Effect of fading, diversity and interference on the performance of multitone CDMA systems

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Summary
The performance of a multitone Code Division Multiple Access (CDMA) system is investigated in this paper by considering the effect of multipath fading, diversity and narrowband interference. The performance is evaluated in terms of the average bit error rate when both multiple access and partial band interference are present. Results are obtained by deriving the statistics of the receiver output parameters and then tested by numerical evaluation. The bit error rate performance is analyzed as a function of bit rate, bandwidth, delay range and different diversity techniques, while considering issues relevant to system implementation. Comparisons were made between conventional single-tone CDMA systems and multitone CDMA systems. The effectiveness of multitone CDMA in suppressing interference compared to conventional CDMA system is analyzed. Results reveal considerable performance improvement of multitone CDMA system over conventional CDMA system.

KEY WORDS
multitone CDMA
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1. Introduction

In mobile communication, multipath fading, multiple access interference and partial band interference limit system performance and capacity. These limitations can be mitigated by means of diversity where the system takes advantage of the multipath channel, or by the means of Direct Sequence Code Division Multiple Access (DS-CDMA) spread spectrum. The use of Spread Spectrum (SS) or CDMA has long been debated. DS-CDMA applications include cellular, microcellular, indoor and satellite communication systems [1–3]. The potential benefits of CDMA include the ability to reduce Multiple Access Interference (MAI), high immunity against multipath distortion, increased system capacity and high flexibility. Although conventional or Single-Tone CDMA (ST-CDMA) systems are currently in use in practical applications, the services they provide are still limited to voice and low data rates. Hence, improvement in system performance is always under investigation. One potential improvement is the combination of CDMA techniques and multicarrier modulation.

One implementation of multicarrier CDMA technique is based on transmitting data by dividing the high data-rate stream into several low data-rate substreams, and using these substreams to modulate different subcarriers. By using a large number of subcarriers, it is possible to achieve high immunity against multipath dispersion. The duration of each substream will be much higher than the channel time dispersion, such that the effect of Intersymbol Interference (ISI) is minimized. Multicarrier CDMA have recently received widespread interest owing to their potential for high-speed transmission, their effectiveness in mitigating the effects of multipath fading and their interference rejection capabilities [4–6]. Multicarrier CDMA schemes have the benefit of bandwidth efficiency, frequency diversity, low-cost signal processing and interference rejection capability in high data-rate applications [7]. The combination of multicarrier modulation with SS will allow one to benefit from the advantages of both schemes.

Different multicarrier techniques have been presented and studied by various researchers [4–15]. In Reference [8], each symbol of each user is first spread using a unique spreading code, and then orthogonal disjoint carriers modulate multiple copies of the spread signal. In this way, multicarrier transmission is used as a way of achieving frequency diversity. In Reference [9], source data stream is serial-to-parallel (S/P) converted to a number of lower-rate data streams and each stream feeds a number of parallel streams with the same rate. The bits of the later streams are then spread by the same Pseudorandom-Noise (PN) spreading code sequence and they modulate orthogonal carriers with successively overlapping bandwidth. However, this model seems to be complicated. In another approach [10], similar to that in Reference [8], an algorithm was proposed to select the best subchannel with respect to the fading process for each substream. In Reference [11], there is no S/P conversion, and each data symbol (the number of which is equal to the length of the chip sequence) is transmitted in several narrowband subcarriers. Before modulation each data bit is spread by only one chip of the total sequence. In References [12] and [13], the performance of the multicarrier system was analyzed when convolutional coded substreams are used. Illustrating the effect of fading correlation between the various substreams for the multicarrier scheme was studied in References [14] and [15].

Another variant of multicarrier modulation proposed in Reference [16] is denoted as multitone CDMA (MT-CDMA). In Reference [16], the data bits are S/P converted and parallel data modulates the orthogonal carriers. The carriers are separated by the inverse of the paralleled data period and are spread by a user specified PN code. Unlike Reference [16], this paper will investigate system performance in the presence of multipath fading, diversity and Partial Band Interference (PBI).

This paper investigates the performance of the multicarrier system in the presence of PBI, for a channel with various Multipath Intensity Profile (MIP). The Bit Error Rate (BER) performance is analyzed in terms of signal to interference ratio, intercarrier interference, number of active users and the effect of diversity. The effectiveness of MT-CDMA in suppressing narrowband interference compared to ST-CDMA is analyzed. It is shown that MT-CDMA systems outperform ST-CDMA systems in the presence of PBI and can therefore be favorably used for CDMA/FDMA (frequency division multiple access) or CDMA/TDMA (time division multiple access) overlay without the need for sophisticated adaptive notch filtering.

In the next section, we present the description of the system model and provide explanations for our assumptions. In Section 3, the performance statistics of the decision variable is derived. The average probabilities of error for several diversity schemes are given in Section 4. Numerical results are presented.
and discussed in Section 5, while concluding remarks are given in Section 6.

2. System Description

The transmitter for the $k$th user is shown in Figure 1, where $b_k$ is the data bits and $c_k$ is the PN sequence. Data streams from the encoder having period $T/S$ are S/P converted to $S$ substreams with period $T$. The data stream of each substream modulates a particular tone. Thus, if the carrier frequency of the 0th tone is $f_0$, then the $x$th tone has carrier frequency

$$f_x = f_0 + \frac{x}{T}, \quad x = 0, 1, 2, \ldots, S - 1$$

Each of the tones is multiplied with each user’s unique code sequence having chip duration $T_c = T/N$, where $N$ is the length of the code. Thus, the transmitted signal for the $k$th user is given by

$$s_k(t) = \sqrt{2P} \sum_{x=0}^{S-1} c_k(t) b_{x,k}(t) \cos(2\pi f_x t)$$  \hspace{1cm} (1)

where $P$ is the signal power and $b_{x,k}(t)$ is the $k$th user’s symbol waveform for the $x$th carrier. The spectrum associated with the different tones overlap in frequency as shown in Figure 2. It is assumed that the tones are orthogonal and hence spectrum overlapping is inconsequential.

We assume a channel with impulse response

$$h(t) = \sum_{l=0}^{L-1} A_{kl} e^{j\phi_l} \delta(t - \tau_l)$$  \hspace{1cm} (2)

where the different tones fade independently. The subscript $kl$ indicates the $k$th user’s $l$th path. The path gains $A_{kl}$ are Nakagami-distributed, while the phase delay $\phi_l$ and the path delay $\tau_l$ are uniformly distributed between $[0, \pi]$ and $[0, T/S]$, respectively. In addition to fading, a zero-mean Gaussian noise $n(t)$ with two-sided power spectral density $N_0/2$, and PBI jammer $J(t)$, are added to the signal. The PBI signal has period $T_j$ and power $J$, and is given by

$$J(t) = \sqrt{2J} j(t) \cos[2\pi(f_0 + \Delta)t + v]$$  \hspace{1cm} (3)

![Fig. 1. Transmitter for MT-CDMA system for BPSK modulation.](image1)

![Fig. 2. MT-CDMA and ST-CDMA spectrum with partial band interference.](image2)
where \( \nu \) denotes the phase, and \( \Delta \) is the offset of the interference center frequency from the 0th tone’s carrier frequency.

The total received signal corrupted by noise and PBI is given by

\[
r(t) = \sqrt{2P} \sum_{k=0}^{K-1} \sum_{l=0}^{K-1} A_{kl} c_k(t - \tau_{kl}) b_{x,k}(t - \tau_{kl})
+ \sqrt{2P} \sum_{l=0}^{L-1} A_{0l} c_0(t - \tau_{0l}) b_{y,0}(t - \tau_{0l})
+ \sqrt{2P} \sum_{l=0}^{L-1} \sum_{n=0, n \neq y}^{L-1} A_{nl} c_l(t - \tau_{0l}) b_{x,0}(t - \tau_{0l})
+ \sqrt{2P} \sum_{l=0}^{L-1} \sum_{n=0, n \neq y}^{L-1} A_{nl} c_l(t - \tau_{0l}) b_{x,k}(t - \tau_{0l})
+ \sqrt{2P} \sum_{l=0}^{L-1} \sum_{n=0}^{L-1} A_{nl} c_l(t - \tau_{0l}) b_{x,k}(t - \tau_{0l})
+ \cos[2\pi f_s (t - \tau_{0l}) + \phi_{0l}]
+ \left[ \sum_{l=0}^{L-1} \sum_{n=0}^{L-1} \sum_{k=0}^{K-1} A_{kl} c_k(t - \tau_{kl}) b_{x,k}(t - \tau_{kl}) \right]
+ \cos[2\pi f_s (t - \tau_{0l}) + \phi_{0l}] + n(t) + J(t)
\]  

We have assumed perfect power control for each user. The received signal is despread using a user-specific code sequence, followed by correlation with a locally generated subcarrier over one bit interval. In the receiver shown in Figure 3, \( M \) out of \( L \) paths are correlated and then combined with a diversity combiner. It is assumed that the amplitude and phase remain constant within one bit interval. The received signal due to symbol transmitted by the carrier \( \nu \), of the reference user \( 0 \), in the reference path \( 0 \) is given by

\[
r(t) = \sqrt{2P} A_{00} c_0(t - \tau_{00}) b_{x,0}(t - \tau_{00})
+ \cos[2\pi f_s (t - \tau_{00}) + \phi_{00}]
+ \sqrt{2P} \sum_{l=0}^{L-1} A_{0l} c_0(t - \tau_{0l}) b_{y,0}(t - \tau_{0l})
+ \cos[2\pi f_s (t - \tau_{0l}) + \phi_{0l}]
+ \sqrt{2P} \sum_{l=0}^{L-1} \sum_{n=0, n \neq y}^{L-1} A_{nl} c_l(t - \tau_{0l}) b_{x,0}(t - \tau_{0l})
+ \cos[2\pi f_s (t - \tau_{0l}) + \phi_{0l}]
+ \sqrt{2P} \sum_{l=0}^{L-1} \sum_{n=0, n \neq y}^{L-1} A_{nl} c_l(t - \tau_{0l}) b_{x,k}(t - \tau_{0l})
+ \cos[2\pi f_s (t - \tau_{0l}) + \phi_{0l}]
+ n(t) + J(t)
\]

\[
U_{x,0} = \int_0^T r(t) c_0(t - \tau_{00}) \cos[2\pi f_s (t - \tau_{00}) + \phi_{00}] dt.
\]

3. System Performance Analysis

The received signal enters the 5 groups of correlators and in each group there are \( M \) correlators to detect \( L \) paths per carrier. The decision variable at the receiver output is given by

\[
U_{x,0} = \int_0^T r(t) c_0(t - \tau_{00}) \cos[2\pi f_s (t - \tau_{00}) + \phi_{00}] dt.
\]

where \( \cos[2\pi f_s (t - \tau_{00}) + \phi_{00}] \) is the recovered carrier and \( c_0(t - \tau_{00}) \) is the spreading sequence of the

\[
R_{x,y,k}(\tau_{kl}) = \int_0^T c_k(t - \tau_{kl} + T) c_0(t) \times f \left[ \frac{2\pi (x - y)t}{T} \right] dt
\]

\[
\tilde{R}_{x,y,k}(\tau_{kl}) = \int_{\tau_{ty}}^T c_k(t - \tau_{kl}) c_0(t) \times f \left[ \frac{2\pi (x - y)t}{T} \right] dt
\]

where \( f[] \) may be a cosine or a sine depending on the value of \( p \). When \( x = y \), one obtains

\[
R_{xy,k}(\tau_{kl}) = \int_{0}^{t_{l}} c_{k}(t - \tau_{kl} + T)c_{0}(t) \, dt
\]

\[
\hat{R}_{xy,k}(\tau_{kl}) = \int_{0}^{t_{l}} c_{k}(t - \tau_{kl})c_{0}(t) \, dt
\]

\[
R_{xy,k}^s(\tau_{kl}) = \hat{R}_{xy,k}(\tau_{kl}) = 0
\]

with the superscript \( c \) and \( s \) denoting sine and cosine. Also, let

\[
G_{xy,k}(pq) = \sum_{l=0}^{L-1} A_{kl} g(\theta_{kl}) R_{xy,k}^{p}(\tau_{kl})
\]

and

\[
\hat{G}_{xy,k}(pq) = \sum_{l=0}^{L-1} A_{kl} g(\theta_{kl}) \hat{R}_{xy,k}^{p}(\tau_{kl})
\]

The first argument of \( G \) indicates the use of sine or cosine inside the partial correlation function, while the second argument indicates whether \( g(.) = \cos(.) \), or \( g(.) = \sin(.) \). This arises for the case when \( q = c \) or \( q = s \), respectively. Using these notations, the in-phase decision variable consisting of the desired signal \( S_{d} \), noise and interference terms may be written as

\[
U_{y,0} = S_{d} + U_{1} + U_{2} + U_{3} + \eta(t) + N_{j}(t) \quad (9)
\]

where

\[
S_{d} = \sqrt{P} A_{00} T c_{0,0}
\]

\[
U_{1} = \sqrt{P} \left[ b_{y,0}^{-1} G_{y,0}(cc) + b_{y,0}^{0} \hat{G}_{y,0}^{'}(cc) \right]
\]

\[
U_{2} = \sqrt{P} \frac{s-1}{2} \sum_{k=0, k \neq y}^{s-1} b_{y,k}^{-1} \left[ G_{y,k}(cc) - G_{y,k}(ss) \right] + b_{y,0}^{0} \left[ \hat{G}_{y,0}(cc) - \hat{G}_{y,0}(ss) \right]
\]

\[
U_{3} = \sqrt{P} \frac{K-1}{2} \sum_{k=0}^{K-1} \sum_{l=1}^{S-1} b_{y,k}^{l} \left[ G_{y,k}(cc) - G_{y,k}(ss) \right] + b_{y,0}^{0} \left[ \hat{G}_{y,0}(cc) - \hat{G}_{y,0}(ss) \right]
\]

In Equation (9), \( U_{1} \) is the interference component when \( K = 0 \) and \( x = y \), which arises because of the multipath effect of the channel for the desired user’s desired tone. \( G^{'} \) and \( \hat{G}^{'} \) are similar to \( G \) and \( \hat{G} \), respectively, with summation from \( l = 1 \) to \( (L - 1) \) instead of \( l = 0 \) to \( (L - 1) \). \( U_{2} \) is the interference component when \( K = 0 \) and \( x \neq y \), which arises because of the loss of orthogonality of other tones with respect to the desired tone of the same desired user. This is primarily caused by multipath propagation through the channel. The \( U_{1} \) term is the MAI component.

To find the BER performance, we examine the variances of the noise and interference terms. The variance of the noise terms is given as \( \sigma_{n}^{2} = N_{0}T/4 \) \[17\]. For the \( U_{1} \) term, the variance can be written as

\[
\sigma_{U_{1}}^{2} = \text{var} \left\{ \sqrt{P} \left[ b_{y,0}^{-1} G_{y,0}(cc) + b_{y,0}^{0} \hat{G}_{y,0}^{'}(cc) \right] \right\}
\]

\[
= \text{var} \left\{ \sqrt{P} \left[ b_{y,0}^{-1} \left( \sum_{l=1}^{L-1} A_{0l} \cos(\theta_{0l}) R_{xy,0}(\tau_{0l}) \right) \right] + b_{y,0}^{0} \left( \sum_{l=1}^{L-1} A_{0l} \cos(\theta_{0l}) \hat{R}_{xy,0}(\tau_{0l}) \right) \right\}
\]

\[
= \frac{P}{2} \left\{ \frac{1}{2} \sum_{l=1}^{L-1} \text{var}[A_{0l}] \text{var}[R_{xy,0}(\tau_{0l})] + \frac{1}{2} \sum_{l=1}^{L-1} \text{var}[A_{0l}] \text{var}[\hat{R}_{xy,0}(\tau_{0l})] \right\}
\]

\[
= \frac{P}{2} \left\{ \text{var}[R_{xy,0}(\tau_{0l})] \sum_{l=1}^{L-1} \text{var}[A_{0l}] \right\} \quad (10)
\]

given that \( \text{var}[\hat{R}_{xy,0}(\tau_{0l})] = \text{var}[\hat{R}_{xy,0}(\tau_{0l})] \) \[16\]. If \( N_{1} \) is the number of chips per data bit before S/P conversion (i.e. \( N_{1} = T/ST_{c} \)), then

\[
\text{var}[R_{xy,0}(\tau_{0l})] = \frac{T_{c}(N_{1}T_{c})}{3} = \frac{T_{c}}{3} \left( \frac{T}{S} \right)
\]

\[
= \frac{T^{2}}{3S} \left( \frac{1}{T/T_{c}} \right) = \frac{T^{2}}{3SN}
\]

The value of the term \( \sum_{l=1}^{L-1} \text{var}[A_{0l}] \) depends on the assumed MIP. If we consider exponential MIP for the path gains, the strength of the \( l \)th path related to the first path is given by \( A_{0l} = A_{00}e^{-\delta l}, \delta \geq 0 \) where \( \delta \) is the decay factor. Thus

\[
\sum_{l=1}^{L-1} \text{var}[A_{0l}] = \sum_{l=1}^{L-1} \text{var}[A_{00}]e^{-\delta l} = \Omega \left( \sum_{l=0}^{L-1} e^{-\delta l} - 1 \right)
\]

\[
= \Omega[Q(L, \delta) - 1]
\]

where $\Omega$ is the second moment of $A_0$, and

$$ Q(L, \delta) = \frac{L-1}{L} e^{-\delta L} = \frac{1 - e^{-\delta L}}{1 - e^{-\delta}} $$

Hence, the variance of $U_1$ is given by

$$ \sigma^2_{U_1} = \frac{P}{2} \left\{ \frac{T^2 \Omega\{Q(L, \delta) - 1\}}{3SN} \right\} $$

(11)

Although the above derivation of $Q(L, \delta)$ uses exponential profile, other profiles can be used. For example, one can use Gaussian profile as well as uniform profile or any other appropriate profile.

The variance of $U_2$ may be written as

$$ \sigma^2_{U_2} = \text{var}\left\{ \frac{P}{2} \sum_{x=0, \neq y}^{S-1} b_{xy}^{-1} [G_{xy,0}(cc) - G_{xy,0}(ss)] + b_{0,0}^0 [G_{xy,0}(cc) - G_{xy,0}(ss)] \right\} $$

$$ = \frac{P}{2} \sum_{x=0, \neq y}^{S-1} \text{var}\left\{ b_{xy}^{-1} \left[ \sum_{l=0}^{L-1} A_{0l} \cos(\theta_{0l}^r) R_{xy,0}(\tau_{0l}) \right. \right. $$

$$ \left. \left. - \sum_{l=0}^{L-1} A_{0l} \sin(\theta_{0l}^r) R_{xy,0}(\tau_{0l}) \right] + b_{0,0}^0 \left[ \sum_{l=0}^{L-1} A_{0l} \cos(\theta_{0l}^r) R_{xy,0}(\tau_{0l}) \right. \right. $$

$$ \left. \left. - \sum_{l=0}^{L-1} A_{0l} \sin(\theta_{0l}^r) R_{xy,0}(\tau_{0l}) \right] \right\} $$

$$ = \frac{P}{2} \sum_{x=0, \neq y}^{S-1} \sum_{l=0}^{L-1} \text{var}[A_{0l}] \text{var}[R_{xy,0}(\tau_{0l})] $$

$$ + \sum_{l=0}^{L-1} \text{var}[A_{0l}] \text{var}[R_{xy,0}(\tau_{0l})] $$

$$ + \sum_{l=0}^{L-1} \text{var}[A_{0l}] \text{var}[R_{xy,0}(\tau_{0l})] $$

$$ + \sum_{l=0}^{L-1} \text{var}[A_{0l}] \text{var}[R_{xy,0}(\tau_{0l})] $$

$$ = \frac{P}{2} \sum_{x=0, \neq y}^{S-1} \Omega Q(L, \delta) \left[ E\left[ (R_{xy,0}^r)^2 \right] + E\left[ (R_{xy,0}^s)^2 \right] \right] $$

(12)

with $E[(R_{xy,0}^r)^2] = E[(R_{xy,0}^s)^2]$ shown in Appendix A.

The variance of the MAI $U_3$ can be derived accordingly. The variance will contain a term similar to Equation (11) for the case when tone $x = y$, and another term similar to Equation (12) for the case when $x \neq y$. Hence, the variance of $U_3$ can be written as

$$ \sigma^2_{U_3} = \frac{P}{2} \left\{ \frac{T^2 \Omega\{Q(L, \delta) - 1\}}{3SN} + \sum_{x=0, \neq y}^{S-1} \Omega Q(L, \delta) \right\} $$

$$ \times \left\{ E[(R_{xy,0}^r)^2] + E[(R_{xy,0}^s)^2] \right\} $$

(13)

At the output of the correlator, the narrowband interference signal is given by

$$ N_j(t) = \int_0^T \sqrt{2J} j(t) \cos[2\pi(f_0 + \Delta)t + \psi] $$

$$ \times c_0(t) \cos(2\pi f_1 t) dt $$

Taking only the envelope will lead to

$$ N_j(t) = \sqrt{\frac{J}{2}} \int_0^T j(t)c_0(t) \cos[2\pi(\Delta - \frac{\psi}{T})t + \psi] $$

(14)

Assuming that the PBI is zero-mean, independent and identically distributed (IID), then the variance becomes

$$ \sigma^2_{N_j} = \text{var}[N_j(t)] = E[N_j^2(t)] $$

$$ = E\left\{ \frac{J}{2} \int_0^T \int_0^T j(t_1)j(t_2)c_0(t_1)c_0(t_2) \right\} $$

$$ \times \cos \left[ 2\pi \left( \Delta - \frac{\psi}{T} \right) t_1 + \phi \right] $$

$$ \times \cos \left[ 2\pi \left( \Delta - \frac{\psi}{T} \right) t_2 + \phi \right] $$

(15)

where $\rho_j(t)$ and $\phi_j(t)$ are the autocorrelation functions of $J(t)$ and $c_0(t)$, respectively. The autocorrelation function is a nonzero triangular function given by [18]

$$ \rho_T(t) = \begin{cases} 1 - \frac{|t|}{T}, & |t| \leq T \\ \phi_0, & |t| > T \end{cases} $$

(16)

The autocorrelation function of $J(t)$ or $c_0(t)$ can be obtained from Equation (16) by replacing $T$ by $T_e$. 

or \( T_j \), respectively. We can manipulate the double integral in Equation (15) by domain transformation. Let

\[
\sigma_{N_j}^2 = \frac{J}{2} \int_{-T}^{T} \rho_j(\mu) \rho_j(\mu) \mu d\mu \int_{|\mu|}^{2T-|\mu|} f(\mu) d\mu \int_{|\mu|}^{2T-|\mu|} f(\mu) d\mu
\]

with \( f(\cdot) \) representing the function within the parentheses in Equation (15). By variables transformation, let \( \mu = t_1 - t_2 \) and \( r = t_1 + t_2 \), such that the Jacobian of the transformation is 2. To determine the new limits, consider Figure 4. When \( t_1 = T \) and \( t_2 = T \), \( r = 2T \) and when \( t_1 = -T \) and \( t_2 = -T \), \( r = 0 \). Hence, the limits of \( r \) are changed from 0 to \( 2T \). Similarly, we can determine the limits of \( \mu \). Now at any point say \( A, r = 2T - \mu \). However, \( r \) cannot be less than \( \mu \) since \( r \) is the sum of \( t_1 + t_2 \), which can be at least equal to \( t_1 - t_2 \), but not less. Thus,

\[
\sigma_{N_j}^2 = \frac{1}{2} \int_{-T}^{T} f(\mu) d\mu \int_{|\mu|}^{2T-|\mu|} f(\mu) (2T-|\mu|) d\mu
\]

\[
= \frac{1}{2} \int_{-T}^{T} f(\mu) (T - |\mu|) d\mu
\]

Equation (15) can be written as

\[
\sigma_{N_j}^2 = \frac{J}{2} \int_{-T}^{T} \rho_j(\mu) \rho_j(\mu) \mu d\mu \int_{|\mu|}^{2T-|\mu|} f(\mu) d\mu \int_{|\mu|}^{2T-|\mu|} f(\mu) d\mu
\]

with \( f(\cdot) \) representing the function within the parentheses in Equation (15). By variables transformation, let \( \mu = t_1 - t_2 \) and \( r = t_1 + t_2 \), such that the Jacobian of the transformation is 2. To determine the new limits, consider Figure 4. When \( t_1 = T \) and \( t_2 = T \), \( r = 2T \) and when \( t_1 = -T \) and \( t_2 = -T \), \( r = 0 \). Hence, the limits of \( r \) are changed from 0 to \( 2T \). Similarly, we can determine the limits of \( \mu \). Now at any point say \( A, r = 2T - \mu \). However, \( r \) cannot be less than \( \mu \) since \( r \) is the sum of \( t_1 + t_2 \), which can be at least equal to \( t_1 - t_2 \), but not less. Thus,

\[
\sigma_{N_j}^2 = \frac{1}{2} \int_{-T}^{T} f(\mu) d\mu \int_{|\mu|}^{2T-|\mu|} f(\mu) (2T-|\mu|) d\mu
\]

\[
= \int_{-T}^{T} f(\mu) (T - |\mu|) d\mu
\]

For \( \mu/T_c = z \), \( d\mu = T_c dz \), and notice that when \( \mu \to \pm T \), \( z \to \pm T/T_c = \pm N \). Therefore, \( |\mu|/T_c = (|\mu|/T_c)/(T_c/T_c) = |z|/T_c = |z|/N \), and \( |\mu|/T_j = (|\mu|/T_j)/(T_j/T_j) = |z|/s \). Since \( s = T_c/T_j = (T/T_j)/(T_c/T_c) = (T/T_j)(1/(T/T_c)) = u/N \), then \( |\mu|/T_j = |z|u/N \), where \( u = T/T_j = (1/T_j) \times [S/(S/T)] = (B_j/B_T)S = (B_j/B_T) \). \( B_j \) is the bandwidth of the narrowband signal, \( B_T \) is the bandwidth of one tone of the MT-CDMA signal and \( B_T \) is the total bandwidth of MT-CDMA system. Substituting values in Equation (19) will give

\[
\sigma_{N_j}^2 = \frac{J T T_c}{2} \int_{-N}^{N} \left(1 - \frac{|\mu|}{N}\right) (1 - |\mu|) \cos \left[ 2\pi \left( \frac{\Delta - \frac{\mu}{N}}{T_c} \right) z T_c \right] d\mu
\]

\[
= \frac{J T^2}{2N} \left( \int_{-N}^{N} \left(1 - \frac{|\mu|}{N}\right) (1 - |\mu|) \right) \left(1 - \frac{|\mu|}{N}\right) \right] d\mu
\]

\[
= \frac{P_T T^2 \lambda}{N} \left( \frac{J}{T_c} \right)
\]

where \( \lambda \) is the integration within the bracket and \( S = P T \) is the average received power per symbol. The total variance can now be written as

\[
\sigma_{\nu,0}^2 = \frac{E_T}{4} \left\{ \frac{2 \lambda}{N} \left| \frac{J}{T_c} \right| + \frac{2 \lambda}{N} \left| \frac{J}{T_c} \right| + \frac{2 \lambda}{N} \left| \frac{J}{T_c} \right| \sum_{x=0}^{S-1} E \left[ (R_{x,y})^2 \right] \right\}
\]

\[
= E \left[ (R_{x,y})^2 \right]
\]

where \( E_x = PT \) is the average received energy per bit. For an AWGN channel, the decision variable is given by

\[
\gamma = \frac{E \left[ U_{y,0} \right]}{\sqrt{2 \sigma_{\nu,0}^2}} = \sqrt{\frac{T}{2} A_{00} T}{\sqrt{2 \sigma_{\nu,0}^2}}
\]

Thus, the probability of error conditioned on the fading amplitude \( A_{00} \) is given by

\[
P_e(A_{00}) = \frac{1}{2} \text{erf} c \left[ \gamma \right]
\]

This conditional probability is removed by averaging \( P_e(A_{00}) \) over the probability density function (PDF) of \( A_{00} \) such that the average BER becomes

\[
P_e = \int_{0}^{\infty} f(A_{00}) P_e(A_{00}) dA_{00}
\]

where \( f(A_{00}) \) is the receiver output PDF of the fading amplitude.

---

Fig. 4. Relationship of changes in limits as variable of integration changes.
4. Diversity Combining

In this section, we present the performance of the system when diversity techniques are employed at the receiver. We assume that the fading amplitude is Nakagami-distributed.

4.1. Selection Diversity

In Selection Diversity (SD), among a group of \( M \) branches, the largest one is selected with output

\[
A_{\text{max}} = \max(A_0, A_1, \ldots, A_{0(M-1)})
\]

where all the \( A_0 \) are IID Nakagami-distributed variables with parameter \( (m, \Omega) \) having PDF [19]

\[
f_{A_0}(R) = \frac{2mR^{2m-1}}{\Gamma(m)\Omega^m} e^{-\frac{(mR^2)}{\Omega}}
\]

The constant \( m \geq \frac{1}{2} \) is the signal-fading parameter that indicates the severity of fading and \( \Omega \) is the average signal power. The distribution function is given by

\[
F_{A_0}(R) = \frac{2}{\Gamma(m)} \Gamma\left(m, \frac{mR^2}{\Omega}\right)
\]

Substituting \( x = (m/\Omega)R^2 \), we get

\[
F_{A_0}(R) = \frac{1}{\Gamma(m)} \Gamma\left(m, \frac{mR^2}{\Omega}\right)
\]

where \( \Gamma \) is the incomplete Gamma function of the second kind [20]. For IID random variables, the output PDF is given by

\[
f_{A_{\text{max}}}(R) = M \left[ \frac{1}{\Gamma(m)} \Gamma\left(m, \frac{mR^2}{\Omega}\right) \right]^{M-1} \times \frac{2m^mR^{2m-1}}{\Gamma(m)\Omega^m} e^{-\frac{(m/\Omega)R^2}}
\]

Using Equations (23) and (25), the average BER in Equation (24) becomes

\[
P_{e}^{\text{sd}} = M \left[ \frac{1}{\Gamma(m)} \Gamma\left(m, \frac{mR^2}{\Omega}\right) \right]^{M-1} \times A^{2m-1} e^{-\frac{(mR^2)}{\Omega}} \textrm{erf} c \left[ \sqrt{\frac{mR^2}{\Omega}} \right] \text{d}R
\]

4.2. Equal Gain Combining

Equal Gain Combining (EGC) is based on combining all the decision variables of the group of signals with equal weight [21] such that

\[
U_{y,0}^{\text{egc}} = \sum_{n=0}^{M-1} U_{y,0,n} = \sqrt{\frac{PT}{2}} b_{q,0}^{\text{egc}} \sum_{n=0}^{M-1} A_{0n} + \sum_{n=0}^{M-1} N_{0n}
\]

where \( N_{0n} \) represents the combined noise and interference components of path \( n \). Note that since \( \text{var}[N_{0n}] \) does not depend on \( n \), hence we have

\[
\text{var} \left[ \sum_{n=0}^{M-1} N_{0n} \right] = M \cdot \text{var}[N_{00}]
\]

Therefore, the error conditioned on \( \xi = \sum_{n=0}^{M-1} A_{0n} \) is

\[
P_{e}^{\text{egc}}(\xi) = \frac{1}{2} \text{erf} c \left[ \frac{\xi}{\sqrt{\Omega Z_{\text{egc}}}} \right]
\]

where \( Z_{\text{egc}} = \sigma^2_{v}/\sqrt{E[T]/4} \). Suppose \( v = A_{00} + A_{01} + \cdots + A_{0(M-1)} \), the random variable \( v \) is Nakagami-distributed with parameters \( (mM, M^2\Omega) [1 - (1/5m)] \) [19]. Hence,

\[
f_{v}(v) \approx \frac{2(mM)^{2m-1}v^{2m-1}e^{-\frac{(m/\Omega)v^2}}}{\Gamma(mM)(M^2\Omega[1-(1/5m)])^{mM}v^{2m-1}e^{-\frac{(m/\Omega)v^2}}}
\]

Finally from Equation (24), the average BER expression can be written as

\[
P_{e}^{\text{mgc}} = \frac{(5m^5/M\Omega^5[5m-1])^{mM}}{\Gamma(mM)} \int_{0}^{\infty} \text{erf} c \left[ \sqrt{\frac{\xi}{M\Omega Z_{\text{egc}}}} \right] \times v^{2m-1} e^{-\frac{(m/\Omega)v^2}} \text{d}v
\]

4.3. Maximal Ratio Combining

In Maximal Ratio Combining (MRC), the received signal is weighted proportionally to the received signal amplitude before summing. The idea of MRC is that components of the received signal with large amplitudes contain relatively little noise. Thus, their effect on the decision process is increased by squaring their amplitudes [22]. In this case, the decision variable is given by

\[
U_{y,0}^{\text{mrc}} = \sqrt{\frac{PT}{2}} b_{q,0}^{\text{mrc}} \sum_{n=0}^{M-1} A_{0n}^2 + \sum_{n=0}^{M-1} A_{0n} N_{0n}
\]

Since the mean value of the signal is

\[
E\left[U_{y}\right] = \sqrt{\frac{PT}{2}} \sum_{n=0}^{M-1} A_{0n}^2
\]
and the variance is

$$\text{var} \{ U_n \} = \text{var} \left[ \sum_{n=0}^{M-1} A_{0n} N_{0n} \right] = \text{var} \{ N_{00} \} \sum_{n=0}^{M-1} A_{0n}^2$$

(33)

the error conditioned on $\xi' = \sum_{n=0}^{M-1} A_{0n}^2$ can be written as

$$p_{e\xi'} (\xi') = \frac{1}{2} \text{erf} \left( \sqrt{\frac{\xi'}{\Omega Z_n}} \right)$$

(34)

Now, if $w^2 = A_{00}^2 + A_{10}^2 + \cdots + A_{0(M-1)}^2$, then $w$ is Nakagami-distributed with parameter $(mM, M\Omega)$ [19], such that

$$f_{\xi} (w) = \frac{2 (mM)^m w^{2mM-1}}{\Gamma(mM) (M\Omega)^m} e^{-\left[ (m/M)w^2 \right]}$$

(35)

Similarly, the average BER for MRC is given by

$$p_{e\xi q} = \frac{(sm^2/2M\Omega[Sm - 1])^m}{\Gamma(mM)} \int_0^\infty \text{erf} \left( \sqrt{\frac{\xi}{M\Omega Z_n}} \right)$$

$$\times e^{2mM-1} e^{-\left[ (m^2)/M\Omega \right]} \, dv$$

(36)

5. Numerical Results and Discussion

To ensure proper comparison, it is relevant to mention some issues regarding delay range, bandwidth, bit rate and power. For comparison purposes, the same channel and delay range are used for both ST-CDMA and MT-CDMA. For ST-CDMA system, the delay range would be 0 to $T/S$. Although S/P conversion of the bit period increases the number of tones, the delay range should be kept constant within $[0, T/S]$. Actually, the period of $S$ increases $S$ times for 1/$S$ S/P conversion. This implies that if we divide $T$ in all cases with $S$, then we will get the same value regardless of the number of tones.

Our next concern is to use the same bandwidth for both cases. The system could be regarded as a cascade of multicarrier modulation followed by SS. In ST-CDMA system, the bandwidth is $S/T$, while in MT-CDMA, the bandwidth for each tone is $1/T$, giving rise to a total bandwidth of $S/T$. Hence, the system bandwidth for both cases is ensured. Also, we must keep the chip period the same irrespective of the value of $S$. This will ensure that the bandwidth of the system is the same. Let $N_1$ be the number of chips per data bit for a single-tone system. The bit period of data increases $S$ times if we use $S$ tones thereby increasing the number of chips per data bit by $S$ times. Keeping $T_C$ constant when $S$ multiplies the symbol duration, the length of PN sequence should be multiplied accordingly [16]. In other words, the ratio of the number of chips and tones should be constant. If $N$ is the chip length of MT-CDMA, then $N/S$ should have the same value as $N_1$ in all cases. In our computation, the bit rates for both ST-CDMA and MT-CDMA are automatically kept the same to ensure equal bandwidth.

Similarly, if the power in a single-tone system is $P$, then power carried by each $S$ substream is $P/S$. So the same overall power is the same in both the cases. And since bit duration of each substream is $S$ times larger than that in ST-CDMA, the energy of each symbol also remains the same.

Finally, the frequency position of PBI signal with respect to wideband signal is kept the same. For the MT-CDMA and ST-CDMA, the center frequencies are spaced $x/T$ apart and $(T/2)(S - 1)$ apart, respectively. Thus, all comparison made for identical $E_s/N_0$ experience identical system resource allocation.

With these considerations in mind, the BER performance of a single-tone system is shown in Figure 5 for tones 0, 3 and 7, with $J/S = 3$ dB, $N = 32$, $\delta = 0$, $\Omega = 10$ dB, $m = 1$ and encoder bit rate of $S/T = 1.0$ Mbps. Also, in determining the variance of the PBI, the bandwidth is taken to be equal to one tone of multitone system and $\Delta = 0$. That is, the center frequency of the jammer is the same as that of the 0th tone of MT-CDMA system. Ideally, if there was no narrowband jamming, tones 0 and 7 would have the same BER performance. From Figure 2, it is observed that both tones experience the same interference due to multipath propagation, which will affect the orthogonality of the different carriers. Under normal circumstances, the middle tone will show worst BER performance since it experiences the most interference. But in this case we have PBI which overlays the first tone. So the angular distance of the last tone ($q = 7$) from narrowband signal is large compared to the first tone. Hence, the first tone ($q = 0$) shows worse BER performance than the 7th tone. The middle tone ($q = 3$) has performance between $q = 0$ and $q = 7$ tones. From Figure 5, we cannot conclude that middle tones will always perform better than first tones, because it depends on the value of $J/S$ and a trade-off between larger angular distance from the PBI and higher intercarrier interference.

In Figure 6, the BER performance of a single-tone system and a multitone ($S = 4$) system is compared for several diversity schemes when $J/S = 5$ dB.
Fig. 5. BER performance of MT-CDMA system for different tones with MRC.

Fig. 6. The effect of diversity on the BER performance of MT-CDMA and ST-CDMA.

Figure 7 shows the BER performance with respect to $J/S$. The robustness of MT-CDMA system in mitigating the effect of PBI is evident from the figure. Observe that while a conventional system can sustain only about $J/S = 5$ dB to give a BER about $2 \times 10^{-3}$, the multitone system can sustain about 13 dB of $J/S$. At higher interference power, MT-CDMA consistently gives better performance than ST-CDMA. Comparing the diversity techniques, we

$N = 128$ and $N_1 = 4$. Clearly, the better performance of the multitone system is observable in Figure 6. The average BER performance of the multitone case is determined by taking the average value at a particular value of $E_b/N_0$. As expected, MRC gives better performance compared to EGC or SD. When ST-CDMA and MT-CDMA are compared, it is evident that the multitone system outperforms the single-tone system for all diversity schemes.
also observe that MRC shows better performance than EGC and SD.

Figure 8 gives system performance as a function of the number of users. The consistent superior performance of MT-CDMA system over ST-CDMA system is evident from the plots. As expected, system performance deteriorates as the number of users increases.

While ST-CDMA approaches a limit of 100 users, MT-CDMA can sustain more users.

6. Conclusion

In this paper, we have presented the performance of a MT-CDMA system overlaid by narrowband
interference. Instead of using filters to suppress the narrowband interference and prevent against fading, the technique of MT-CDMA was applied. With its larger symbol duration, and hence improved cross-correlation property and more equal distribution of power between its frequency spectrum, the MT-CDMA system performs better than a conventional ST-CDMA system in reducing both narrowband and multipath interferences. It was shown that multitone CDMA could effectively reduce the effect of interference and improve system performance. Also, the effect of narrowband signal in various adjacent tones of the multicarrier system has been analyzed. It has been shown that for a high data-rate transmission, the effect was consistently different because of gradually increasing angular distance of successive tones from the interfering signal. In addition, the performance of the MT-CDMA system has been compared to conventional single-tone CDMA system with respect to the same bit rate, bandwidth, power, delay range and location of system bandwidth with respect to bandwidth of narrowband signal. Without complicated filters, MT-CDMA system achieves considerable performance improvement over the ST-CDMA system.

Appendix A

Derivation of $E[(R_{y,x}^c)^2]$ 

In this appendix, we show how $E[(R_{y,x}^c)^2]$ or $E[(R_{x,y}^c)^2]$ in Equation (12) is derived. We know from Equation (8) that

$$R_{y,x}^c(t_{ul}) = \int_0^{T_c} c_k(t - t_{ul} + T)c_0(t) \times f \left[ \frac{2\pi(x - y)t}{T} \right] dt \quad (A-1)$$

Notice that after S/P conversion, one data bit will contain $N$ chips such that

$$c_0(t) = \sum_{g=0}^{N-1} c_0^g \tilde{P}(t - gT_c), \quad \text{and}$$

$$c_k(t) = \sum_{h=0}^{N-1} c_k^h \tilde{P}(t - hT_c) \quad (A-2)$$

where $\tilde{P}(t)$ is a rectangular pulse of duration $T_c$. Hence,

$$R_{y,x}^c(t) = \sum_{g=0}^{N-1} c_0^g \sum_{h=0}^{N-1} c_k^h \int_0^{T_c} \tilde{P}(t - \tau - hT_c - T) \tilde{P}(t - gT_c) \times \cos \left[ \frac{2\pi(x - y)t}{T} \right] dt \quad (A-3)$$

where $\tau$ is uniformly distributed between $[0, N_1T_c]$ and $T/S = N_1T_c$. Note also that $N_1$ is the number of chips before S/P conversion. For a particular value of $\tau$, say $\tau = iT_c + \beta$, $0 \leq \beta \leq T_c$, and for $0 \leq i \leq N_1$ Equation (A-3), can be written as

$$R_{y,x}^c(t) = \sum_{g=0}^{N-1} c_0^g \sum_{h=0}^{N-1} c_k^h \int_0^{iT_c + \beta} \tilde{P}(t - (h + i - N)T_c - \beta) \quad (A-4)$$

Fig. 9. Illustration of overlapping of $(g' - 1)$th and $(g')$th coding chips.

\[ R_{xy}^c(t) = \sum_{g=0}^{i} C_g e^{i \frac{2 \pi (x - y) g}{N}} \]

Taking the expectation of Equation (A-7), we obtain

\[ E \left[ R_{xy}^c(t)^2 \right] = E \left[ R_{xy}^0(t)^2 \right] = 1 \sum_{g=0}^{i-1} \left[ |F_1(g, \beta)|^2 + |F_2(g, \beta)|^2 \right] \]

\[ = \frac{T^2}{4 \pi^2 (x - y)^2 N_1} \times \mathcal{F}(x) \quad \text{(A-8)} \]

The expression for \( E \left[ \left( R_{xy}^c(t) \right)^2 \right] \) is similarly obtained and is given by

\[ E \left[ \left( R_{xy}^c(t) \right)^2 \right] = E \left[ \left( R_{xy}^0(t) \right)^2 \right] \]

\[ = \frac{T^2}{4 \pi^2 (x - y)^2 N_1} \times \mathcal{F}(x) \quad \text{(A-10)} \]

where

\[ \mathcal{F}(x) = \sum_{g=0}^{i-1} \sin \left( \frac{2 \pi (x - y) g}{N} \right) \]

\[ + \frac{1}{4 \pi(x-y)} \sin \left( \frac{4 \pi (x - y) g}{N} \right) \]

References


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