A Multi-Criteria Approach for Ranking Schedules for Multi-Mode Projects

Utkan Eryılmaz¹, Öncü Hazır², and Klaus Werner Schmidt²

¹Department of Business Administration, TED University, Ankara
e-mail: {utkan.eryilmaz;oncu.hazir}@tedu.edu.tr
²Department of Mechatronics Engineering, Çankaya University, Ankara
e-mail: schmidt@cankaya.edu.tr

Keywords: Scheduling, Robustness, Multi-Criteria Decision Analysis, Data Envelopment Analysis

1. Introduction

In this study, we investigate a multi-mode project scheduling problem that considers a single non-renewable resource (money), namely, the discrete time/cost trade-off problem (DTCTP). The deadline version of the problem assigns modes to each activity so that the total cost is minimized while respecting the project deadline; whereas the budget version minimizes the project duration given a constraint on total spending. Both versions have application areas in practice as they model the time/cost relationship in processing activities. Small and medium size problems could be solved by exact methods (Demeulemeester et.al. 1996, 1998); whereas approximate solution methods are needed for large size problems (See Hazır et al. 2010a, for a comprehensive literature review and the problem formulation).

Combining the two versions of the DTCTP, we propose to construct non-dominated time/cost solutions and rank them regarding robustness. We define robust solutions as the schedules which are insensitive regarding deviations in activity durations. To evaluate robustness, we propose a new measure in section 2.2. Even if cost and completion time are the most widely investigated criteria in academic literature; in real life, developing robust schedules becomes critical to hedge against uncertain events (Hazır et al. 2010b).

To rank the solutions we use super-efficiency models (Andersen and Petersen 1993) of data envelopment analysis (DEA) [Charnes, Cooper and Rhodes (CCR), 1978]. DEA models do not require defining explicit weights for criteria, which are difficult to obtain in real life and may not be uniform in the feasibility region. In the literature, there are multi-objective resource constrained project scheduling studies. Viena and Sousa (2000) applied metaheuristics to optimize project completion time, mean weighted lateness and resource availability. Balestin and Blanco (2011) obtained the set of non-dominated solutions based on completion time and tardiness. They also proposed measures to express quality of these sets and compare the techniques based on these measures. Differently, we focus on ranking. To the best of our knowledge, this study is the first multi-criteria optimization research that integrates robustness in multi-mode project scheduling.

2. Methodology

We develop a two-stage algorithm. First, we obtain a set of approximately efficient solutions and then order them based on total cost, completion time and robustness.

2.1. Approximately Non-Dominated Solutions

Hapke et al. (1998) used the term approximately non-dominated solutions to describe the set obtained through their heuristic search. We use that term for schedules in \( (B, C_{max}) \); where \( \varepsilon_b \) and \( \varepsilon_c \) represent tolerance levels.

According to our definition, for an approximately efficient solution \((B, C_{max})\) there exist no feasible solutions with \(\{ (B, C_{max}'), C_{max}' < C_{max} - \varepsilon_c \} \) and \(\{ (B', C_{max}), B' < B - \varepsilon_b \} \). To generate solutions to the DTCTP, we use direct CPLEX modeling and Benders Decomposition for larger instances. Setting the optimality gap around 2% and truncating the branch & bound, solving the problems iteratively produces high quality solutions quickly (Hazır 2010a).
2.2. Robustness Measure

To measure robustness, we use total slack ($TS_i$), the amount of time by which the activity completion can be delayed without delaying the project completion time, based measures. For DTCTP, several slack based functions have been already formulated and tested using simulation (Hazir et al. 2010b). We formulate a new robustness measure (RM), an exponential utility function that uses the ratio of slack ($TS_i$) to activity duration ($p_i$), and represents the average value over n activities. Figure 1 shows that when $TS_i$ is equal to the activity duration (SDR=1), the contribution of additional slack becomes minimal. Hence, to maximize RM, considering the durations, slacks should be distributed evenly among activities.

$$RM = \frac{\sum_{i=1}^{n} 1 - e^{-2 SDR_i}}{n}$$

$$SDR_i = \frac{TS_i}{p_i}$$

![Utility Function](image)

2.3. DEA as a Multi Criteria Ranking Tool for Alternatives

DEA is used for ranking decision making units that include multiple inputs and/or outputs. Every unit maximizes its own ranking based on most favorable weights for inputs and outputs for itself rather than fixed values (CCR, 1978). DEA has been applied to evaluate the performance of different types of organizations in different sectors such as education, health care, banking and services. Efficiency of a unit is calculated by dividing the aggregated value of all outputs ($y_kj$; $j$th output of the $k$th unit) to the aggregated value of inputs ($x_ks$; $s$th input of the $k$th unit). None of the units will have efficiency value greater than 1. The DEA CCR model is formulated below:

$$\max h_k = \frac{\sum_{j=1}^{J} v_j y_{kj}}{\sum_{s=1}^{S} u_s x_{ks}} \quad (1)$$

subject to:

$$\sum_{j=1}^{J} v_j = 1; \quad i = 1,...,I \quad (2)$$

$$v_j, u_s \geq 0; \quad j = 1,...,J, \quad s = 1,...,S \quad (3)$$

For each unit $k = 1,...,I$, this model is solved maximizing the efficiency ($h_k$). Input ($x_{ks}$) and output values ($y_{kj}$) are assumed to be known. Decision variables of the model are the weights for $S$ inputs ($u_s$) and $J$ outputs ($v_j$). Note that the model can easily be converted to a linear program (LP) by equating the denominator of the objective function to unity.

The interactive approach of Belton and Vickers (1999) was one of the first studies using DEA as an MCDM tool. Stewart (1996) analyzed the correspondence between the efficiency definition and distance to the Pareto frontier. Joro et al. (1998) showed the connection between DEA and LP by formulating DEA as a reference point model. Eryilmaz and Karasakal (2006) combined DEA with outranking methods and ranked MBA programs using published data.

We use the super efficiency approach to obtain distinct scores for efficient units. In the original DEA all efficient units are scored 1 and cannot be differentiated. However, super efficiency score is calculated using the radial distance of the current unit to the efficiency frontier excluding itself; the constraint is valid for $i \neq k$ (Eq.4); the unit under evaluation can get a score greater than 1.

$$\frac{\sum_{j=1}^{J} v_j y_{ij}}{\sum_{s=1}^{S} u_s x_{is}} \leq 1; \quad i \neq k, i = 1,...,I \quad (4)$$

DEA serves as a practical approach for selection among schedules: First, there is a non-linear and complex relation among duration, cost and robustness which is difficult to formulate and solve analytically. Moreover, marginal improvement in robustness (utility) is a decreasing function of
slacks, which depend on the schedule (mode selection). Secondly, there is positive correlation between input and output variables: given the budget (completion time), robustness could be increased by increasing the completion time (budget). In both cases, some of the activities would include more slacks hence flexibility. However inputs and outputs cannot be easily measured by a common concrete unit (such as money) and therefore cannot be aggregated.

2.4. Algorithm for Ranking the Non-Dominated Schedules

**Stage 1: Generating the set of approximately non-dominated schedules**

We use the data set of Akkan et al. (2005). Given the budget, solutions to the DTCTP (budget version) instances are solved. In each iteration, for a given budget, 10 feasible solutions within an optimality gap [0.25-2.5 %] and the optimal one are recorded. This process is replicated for budget values above [10%, 20%] the minimal total cost, with discrete steps of 0.5 %). Therefore, 231 (21*11) schedules with associated duration and cost ($B_k$, $C_{max}^k$) are recorded.

**Stage 2: Ranking the Schedules using DEA**

In this stage, the $RM$ is calculated and integrated as output score for each schedule generated in stage 1. Cost and project completion time are taken as input values and robustness as output (for the $k^{th}$ schedule, $x_{ij} = B^k$ and $x_{i2} = C_{max}^k$, and $y_i = RM^k$). The super-efficiency method is then suitable to present the decision maker schedules that have distinct properties compared to other solutions. We use the tool EMS tool [Scheel, 2000] for calculations.

The first 10 ranked solutions for a problem instance with 136 activities (complexity index, CI=14, coefficient of network complexity, CNC=8, concave cost function and between 2 and 10 modes per activity) are given in Table 1. The efficient solutions have diverse characteristics: the 4th schedule has a favorable budget value, whereas the completion time for 1st, 3rd places are balanced in both criteria. Even the 2nd schedule is worse than the 1st schedule regarding the completion time, it is significantly better in budget and slightly in robustness. Figure 2a illustrates the time/cost trade-off and the generated schedules. The efficient ones are highlighted as red in Figure 2b and 2c. Note that as the budget increases, we can generate left shifted schedules, which results in smaller slacks regarding project completion time and hence the RM decreases.

<table>
<thead>
<tr>
<th>Rank</th>
<th>B</th>
<th>$C_{max}$</th>
<th>RM</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>29817</td>
<td>523</td>
<td>0.657</td>
<td>104.35</td>
</tr>
<tr>
<td>2</td>
<td>23022</td>
<td>590</td>
<td>0.686</td>
<td>101.41</td>
</tr>
<tr>
<td>3</td>
<td>17386</td>
<td>704</td>
<td>0.727</td>
<td>100.86</td>
</tr>
<tr>
<td>4</td>
<td>15928</td>
<td>741</td>
<td>0.730</td>
<td>100.69</td>
</tr>
<tr>
<td>5</td>
<td>19568</td>
<td>654</td>
<td>0.710</td>
<td>100.47</td>
</tr>
<tr>
<td>6</td>
<td>19498</td>
<td>651</td>
<td>0.704</td>
<td>99.53</td>
</tr>
<tr>
<td>7</td>
<td>15902</td>
<td>739</td>
<td>0.724</td>
<td>99.39</td>
</tr>
<tr>
<td>8</td>
<td>20333</td>
<td>639</td>
<td>0.697</td>
<td>99.04</td>
</tr>
<tr>
<td>9</td>
<td>16441</td>
<td>723</td>
<td>0.718</td>
<td>98.94</td>
</tr>
<tr>
<td>10</td>
<td>17392</td>
<td>702</td>
<td>0.717</td>
<td>98.92</td>
</tr>
</tbody>
</table>

Figure 2a. Budget vs Completion Time  Figure 2b. Robustness Measure vs. Completion Time  Figure 2c. Robustness Measure vs. Budget
3. Conclusion

In this research, time/cost relations in project scheduling are investigated and robust solutions are sought. An algorithmic basis for a decision support system (DSS) that support project managers is under development. Using the algorithm, a set of high quality project schedules are determined and ranked using DEA analysis. Therefore project managers can concentrate on only a few alternative schedules that will be the basis in planning to complete projects within time and cost targets. To validate the effectiveness and efficiency of our approach, extensive computational experiments and statistical analysis will be performed in future work.

Acknowledgements

This study was supported by The Scientific and Technological Research Council of Turkey (TUBITAK) under grant SOBAG 113K245.

References