Efficient Limited Feedback Schemes for Network MIMO Systems

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Abstract—In this paper, a feedback channel selection scheme and a bit allocation scheme are proposed for a network MIMO system with limited feedback. The feedback channel selection is performed by comparing the gain of each channel vector with a certain threshold. In particular, we derive an optimal threshold that maximizes a utility function, which is defined in terms of the expected rate of each user. In addition, we also develop an unequal feedback bit allocation scheme over the selected set of channels to further improve the system performance. It is shown that the joint feedback channel selection and bit allocation can provide significant gain in the average sum rate for a network MIMO system with limited feedback.

I. INTRODUCTION

Inter-cell interference (ICI) is one of the most challenging problems in cellular systems, since cells usually operate independently of one another. Even though many solutions, such as fractional frequency reuse, power control, and spread spectrum, have been developed to solve the ICI problem, these solutions cannot fully utilize wireless resources, resulting in limited performance gain. Recently, network multiple-input multiple-output (MIMO), which is also known as multi-cell MIMO, coordinated multi-point (CoMP) transmission, and multi-cell cooperative processing has attracted a lot of interest due to its promising capability of mitigating the ICI and improving network spectral efficiency at the same time [1]-[4].

The main idea of network MIMO is to allow adjacent base stations to cooperate in serving users by sharing their resources and user-specific data and/or channel state information. When user data as well as the channel state informations are shared among the cooperating base stations, in particular, the base stations form a virtual base station and the corresponding cells comprises a cluster. A network MIMO system looks quite similar to a multi-user MIMO (MU-MIMO) system, in that both systems serve multiple users simultaneously by using multiple transmit antennas. Differently from a MU-MIMO system that has colocated antennas at a single base station, however, a network MIMO system has transmit antennas distributed over several base stations, which cause non-uniform channel gains, referred to as channel asymmetry [5].

Many previous works have shown that the network MIMO provides significant gain in terms of capacity [1], [2] and outage probability [6]. However, there are still many practical issues, such as scalability, limited feedback, limited backhaul capacity, and synchronization problem. Among these issues, we focus on the limited feedback aspect with consideration on the scalability. The impact of limited feedback has been thoroughly studied in the context of point-to-point and broadcast MIMO [7], [8]. However, the impact on network MIMO has been less investigated.

The channel asymmetry inherent to network MIMO results in selectivity of feedback channels between each user and cooperating base stations. This implies that channels associated with relatively small gain can be ignored, which brings an opportunity of feedback and backhaul overhead reduction [9]. In other words, more accurate channel state information can be fed back for the selected channels, while keeping the feedback overhead the same. In particular, a selective feedback scheme was proposed in [9], where the channel corresponding to each base station is selected only when the channel gain exceeds a certain threshold. In [9], the authors assumed that the coefficients of selected channels are fed back to base stations without quantization and defined the feedback load as the number of channel coefficients to be fed back. Thus, the threshold was merely determined to meet the feedback load constraint. However, in practical limited feedback system, there is a tradeoff between the feedback accuracy and the number of feedback channels and this tradeoff also should be considered when the threshold is determined. The feedback channel selection scheme in [5] selects feedback channels so as to maximize the individual expected signal-to-interference-plus-noise ratio (SINR) of each user with the fixed number of feedback bits per user. However, the number of feedback bits needs to be increased in accordance with the received SINR and the number of transmit antennas [7]. Though a feedback bit allocation scheme proposed in [10] allocates unequal number of feedback bits to users considering such heterogeneous received SNRs between users, the number of transmit antennas(equally, the number of channels) was not considered due to its single cell environment.

In this paper, we propose a feedback channel selection scheme and a feedback bit allocation scheme suitable for a network MIMO system. We first extend the threshold-based feedback channel selection in [9] to the case of limited feedback system based on channel codebook and channel vector quantization. Specifically, we define a utility function based on the expected rate of each user and derive an optimal threshold that maximizes the utility. Then, a feedback bit allocation scheme in [10] is modified and incorporated into the channel selection scheme. Through computer simulations, it is shown that the proposed scheme significantly improves the average sum rate.

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This paper is organized as follows. The system model of a network MIMO is described in Section II. In Section III, we propose a threshold-based feedback channel selection scheme and a feedback bit allocation scheme. Simulation results are presented to validate the performance of the proposed schemes in Section IV. Finally, conclusions are drawn in Section V.

II. SYSTEM MODEL

A. Network MIMO System

We consider the downlink of a cellular system, where each base station is equipped with $N_t$ transmit antennas and each user has a single receive antenna. $M$ cells or sectors associated with adjacent base stations are grouped into a cluster for network MIMO, as illustrated in Fig. 1 for $M = 3$. We assume that clusters are disjoint so that scheduling and beamforming are performed in each cluster independently of other clusters. The channel state information and data for every user in a cluster are shared among the cooperating base stations through a backhaul coordinator. $K$ users are assumed to be uniformly distributed in each cluster and zero-forcing beamforming is considered to serve $MN_t$ users simultaneously among $K$ users. The received signal of the user $k$ in the cluster $i$ can be expressed as

$$y_k^{(i)} = h_k^{(i)}w_k^i x_k^{(i)} + h_k^{(i)} \sum_{\ell \neq k} w_k^i x_{\ell}^{(i)} + \sum_{j \neq i} h_k^{(j)} w_j^i x_{j}^{(j)} + n_k^{(i)}$$  \hspace{1cm} (1)

where the first term in the right hand side denotes the desired signal, the second and third terms represent intra-cluster interference due to limited feedback and inter-cluster interference, respectively, and the last term denotes the noise. From now on, we ignore the inter-cluster interference and omit the cluster index to focus on the impact of limited feedback.

In (1), $h_k = [h_{k,1} \ h_{k,2} \ \cdots \ h_{k,M}]$ denotes a $1 \times MN_t$ composite channel vector from all $M$ base stations in a cluster to the user $k$, and $w_k = [w_{k,1}^T \ w_{k,2}^T \ \cdots \ w_{k,M}^T]^T$ denotes a beamforming vector with each element $w_{k,m}$ being an $N_t \times 1$ vector corresponding to the base station $m$. The $1 \times N_t$ channel vector $h_{k,m}$ can be decomposed as $h_{k,m} = \sqrt{\rho_{k,m}}h'_{k,m}$, where $\rho_{k,m}$ is the average SNR between the base station $m$ and user $k$, and $h'_{k,m}$ represents short-term fading with independent and identically distributed (i.i.d.) complex Gaussian entries of zero-mean and unit variance. Assuming the equal power allocation, the transmit power of each base station is computed to satisfy per base station power constraint as [9]

$$P = \min_m \frac{P_m}{\text{tr} \left( \sum_k w_{k,m}w_{k,m}^H \right)}$$  \hspace{1cm} (2)

where $P_m$ denotes the maximum allowed power of the base station $m$, and $\text{tr}(\cdot)$ denotes the trace operation.

B. Limited Feedback with Channel Selection

Each user needs to estimate the channel state information and feedback it to the base stations to compute the joint precoding vector $w_k$. For a frequency division duplex (FDD) operation, the channel vector is decomposed into the magnitude and direction components, which are quantized to form the channel quality information (CQI) and channel direction information (CDI) respectively and then reported to base station(s) using allocated feedback resource. Especially in network MIMO system, the CQI’s and CDI’s feedback to the base stations are collected to the coordinator to reconstruct the composite channel vector. Since the CDI usually incurs more feedback overhead than the CQI [8], we assume that the CDI is ideally delivered to the coordinator without quantization, and consider quantization of the CDI with limited number of bits: $b_k$ bits for the user $k$.

In addition to limited feedback, we consider the channel selection which selects a few channel vectors among $M$ channel vectors and aggregates selected channels to a single effective channel to be quantized and fed back. The CDI corresponding to a selected and aggregated channel vector $\hat{h}_k(S_k)$ is generated from a predefined codebook of size $2^{b_k}$ where $S_k$ is the set of selected base station channels or the corresponding channel vectors with cardinality $|S_k| \leq M$.

Let the codebook be $F = \{f_1, f_2, \ldots, f_{2^{b_k}}\}$, then the nearest codewector to the normalized channel vector is chosen as the CDI: $\hat{h}_k(S_k) = \arg\max_{f \in F} \|\hat{h}_k(S_k)/\|\hat{h}_k(S_k)\|\| f \|$, where $\hat{h}_k(S_k) = h_k(S_k)/\|h_k(S_k)\|$ denotes the normalized version of $h_k(S_k)$. As for the codebook design, we assume the random vector quantization (RVQ) [7], in which the codevectors in the codebook are independently and isotropically distributed on the $|S_k|N_t$-dimensional unit sphere.

An example of the feedback procedure described above is given below for the case when $M = 3$ and the user $k$ is assumed to select the channels for the base stations 1 and 3.

- **Original channel vector:** $h_k = [h_{k,1} \ h_{k,2} \ h_{k,3}]$
- **Selected channel vector:** $h_k(S_k) = [h_{k,1} \ h_{k,3}]$
- **Codevector selection:** $\hat{h}_k(S_k) = \arg\max_{f \in F} \|\hat{h}_k(S_k)/f\|$
- **Channel feedback from user $k$ to the coordinator:** CQI ($\|\hat{h}_k(S_k)\|$ or SINR[8], ideal), CDI ($\hat{h}_k(S_k)$, $b_k$ bits)
- **Reconstructed channel at the coordinator:** $\hat{h}_k = CQI \cdot [\hat{h}_k(S_k)(1:N_t) \ 0_{1 \times N_t} \ \hat{h}_k(S_k)(N_t+1:2N_t)]$

Note that either channel magnitude or expected SINR [8] can be used as a CQI. Although the SINR is superior to channel magnitude, we adopt channel magnitude for simplicity.
\[
E [R_k|\delta] = E \left[ \log_2 \left( 1 + \frac{\|h_k\|^2 \beta (1, MN_t - 1) + \|h_k(S'_k)\|^2 + \|h_k(S'_k)\|^2 M_{N_t - 1}}{1 + \left( \sin^2 \theta_k \|h_k(S'_k)\|^2 \right) \sum_{m \in S_k} \beta (1, MN_t - 2)} \right) \right]
\]

\[
E [R_k|\delta] \geq r_k(\delta) = \log_2 \left[ \frac{\sum_{m \in S_k} \rho_{k,m}}{1 + N_t \left( 2 + \frac{\sum_{m \in S_k} \rho_{k,m}}{\sum_{m' \in S'_k} \rho_{k,m'}} \right)} \right]
\]  

III. FEEDBACK CHANNEL SELECTION AND BIT ALLOCATION

In Section III-A, we propose a threshold-based channel selection scheme similar to [9], so that only the channel vectors associated with \( \rho_{k,m} \geq \delta \) are selected, where \( \delta \) denotes a certain threshold. First, in Section III-A, we define a utility as a function of the threshold and derive an optimal threshold that maximizes the utility, under the assumption that the number of feedback bits is fixed. Then, in Section III-B, we generalize the feedback bit allocation scheme proposed in [10] to be suited for network MIMO systems and incorporate it into the feedback channel selection.

A. Threshold-Based Feedback Channel Selection

We define a utility as a function of the threshold \( \delta \)

\[
U(\delta) = \sum_k \omega_k E [R_k|\delta]
\]

which represents the sum of weighted average rates of users for a given threshold \( \delta \). \( \omega_k \) in (5) is the weight for each user related to the scheduling policy, and the conditional average rate of each user \( E [R_k|\delta] \) is given in (3), which can be derived as in the Appendix A. In (3), the selected base station \( S_k \) can be expressed as \( S_k = \{ m : \rho_{k,m} \geq \delta, m = 1, 2, \ldots, M \} \), and \( h_k(S_k) \) is the corresponding channel vector. \( S'_k \) is the complementary set of \( S_k \), and \( \beta(x,y) \) represents a beta-distributed random variable with parameter \((x,y)\). A lower bound \( r_k(\delta) \) of the expected rate in (3) can be derived as (4). The detailed derivation is given in the Appendix B.

The lower bound in (4) shows the tradeoff that comes from channel selection in limited feedback systems. If user \( k \) choose more channels, then the second term in the denominator inside the log, which represents the interference caused by quantization error, increase. On the other hand, the third term in the denominator inside the log, which represents the interference caused by non-selected channels, becomes dominant when user \( k \) choose a small number of channels. Thus, we need to find a proper threshold for channel selection that can enhance the system performance or utility of the system.

We assume that the number of feedback bits per user is fixed, i.e., \( b_k = B, \forall k \), to account for the limited feedback. Then, \( h_k(S_k) \) needs to be quantized accordingly, and the constraint can play as an additional condition in the utility function. Hence, the problem of determining an optimal threshold \( \delta^* \) in a limited feedback scenario can be formulated as

\[
\delta^* = \arg \max_{\delta} U(\delta|b_k = B) = \arg \max_{\delta} \sum_k \omega_k E [R_k|\delta, b_k = B]
\]

If we replace the average user rate with its lower bound in (4), (6) is converted to

\[
\delta^* = \arg \max_{\delta} \sum_k \omega_k r_k(\delta|b_k = B)
\]

In a network MIMO system, it is hard to derive proper distribution of average SNR’s. Due to this reason, we will find the optimal threshold through numerical search over \( \rho_{k,m}, \forall k, m \).

As compared with the previous work in [9] where channel quantization is not considered, the proposed scheme in (7) accounts for limited feedback system based on channel quantization associated with channel dimension scalability. Moreover, the threshold is devised to improve the system performance under the limited feedback constraint.

B. Joint Feedback Channel Selection and Feedback Bit Allocation

It is desired to increase the number of feedback bits with the channel quality and the number of transmit antennas [7]. However, the feedback channel selection scheme in Section III-A does not consider the scalability of feedback bits by allocating the same number of bits to every user. In this section, we relax the constraint and allow different number of bits to be allocated to different users, while keeping the total number of feedback bits fixed, i.e., \( \sum_k b_k = B_T \). A feedback bit allocation with the constraint of total number of bits was presented in [10] in the MU-MIMO context. The scheme was developed to account for different received SNRs due to random user locations. We generalize the scheme in [10] so that it can handle the varying number of transmit antennas due to feedback channel selection in a network MIMO system with feedback channel selection.

With a given threshold value \( \delta \) and the average SNR informations, \( \rho_{k,m} \) sent from users under certain scheduling policy, the coordinator can acquire the selected channel information \( S_k \) of each user, and corresponding average channel gains, \( \sum_{m \in S_k} \rho_{k,m} \) and \( \sum_{m' \in S'_k} \rho_{k,m'} \). Using these informations and
where $s_k$ = $\sum_{m \in S_k} \rho_{k,m'}$, $\eta_k = 1 + \sum_{m' \in S_k^c} \rho_{k,m'}$, $\gamma_k = 2^{-(|S_k| - 1)/\gamma}$, and $\beta_k = 1/(|S_k| N_t - 1)$. $b_0$ is the minimum number of feedback bits per user. In this case, total bit constraint becomes $B_T = B_T - K b_0$.

From the KKT conditions, we can derive a solution in (9) for modified problem with continuous relaxation. Differently from the solution in [10], our solution is function of the number of selected channels, $|S_k| N_t$ and average channel gains of both selected channels and non-selected channels. If the threshold is low enough that all the channels are selected, then the derived solution will converge to the solution of [10]. It is also worth mentioning that not only the received SNR and the number of transmit antennas but also the scheduling policy is considered in the feedback bit allocation. This allows to allocate more feedback resource to the users with higher priority.

Note that the solution of feedback bit allocation has been derived for a certain given threshold $\delta$. We can compute the optimal threshold $\delta^*$ and the corresponding optimal bit allocation $b^*$ as

\[
(\delta^*, b^*) = \arg \max_{\delta, b} U(\delta, b)(\delta, B_T) = \arg \max_{\delta, b} \sum_k \omega_k r_k(\delta, b_k^*(\delta)).
\]

In this case, unlike (6), the feedback rate constraint is given in the form of the total number of bits $B_T$ rather than the number of bits per user. The overall procedure of feedback channel selection and bit allocation is depicted in Fig. 2. As seen from (4), (7), and Fig. 2, the proposed feedback channel selection requires information on the average SNRs, not on the instantaneous channels, which relieve the overhead of information exchange between BSs and MS in Fig. 2.

The complexity of the proposed scheme can be measured by the number of $\delta$’s tried for optimization in (12). Intuitively, the minimum number of trials will be $K M$, i.e., the total number of channels, not to cause any performance loss. Meanwhile, the complexity of each trial will be determined mainly by the bit allocation.

IV. Numerical Results

In this section, we evaluate the performance of the proposed schemes in Section III. A cluster of three adjacent sectors is considered, as depicted Fig. 1. The average received power of the channel between the base station $m$ and the user $k$ in a cluster is defined as

\[
\rho_{k,m} = \rho_0 \left( \frac{d}{d_r} \right)^{-\alpha}
\]

where $d$ is a distance from the base station to the user, and $\rho_0$ denotes the reference SNR (=15dB), i.e., the average received SNR of the user located at distance $d_r$ from the base station. The value of $d_r$ is set to the cell radius, which is assumed to be 500m. The pathloss exponent $\alpha$ is set to 3.7. It is assumed that the zero-forcing beamforming is adopted along with the power allocation in (2), and the users are served in the round robin fashion, which implies $\omega_k = 1$, $\forall k$. The average cluster rate is evaluated by taking the average of 1,000 drop events.

We first evaluate the performance of the proposed threshold-based feedback channel selection, when the number of feedback bits is equal and fixed for every user. In Fig. 3, the average cluster sum-rate is depicted, when the number of feedback bits per user varies. The performance of proposed
channel selection is compared with the distributed channel selection proposed in [5]. In [5], each user chooses the channels to be fed back so as to maximize the individual expected SINR, and reports the selected base station set $S_k$. The proposed threshold-based feedback channel selection without feedback bit allocation shows slightly better performance than the proposed scheme in [5]. However, the proposed channel selection in [5] is not able to allocate unequal number of feedback bits at all since its criterion of selecting channels is based on fixed feedback bit constraint per user. Thus, In Fig. 4, we investigate the performance of proposed scheme which jointly performs feedback channel selection and feedback bit allocation. Fig. 4 compares the proposed scheme with the full channel feedback with feedback bit allocation similar to [10]. This figure shows that the performance gain due to feedback bit allocation is limited without feedback channel selection. The performance gain due to channel selection is shown to diminish as the number of feedback bits increases, since more channels are selected with larger amount of feedback bits. It is easy to guess that all the channels will be selected if sufficiently many feedback bits are available.

Fig. 5 shows the relationship between the threshold derived from utility function and number of feedback bits when a feedback bit allocation is not adopted. As the number of feedback bits increases, the threshold decreases so that each user is allowed to choose more channels.

V. CONCLUSION

In this paper, we have proposed a joint feedback channel selection and feedback bit allocation scheme for a network MIMO system. The threshold-based feedback channel selection can reduce the feedback overhead by exploiting channel asymmetry nature of the network MIMO system. The feedback bit allocation scheme allows different number of bits to be assigned to different users. The proposed scheme is shown to improve the system performance while limiting feedback overhead.

APPENDIX A

DERIVATION OF EXPECTED USER RATE

The original expected rate of user $k$ is formulated as

$$
E[R_k] = E \left[ \log_2 \left( 1 + \frac{|h_k w_k|^2}{1 + \sum_{l \neq k} |h_l w_l|^2} \right) \right]
$$

When the threshold $\delta$ is given, the selected channel vector $h_k(S_k)$ where non-selected channels are replaced $1 \times N_t$ zero vectors is decomposed as

$$
h_k(S_k) = |h_k(S_k)| \left( (\cos \theta_k) \hat{h}_k(S_k) + (\sin \theta_k) g_k \right)
$$

(15)

$\theta_k$ is the angle between $h_k(S_k)$ and $\hat{h}_k(S_k)$, and $g_k$ is a unit random vector. Thus, without loss of generality, the original channel vector $h_k$ can be expressed as

$$
h_k = |h_k(S_k)| \left( (\cos \theta_k) \hat{h}_k(S_k) + (\sin \theta_k) g_k \right) + h_k(S_k^c)
$$

(16)

Then, the intra-cluster interference $\sum_{l \neq k} |h_l w_l|^2$ becomes

$$
\sum_{l \neq k} |h_l w_l|^2 = \sum_{l \neq k} \left( |h_k(S_k)| (\sin \theta_k) g_k + h_k(S_k^c) \right) w_l^2
$$

(17)

since $w_l$ is orthogonal to $\hat{h}_k(S_k)$.

If we let $g_k' = |h_k(S_k)| (\sin \theta_k) g_k + h_k(S_k^c)$ then the intra-cluster interference is replaced as

$$
\sum_{l \neq k} |h_k w_l|^2 = \sum_{l \neq k} |g_k' w_l|^2 = \left\| g_k' \right\| \sum_{l \neq k} |g_k' w_l|^2
$$

(18)
where $\| g_k \| = \| h_k (S_k) \| \sin^2 \theta_k + \| h_k (S^c_k) \| ^2$ and $\| g_k w_{lk} \| ^2$ is equivalent to beta distributed random variable with parameter $(1, MN_t - 1)$[7,8]. The nominator $\| h_k w_{lk} \| ^2$ is decomposed as

$$| h_k w_{lk} | ^2 = | h_k |^2 | h_k w_{lk} |^2$$

where $| h_k w_{lk} |^2$ is also beta distributed random variable with $(1, MN_t - 1)$ since we do not assume any specific user selection scheme such as SUS in [11]. Finally, by using (18) and (19), the expected rate of user $k$ is derived as (3).

$$E[R_k | \delta] \geq E \left\{ \log_2 \left( 1 + \frac{(\sin^2 \theta_k) \| h_k (S_k) \| ^2 + \| h_k (S^c_k) \| ^2}{\beta (1, MN_t - 2)} \right) \right\}$$

$$= \log_2 \left( \sum_{m} \rho_{k,m} N_t \right) \frac{1}{M N_t}$$

$$= \log_2 \left[ \frac{1}{M} \sum_{m} \rho_{k,m} \right]$$

$$+ \log_2 \left[ 1 + \left( 2 \| h_k \|^2 \right)^{1/2} \frac{\rho_{k,m}}{\rho_{k,m}} \right]$$

For $k \in S_k$, $\| h_k \| ^2 = \| h_k'' \|^2$ and $\| h_k \| ^2 = \| h_k' \|^2.$ The expectation of beta distributed random variable with parameter $(a,b)$ is $\frac{a}{a+b}$. And the expectation of quantization error $(\sin^2 \theta_k)$ is tightly bounded as $2 \| h_k \|^2$.

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**References**


**Appendix B**

**Derivation of Lower-Bounded Expected User Rate**

From (3), ignoring 1 in logarithm function of leads to inequality (a) in (20). Approximation (b) is valid when the term inside the first logarithm function is sufficiently large and the average channel gain of $h_k$ is large enough since it is the aggregated channel of all channels from $M$ base stations to user $k$. Inequality (c) comes from Jensen’s inequality and equality (d) comes from

$$E[\| h_{k,m} \|^2] = E[\| \sqrt{\rho_{k,m}} h_{k,m} \|^2] = \rho_{k,m} N_t$$

$$\rho_{k,m} E[\| h_{k,m} \|^2] = \rho_{k,m} N_t$$

$$E[\| h_{k,m} \|^2] = E[\| h_{k,m} \|^2] = \rho_{k,m} N_t$$

The expectation of beta distributed random variable with parameter $(a,b)$ is $\frac{a}{a+b}$. And the expectation of quantization error $(\sin^2 \theta_k)$ is tightly bounded as $2 \| h_k \|^2$.