.

Most countries have norms for well construction that define a minimum distance between neighbors wells to avoid interference. We have shown using Thies’s equation that some interference will always exist. However, the magnitude of this interference (probabilistic entanglement) for wells constructed following the regulations is small enough for normal users to make local decisions, choosing defec-

![Table 3](https://example.com/table3.png)

**Table 3**

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<td>R</td>
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<td>T</td>
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as a cultivated and fertile terrain [78], is a major crop-producer and livestock area; and the richest territory throughout the state. The most important crops cultivated here are: strawberry, tomato, potato, onion, wheat, beans, forage sorghum, oats, and corn [79]. These economic factors and the complex geomorphology of the site makes of the Zamora aquifer an important site for the study of groundwater.

We coupled [80] a MODFLOW [81] model of the aquifer with a genetic algorithm optimization tool [82] and a post optimization analysis with game theory [83]. The basic but instructive optimization problem consisted in: extract the greatest possible volume of water, but at the same time minimizing the drawdown and drawdown velocity.

After 1000 generations, the genetic algorithm optimization converges to four different types of optimal solutions (Figure 3). The extreme right bar corresponds to a privileged extraction type of solution, whereas the extreme left bar matches up a solution that favors conservation. The two bars in the middle correspond to intermediate behavior where extraction and conservation are in equilibrium. All values are standardized to the unity to be interpreted directly as payoffs.

The solutions of the optimization problem were used to construct the payoffs of a hypothetical game among the aquifer users, the resource’s administrator, and a resource protection entity (Figure 4).

The first move in the aquifer game is made by the administrator (admin), who must decide whether a management plan must be implemented or not. If admin decides to take no action, an arbitrary use of the aquifer occurs and user can decide whether to continue with its exploitation unaf- fectedly or to make an undefined change. In the first case, payoffs are calculated from original water extraction rates data, in the second, no payoff can be predicted because of the obvious uncertainty. In case admin decides to implement a management plan, user could choose to cooperate or not. This is represented as a chance move with a probability of occurrence called convincing index. Convincing index is the probability of a user decides to cooperate given that an optimal management has been presented to him. This index is a measure of how successfully
was the administrator to present the management plan. If user does not cooperate, as in the preceding case, two options would be feasible: to pursue with the usual exploitation of the aquifer or to make an undefined change. Supposing user cooperates, then four kinds of calculated optimal solutions are available for selection. Finally, a chance move takes place with a probability called confidence index. This confidence index as the

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**FIGURE 3**

[Graph showing histograms for the total maximum extraction objective function (left) and drawdown minimum objective function (right). Four modes are identified with their corresponding payoff values, where one corresponds to the best payoff and zero to worst.]

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**FIGURE 4**

[Graph representing the decision-making process for the problem where conflicted interests compete. The order in the payoffs is admin, user, and aquifer. Color red (left) stands for admin, blue (up) for user and black (down) for chance. In the second part of the extended game, the order in the payoffs is admin, user and aquifer. Red color (middle-up) stands for admin, blue (middle-down) for user and black (left and right-down) for chance.]

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**FIGURE 3**

Histograms for the total maximum extraction objective function (left) and drawdown minimum objective function (right). Four modes are identified with their corresponding payoff values, where one corresponds to the best payoff and zero to worst.
name implies, try to capture the fact that even when a user decide to cooperate, he might have doubts about the veracity of the payoffs presented.

Game theory analysis for the problem shows the existence of two different Nash equilibria. The first one consists in admin choosing not to plan and user maintaining his current use of aquifer; in the second one, admin chooses to planning. When user decide to cooperate, he must choose the optimal solution number three, and when he do not cooperate then he must opt to carry on with the current use of the aquifer. After repeated game realization covering the whole range of convincing and confidence indexes, we found that whenever they are small, the two types of Nash equilibria are present. If any of these indexes is greater than 0.5, then only the second type of Nash equilibria is found. Bearing these results in mind, an information index \( I \) is proposed as the product of the convincing and confidence indexes. This global index would represent the quality of the project’s information, the way in which it has been presented, and the degree of confidence that society has on the proponent and authorities. In this way, the emergence of cooperation can be evaluated as a function of information.

In the context of this new framework based on quantum game theory, the information index \( I \) may be reinterpreted as a quantum decoherence operator that would be operating in any observation of the groundwater system.

3. DISCUSSION AND CONCLUSIONS

The SMORN system provides a set of Pareto optimal solutions that converge to four different types of optimal solution. The first one privileges extraction, the second one conservation, and the others favor intermediate behavior where extraction and conservation are in equilibrium. With almost every of this interchangeable Pareto solutions, more extractable volumes with less aquifer drawdown are achieved. This is accomplished by modifying the well pumping volumes distribution. It is clear that we should incorporate a new ecological objective function into the system that served to prevent the occurrence of undesired solutions. Game theory analysis seems to be very useful in conflict scenarios, but it only provides one possible solution. Conversely, GA techniques which produces a set of Pareto interchangeable optimal solutions, offers the possibility of choosing. It represents a great opportunity for negotiation when some users, perceiving some of their interests as affected by some of the optimal solutions, appear reluctant to cooperate. Besides, to increase users cooperation more precise parameters could be introduced. Different values range could be imposed to urban, farming, and industrial wells to take into account particular restrictions and user interests; and a time discretization that reflected the required water volumes for each agriculture cycle could be also used. These improvements would allow a wider and more accurate objective functions.

Many natural resources, and particularly groundwater basins, are managed as common property, known in modern economics as CPRs. In other words, many users have access to the resource in question. As the consumption of the natural resource progresses, conflict emerges and management plans need to be devised. This competition for the CPRs give rise to what has been called the tragedy of the commons; that is, individuals tend to increase selfish behavior when faced with shortages of any natural resource or public good. Unfortunately, that uncooperative behavior plunges into an accelerated degradation of the resource. In Hardin’s words, “The ruin is the destination toward which all men rush, each pursuing his own interest in a society that believes in freedom of the commonwealth" [84]. However, as suggested by Elinor Ostrom, Harding’s theory is not always true. Being a version of prisoner dilemma [85], the validity of The tragedy of the commons depends on the fact that strategies of the players are statics. In this way, “... not all natural resource users are unable to change its restrictions, while individuals are seen as prisoners, the policy prescriptions shall refer to this metaphor (The tragedy of the commons). I therefore prefer to address the issue of how to improve the skills of participants to change the rules of the game coercive to achieve results different from the relentless tragedies.” [86]. In Ostrom’s line of thought, we introduced an Information index pretending to change user’s strategies and make cooperation possible.

This information index takes into account the quality of the information presented to society by the authorities, as well as the way in which it was presented and the confidence over the proponent. We found that the only way in which cooperation could be ensured was to provide an information index big enough to increase users trust, which we think can be achieved by considering the following key points: (a) a scientific approach to environmental problems and the use of computer simulation promotes cooperation. Still an adequate translation must be accomplished to make this scientific knowledge accessible to all participants; (b) Because the proponent’s credibility is part of the information index, mechanisms for tracking their reputation like the aforementioned are highly recommended.

We also conceptualized the groundwater flow process as a spatially extended game in a heterogeneous media and using quantum game theory, we obtain nonclassical equations that allow mixed states, probabilistic entanglement, and therefore quantum strategies with the subsequent resolution of the tragedy (Prisoner’s dilemma game). This new
capabilities may be specially important to understand and predict the groundwater flow in highly heterogeneous conditions as in karstic aquifers or fractured media and may be applied even for nonconventional petroleum flow problems which are of a great scientific and economical interest.

From a macroscopic perspective, we use a quantum game theory approach to resolve the classic problem of the tragedy of the commons in a study case. The study site, Zamora aquifer, is in the Duero River's basin in the northwest of Michoacan state (Mexico). A groundwater model in MODFLOW was used as input of a basic but instructive optimization problem: extract the greatest possible volume of water, but at the same time minimizing the drawdown and drawdown velocity. Optimization converges to four different types of optimal solution: the first one corresponds to an extraction privileged type of solution; the second, privileges the aquifer conservation and the two others offer an intermediate solution where extraction and conservation are in equilibrium. The solutions of the optimization problem were used to construct the payoffs of a hypothetical game among the users of the aquifer, the administrator of the resource, and a resource protection entity. Game theory analysis showed that the success in cooperation depends heavily on the information that the users have about the resource. This result is interpreted as the action of a quantum decoherence operator (the information index $I$) over the entangled groundwater system. This operator was constructed as the product of the convincing and confidence indices. The information index reflects the grade of knowledge the users have about groundwater dynamics and optimal management plan. Depending of this, users may have a nonlocal (entangled) or local (nonentangled) perspective of the resource and then preexisting entanglement in the system may be maintained or lost. Whenever information index is big enough (strong entanglement), quantum strategies are available and then the tragedy of the commons may be avoided.

We consider that our results suggest that a conceptual leap must be done in groundwater management. We should not talk any more about aquifer dynamics or exploitation policies by themselves. Physical and social systems are coupled in a fundamental way (through the decoherence operator), and a sociohydrogeological theory should be developed. Some institutions, as the Global Institute for Water Security from Saskatchewan University (Canada), have a social-hydrology research program. Nevertheless, they have an interdisciplinary vision of the problem, meanwhile we propose a complete change in the subject of study. The new basic unity of analysis should be an indivisible pair (users, aquifer), which resembles to the contextual formulation of quantum mechanics [47]. At this regard, science in general and hydrogeology in particular are being challenged by the paradigm shift of multidisciplinarity, interdisciplinarity, and transdisciplinarity. In hydrogeology, this is leading to more interdisciplinary research and applications, and to more complexity at the coupling interfaces of related disciplines [87]. It has been recognized that hydrogeology emphasizes the societal importance of water resources and is expected that this science be able to solve the problems of supply, security, and management of this resource. In all this problems, the interdisciplinary approach will most likely needed [88–90]. But even when this is true, it might be not sufficient to face the complexity of the actual problems in hydrogeology. It is possible that a whole new theory of groundwater be needed. In this sense, Tartakovsky and Winter [91] suggest that the classic deterministic statement of groundwater theory is incomplete by necessity. For this reasons, we are now working in a contextual statistical theory of groundwater that will be presented in future work.

Finally, contrary to World Bank's [6] guidelines and traditional approach [92,93], we propose that the administrator role must not always be represented by an authority, but can also be taken by user organizations. This kind of groundwater user organization already exists in México under the name of COTAs, and we plan to transfer SMORN technology and knowledge to them. With this purpose, we designed SMORN as a highly modular, end-user-oriented free software. At the moment, we are in the phase of making a graphic user interface in GTK that allows the use of any MODFLOW model and resolves the multiobjective management problem described in this work. In a later version, it will be possible to select from different kinds of management problems. The source codes of the system, as well as the binary files and documentation, will be available on the research group web site.

The fundamental information related to water that the population needs to know to take into consideration the central and immediate implications, as a consequence of certain decisions made by the experts, feeds the discussion with a healthy and productive debate that derives from a consensual resolution. Decisions must therefore, take on a democratizing nature in order for society to feel it has been taken into account and has been actively involved in the decision-making process, in this particular case, of a technological nature. It is also clear that the necessity of educating the population through the appropriate means to be able to translate technical information to common usage, the common citizen can take into account the relevance of water as a fundamental element of the ecosystem, and then be able to make a decision in consensus with the specialists and legislators on how to use and administer water.
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