AN UNBIASED CHANNEL ESTIMATOR FOR SPACE-TIME BLOCK CODING UNDER RAYLEIGH FADING

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This paper presents the use of an unbiased channel estimator for space-time block coding under Rayleigh fading. The channel being considered is a Rayleigh fading channel with Gaussian shaping. The channel is modeled as an MIMO channel with a single receive antenna. The channel is modeled as an MIMO channel with a single receive antenna. The channel is modeled as an MIMO channel with a single receive antenna.

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The performance of a supervised LMS adaptive channel estimator for space-time block coded systems (STBC) with two transmit antennas and one receive antenna is demonstrated. The wireless channel statistic is Rayleigh with additive white Gaussian noise (AWGN) present at its output. It is shown that it is possible to achieve an unbiased estimate when a single-tap estimator is used. The two adaptive coefficients (due to employing transmit diversity) converge to the true channel gains owing to the orthogonality of the space-time block code. This adaptive channel estimator results in zero mean square error (MSE) in the absence of noise. The estimated channel gains are used in the STBC decoder to achieve decoding. The algorithm is tested in the context of recovering images transmitted over the wireless channel with modulation and space-time block coding, and is characterized mainly by its simple implementation. In addition, the single-tap estimator is shown to have a convergence time that is almost comparable to that of an N-tap estimator. Although the latter converges somewhat faster, it has a non-zero MSE upon convergence.

**Key words:** Rayleigh fading, channel estimation, unbiased estimator, orthogonal space-time block coding (STBC), least mean square (LMS) adaptive filter, convergence factor
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1. INTRODUCTION

The wireless channel statistic is often Rayleigh, suffering attenuation due to destructive addition of multipaths [1,2]. Therefore, it is difficult for the receiver to reliably determine the transmitted signal unless some less attenuated replica of the signal is provided to the receiver. Transmit diversity has been studied extensively as a method of combating detrimental effects in wireless fading channels because of its simplicity of implementation and the feasibility of having multiple antennas at the base station [3]. In terms of decoding complexity, Alamouti [4] discovered a remarkable scheme for transmission using two transmit antennas. Space-time block coding, introduced in [5], generalizes the scheme discovered by Alamouti to an arbitrary number of transmit antennas and is able to achieve the full diversity promised by the transmit and receive antennas. Space-time block codes retain the property of having a very simple maximum likelihood decoding algorithm based only on linear processing at the receiver due to the orthogonal structure of the space-time block code.

In this paper, a space-time block coded system is considered for the transmission of images over a Rayleigh channel using two transmit antennas and one receive antenna. Prior to space-time block coding and transmission, the elements of the concatenated rows of the image matrix are quantized and M-QAM-encoded. This wireless communication system is shown in Figure 1. The channel is assumed to be a flat-fading Rayleigh channel with a single path gain from each of the two transmit antennas to the receive antenna. The path gains are modeled as samples of independent complex Gaussian variables. The wireless channel is assumed to be quasi-static so that the path gains are constant per frame. The STBC decoder needs an estimate of the two channel gains. After detection of the QAM symbols, they are QAM-demodulated to reconstruct the image with possible errors due to the presence of AWGN.

Figure 1. The wireless communication system for transmission of M-QAM modulated symbols over a flat-fading Rayleigh channel with space-time block coding and maximum likelihood decoding at the receiver

\[
\begin{bmatrix}
\bar{x}_1 & x_2 \\
-x_2 & \bar{x}_1
\end{bmatrix} = \begin{bmatrix}
g_1 \\
g_2
\end{bmatrix}
\]

\[
\hat{(x)_1, (x)_2} = \arg \min_{x, x_2} \{ D \}
\]

\[
D = |a|^2 + |b|^2
\]

\[
a = r_1 - g_1 \bar{x}_1 - g_2 x_2
\]

\[
b = r_2 + g_1 \bar{x}_2 - g_2 \bar{x}_1
\]
A least mean square (LMS) 1-tap adaptive channel estimator is considered for the application under consideration (single tap for each of the two path gains), and the effect of the LMS convergence factor on the reconstructed image quality is demonstrated. Since, in a system identification problem, the number of taps of the adaptive filter is usually large enough, the performance of the single-tap estimator is compared to that of an N-tap estimator. It is found that, although the N-tap estimator converges faster, the unbiased single-tap estimator outperforms it as it results in zero MSE upon convergence in the absence of noise. The single-tap estimate is unbiased owing to the orthogonality of the space-time block code. The N-tap estimator is found to result in non-zero MSE upon convergence in the absence of noise.

Other iterative algorithms, such as the expectation-maximization (EM) algorithm, are reported in the literature in the context of iterative channel estimation for STBC [6,7]. Because of its generality and guaranteed convergence, the EM algorithm is a suitable choice for many estimation problems. Although the EM algorithm is not subject to the limitation of the convergence rate parameter setting that characterizes the steepest descent algorithms, such as the LMS algorithm, it has problems of its own; its convergence rate is usually slower than that of the LMS algorithm [8].

The work in [9] also addresses the problem of channel estimation and tracking for STBC using recursive algorithms. However, a study and analysis concerning the important issues of estimate bias and convergence speed are lacking.

The paper is organized as follows: Section 2 presents the LMS adaptive filter in the Rayleigh channel identification setting within the block coded system; Section 3 presents and discusses simulation results; and finally, Section 4 concludes the paper.

2. THE ADAPTIVE CHANNEL ESTIMATOR

A supervised adaptive channel estimator that estimates the two path gains $g_1$ and $g_2$ indicated in the wireless communication system block diagram of Figure 1 is depicted in Figure 2. Using the complex LMS algorithm for adaptation [10–12], the filtering, error formation, and coefficient updating formulas are:

![Figure 2. Supervised LMS adaptive channel estimator for the wireless communication system of Figure 1](image-url)
\[
e(n) = r(n) - \hat{g}_1(n-1)s_1(n) - \hat{g}_2(n-1)s_2(n) \\
\hat{g}_1(n) = \hat{g}_1(n-1) + \mu s_1(n)e(n) \\
\hat{g}_2(n) = \hat{g}_2(n-1) + \mu s_2(n)e(n)
\]  

(1)

\( \hat{g}_1(n) \) and \( \hat{g}_2(n) \) are the adaptive filter gains, \( r(n) \) is the received (desired) signal, \( e(n) \) is the error signal, \( s_1(n) \) and \( s_2(n) \) are the inputs to the array adaptive filter (training sequences), and \( \mu \) is the convergence factor or step size. The overbar denotes complex conjugation.

In Figure 1, \( x_1 \) and \( x_2 \) represent two successive QAM symbols. The 2x2 orthogonal matrix, which is the STBC codeword, is

\[
\begin{bmatrix}
x_1 & x_2 \\
-x_2 & x_1 
\end{bmatrix}
\]

(2)

This ensures that the two data streams that are output from the space-time block encoder are orthogonal [13].

For the case of perfect knowledge of the underlying channel state information (CSI), the known path gains \( g_1 \) and \( g_2 \) are used in the STBC decoder to produce the estimates \( \hat{x}_1 \) and \( \hat{x}_2 \) that minimize the sum of the two consecutive squared errors or the following decision metric:

\[
(\hat{x}_1, \hat{x}_2) = \underset{x_1, x_2}{\text{arg min}} \left\{ D \right\} \\
D = |a|^2 + |b|^2 \\
a = r - g_1 x_1 - g_2 x_2 \\
b = r_2 + g_1 x_2 - g_2 x_1
\]

(3)

When the CSI is not known at the receiver, which is the case in this work, \( g_1 \) and \( g_2 \) in Equation (3) are replaced by the adaptive gains \( \hat{g}_1 \) and \( \hat{g}_2 \). The maximum likelihood detection of Equations (3) amounts to minimizing the decision metric over all possible values of \( x_1 \) and \( x_2 \). Due to the quasi-static nature of the channel, the path gains are constant over two transmissions. Initially, \( s_1(n) \) and \( s_2(n) \) take on the values of \( x_1 \) and \( x_2 \), respectively, as shown in Figure 1. In the next time slot or iteration, \( s_1(n) \) and \( s_2(n) \) assume the values of \( -x_2 \) and \( \bar{x}_1 \), respectively, in accordance with the space-time block code in Figure 1. Since \( x_1 \) and \( x_2 \) come from a finite alphabet, the STBC decoder performs the function of an M-ary slicer.

**Convergence considerations:**

We now discuss how the LMS adaptive channel estimator of Figure 2 results in an unbiased estimator upon convergence. Convergence of the iterative algorithm described by Equation (1) in the mean value requires that the tap weights \( \hat{g}_1(n) \) and \( \hat{g}_2(n) \) approach constant values as the number of iterations \( n \) approaches infinity [10]. Therefore, the following relations must hold:

\[
E[s_1(n)\bar{e}(n)] = 0
\]

(4)

\[
E[s_2(n)\bar{e}(n)] = 0
\]

(5)

Substituting for \( e(n) \) in Equation (4), and assuming noise-free conditions

\[
E[x_1(n)(\bar{r}(n) - \bar{x}(n)\hat{g}_1(n-1))] = 0
\]

assuming that \( E[x_1(n)s_2(n)] = 0 \) (orthogonal STBC).

Substituting for \( \bar{r}(n) \), and using the above assumption, we obtain
\[
E[s_1(n), (s_1(n) - \hat{s}_1(n)) \hat{g}_1(n-1)] = 0.
\] (6)

On convergence, we obtain
\[
E[|s_1(n)|^2] \hat{g}_1 = E[|s_1(n)|^2] g_1.
\] (7)

Therefore, the final value of \( \hat{g}_1 \) is equal to \( g_1 \). That is, we have unbiased convergence (zero mean square error in the absence of noise), provided that \( \hat{g}_1 \) is conjugated before using it in the STBC decoder decision metric of Equation (3) as a substitute for \( g_1 \). The above analysis equally applies to Equation (5).

If the number of coefficients of the adaptive filter is greater than one, the least squares problem becomes underdetermined and its solution is not unique. This case is important in practical applications, since often we do not know the number of channel gains in advance, and, therefore, a sufficiently large number of adaptive filter taps is employed. For this general case of N-tap estimation of a single-tap channel, it is straightforward, though somewhat lengthy, to show that the expression for the converged mean square error contains non-zero correlation terms that are uncancelable by other terms, thereby resulting in non-zero mean square error upon convergence.

3. SIMULATION RESULTS

The supervised single-tap LMS adaptive algorithm is implemented in MATLAB and tested in the context of transmitting images over a Rayleigh channel with modulation and space-time block coding (Figure 1). The STBC decoder at the receiver uses the estimated path gains provided by the adaptive channel estimator, and the decision metric to be minimized involves the squared errors, as discussed in Section 2. The actual path gains are modeled as samples of iid complex Gaussian variables with zero mean and unit variance. This flat-fading Rayleigh channel is assumed constant per frame. AWGN is present at the channel output at the front end of the receiver. The signal-to-noise ratio at the receiver is taken to be 25dB.

The image to be transmitted is MATLAB’s “earth”. The size of this image is 256x256 pixels. Prior to transmission, the image is quantized, PCM-Gray-encoded, and 16-QAM-modulated. After STBC decoding at the receiver, all these operations are reversed to reconstruct the image.

If the convergence factor \( \mu \) is small, convergence is slow and more exact final values of \( \hat{g}_1 \) and \( \hat{g}_2 \) are obtained. If the image itself is used to provide the training signals, the reconstructed image may show some deterioration even for very high SNR’s since the received image data will take time before it assumes the correct values. The convergence factor \( \mu \) is set to 0.0001. The original and reconstructed images are shown in Figure 3. The symbol error rate (SER) is 0.0021. The magnitudes of the path gains vs the number of iterations are demonstrated in Figure 4, together with the true values of the path gains represented by straight lines.

![Original Image](image1.png)

![Reconstructed Image](image2.png)

Figure 3. Original and reconstructed images, SNR=25 dB. Step size \( \mu=0.0001 \).
When $\mu$ is set to 0.01, the original and reconstructed images will be as shown in Figure 5. Due to faster convergence, a better image should be obtained. The SER is $1.8 \times 10^{-5}$. The convergence behavior of the path gains for this value of $\mu$ is shown in Figure 6. However, an inspection of Figures 3 and 5 shows that the reconstructed images are of comparable quality in the two figures (different values of $\mu$). This is due to the large size of the image (256x256) resulting in errors being mainly due to AWGN rather than convergence delay. Figures 4 and 6 do not show the full range of the number of iterations. The convergence interval is only a small fraction of the total image transmission time. Therefore, the effect of convergence factor on image quality is manifest only for small image sizes. To illustrate this, the original image is down-sampled by 8 twice, resulting in an image size of 32x32, and the simulations are repeated. The results are shown in Figures 7 and 8 for values of $\mu$ set to 0.0001 and 0.01, respectively. The difference in reconstructed quality image for different $\mu$ is obvious.

**Figure 4.** Magnitudes of the adaptive channel gains together with the true values (straight lines). SNR=25dB, $\mu$=0.0001.

**Figure 5.** Original and reconstructed images, SNR=25 dB. Step size $\mu$=0.01.
Figure 6. Magnitudes of the adaptive channel gains together with the true values (straight lines). SNR=25dB, \( \mu=0.01 \).
Usually, training signals other than those provided by the transmitted image are used. When convergence is attained, the adaptive filter is then switched to the decision-directed mode. In the present simulation, however, the images are used to provide the training signals to mimic and demonstrate the effect of a time-varying channel on the convergence behavior, and, consequently, on the image quality.

The mean square of the estimation error signal (Figure 2) is plotted in dB vs SNR for different µ as shown in Figure 9. The figure shows that the optimum µ found by simulation is 0.01. For a single-tap LMS adaptive filter, µ is confined between zero and twice the reciprocal of the input signal power to ensure convergence [11]. The optimum µ is some middle value in this range. Therefore, a suitable choice of µ will depend on the ability to estimate the input signal power.

![Figure 9. MSE (dB) vs SNR (dB) for the 1-tap channel estimator for different µ](image)

Figure 9 is a plot of MSE(dB) vs SNR for the single-tap and a 3-tap adaptive channel estimator for the problem under consideration. Both curves are plotted for µ=0.01. As discussed in Section 2 on convergence considerations, the MSE is larger for the 3-tap estimator. However, the convergence time for the latter estimator is smaller, as shown in Figure 11. This figure is a plot of learning curves for both estimators. The convergence time is expected to decrease further for larger numbers of taps. In [14], the convergence analysis of the LMS algorithm is presented for the case of sinusoidal signal cancellation when the adaptation problem is underdetermined (the number of adaptive filter taps exceeds the number of taps for the system to be identified). The convergence time was also shown to be influenced by the number of adaptive taps, and the larger tap number produced the smallest convergence time. In [14], this result was also arrived at by simulation. The explanation of this result requires further studies. Figures 9–11 were obtained by ensemble-averaging over ten realizations. To sum up, it is clear from Figures 10 and 11 that the performance of the single-tap estimator is comparable to that of an N-tap estimator, with the advantage of being unbiased and simple to implement.
Figure 10. MSE (dB) vs SNR (dB) for 1-tap and 3-tap channel estimator for $\mu=0.01$

Figure 11. Learning curves (MSE, dB vs iteration number) for $\mu=0.01$ and SNR=20 dB
4. CONCLUSIONS

We have simulated and discussed the performance of an LMS adaptive channel estimation algorithm to be used in a space-time block coded system with two transmit antennas and one receive antenna. The orthogonal structure of the space-time block code enables unbiased convergence of the algorithm when a single adaptive gain is used under flat-fading Rayleigh channel conditions. The effect of the convergence factor, or step size, on the received image quality is almost negligible for large image sizes. This is due to the relatively small convergence delay as compared to the total number of iterations when the whole image is transmitted within a single channel frame. In this case, the errors in reconstructed images are mainly due to AWGN. By choosing a suitable value of step size, especially for small image sizes, we can obtain acceptable reconstructed image quality, even when the channel is time-varying (albeit slowly due to Rayleigh fading), with possible errors mainly due to the presence of AWGN.

On comparing performances of single-tap and N-tap adaptive channel estimators for the transmission problem under consideration, it is found that a single-tap estimator is comparable in performance to the N-tap estimator in terms of convergence speed, and has the important additional advantage of being unbiased and simple to implement.

To the best of the authors' knowledge, the results and conclusions presented here are missing from previous work on channel estimation for STBC under flat Rayleigh fading, such as [6,7,9]. The aforementioned features and benefits of the present method and analysis of channel estimation defined by its corresponding scenario, together with the low computational complexity of the LMS algorithm, make it an attractive solution for this application.

The work in this paper has been done for flat fading Rayleigh channels. However, the wireless channel is usually frequency-selective. For this reason, future perspectives involve the extension of the present study of channel estimation for space-time block coding to frequency selective channels for which the performance evaluation of STBCs has been considered in [15] and [16]. The work in these references assumes perfect knowledge of CSI. Investigations into adaptive estimation of frequency-selective channels are expected to exhibit slower convergence due to the increase of eigenvalue disparity of the autocorrelation matrix of the adaptive filter input, owing to the effect of the frequency selective channel. Faster algorithms such as the recursive least squares (RLS) algorithm are expected to improve performance in such cases despite the computational complexity of these algorithms as compared to the LMS algorithm.

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REFERENCES


