LMI-Based Design of Fuzzy Controller and Fuzzy Observer for Takagi-Sugeno Fuzzy Systems: New Non-Quadratic Stability Approach

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Abstract – This paper deals with design procedures of fuzzy controller and fuzzy observer, one of the most important and basic concepts for fuzzy control system design. Fuzzy controller guarantees stability of the whole system i.e. fuzzy controller and fuzzy observer by the Lyapunov stability approach, while fuzzy observer estimates states of the fuzzy dynamic plants. The two designs are formulated as two separate LMI feasibility problems using new non-quadratic stability conditions based on non-quadratic Lyapunov function and Parallel distributed Compensation scheme to stabilize Takagi-Sugeno (T-S) fuzzy systems. Then a separation property based on a vector comparison principle is applied to check the stability of the whole system. Whereas, the obtained results are less conservative. Finally, numerical examples are presented to illustrate the effectiveness of our proposal by showing very satisfactory results.

Keywords: T-S fuzzy systems; Non-quadratic stability conditions; Linear Matrix Inequalities; Fuzzy controller; Fuzzy observer; Separation property

I. INTRODUCTION

During the last years, there has been a great interest in using Takagi-Sugeno fuzzy models to approximate nonlinear systems [1-3]. Many researches have been developed to demonstrate this concept [4-5], where the main idea is the use of models that consist of fuzzy IF-THEN rules with linguistic terms in antecedents, and analytic dynamical equations in the consequents. Also, several researchers in the control community have come up with different techniques for designing control systems. In fact, Takagi and Sugeno [3] proposed a multimodel based approach to overcome the difficulties of the conventional modeling techniques [6]. For this purpose, a nonlinear plant is represented by the T-S fuzzy model, where local dynamics in different state regions are represented by linear models. The overall model of the system is obtained by a fuzzy blending of these local models. This same fuzzy structure is used to control [3, 7-10] and to study the stability of the T-S fuzzy system using Lyapunov method [7, 11] and Linear Matrix Inequalities (LMI), where the problem can be numerically solved by convex optimization techniques [28]. It can provide an effective solution to the control of many complex systems that are difficult to describe using the linearization or identification techniques. The fuzzy control design is carried out using the Parallel Distributed Compensation (PDC) scheme [7-8]. The main idea of the PDC controller design is to derive each control rule from the corresponding rule of T-S fuzzy model so as to compensate it. The resulting overall fuzzy controller, which is nonlinear in general, is a fuzzy blending of each individual linear controller. Wang et al. [8] used this concept to design fuzzy controllers to stabilize fuzzy systems. The advantage of the T-S fuzzy model lies in that the stability and performance characteristics of the system represented by a T-S fuzzy model can be analyzed using Lyapunov function approach [7, 11] by satisfying a set of Lyapunov inequalities [8, 12]. This has been discussed in a huge number e.g. [7-9, 13-17] and [23], who used multiple Lyapunov functions, due to their properties of conservatism reduction.

The states of a system are not always available for measurement which is not the case in a lot of practical problems. To overcome this problem, the notion of observer was introduced. The concepts of linear regulator and linear observer were introduced by Kalman [19] for linear systems in stochastic environment, whereas for nonlinear systems different designs of observer were proposed [20-21], that have the disadvantage of design complexity. To overcome these disadvantages, fuzzy observer concepts were introduced, whose the most popular is the T-S fuzzy observer that was introduced by several authors in the literature. Feng et al. [30] and Tanaka [31] give a certain form for the T-S fuzzy observer with an asymptotic
convergence. Also, Tanaka proposed in his papers [27, 31] a globally exponentially stable fuzzy controllers and fuzzy observers designs for continuous and discrete fuzzy systems for both measurable and non measurable premises variables. Another approach was proposed by Ma and Sun [32] for a fuzzy observer analysis and design of reduced-dimensional and fuzzy functional observer with a separation property.

In this paper, we extend the stability results given in [16] to the case when the states are not available for measurement and feedback in other terms fuzzy observer, by guarantying the stability of the whole system. The observer design is based on the T-S fuzzy model. Each fuzzy rule is responsible for observing the states of a locally linear subsystem [24-25], then a separation property is used to check the stability of the global system. The separation property was introduced by Jadabaie et al. [25] and Ma et al. [26] by two different approaches to assure an independent design for the controller and the observer and the stability of the global T-S system. Whereas, we applied in our proposal the Ma et al. method due to its simplicity, since it does not depend on the stability conditions but rather on the fuzzy Lyapunov functions. Indeed, the separation principle design proposed in [25] is not appropriated for the case of several stability conditions.

The reminder of this paper is organized as follows. Section II presents a short outline of the fuzzy controller design proposed in [16] based on the PDC concept for the stabilization of T-S fuzzy systems. Section III and section IV discusses the proposed fuzzy observer and the separation property principle used for assuring the stability of the global system. Finally, in section V, a simulation example shows the effectiveness of the new observer/controller design. Concluding remarks are given in section VI.

II. FUZZY CONTROLLER DESIGN

Fuzzy controllers are required to satisfy $x(t) \to 0$ when $t \to \infty$, that implies stabilization of the fuzzy control system. A T-S fuzzy system is described by fuzzy IF-THEN rules that represent locally linear input-output relations of a system. The $i$th rule of this fuzzy system is of the following form:

Model Rule $i$:

IF $z_i(t)$ is $M_{i1}$ and... $z_p(t)$ is $M_{ip}$

THEN 

$$
\begin{align*}
  x(t) &= A_i x(t) + B_i u(t) \\
  y(t) &= C_i x(t), \quad i = 1, 2, \ldots, r
\end{align*}
$$

(1)

where $z(t) = [z_1(t), \ldots, z_p(t)]$ is the premise variables vector, $x^T(t) = [x_1(t), \ldots, x_r(t)]$ is the state vector, $u(t) = [u_1(t), \ldots, u_n(t)]$ is the input vector, $r$ is the number of fuzzy rules, and $M_{ip}$ is a fuzzy set. The final outputs of the fuzzy system are inferred as follows:

$$
\begin{align*}
  x(t) &= \sum_{i=1}^{r} h_i(z(t))(A_i x(t) + B_i u(t)) \\
  y(t) &= \sum_{i=1}^{r} h_i(z(t))C_i x(t)
\end{align*}
$$

(2, 3)

where $h_i(z(t))$ is the normalized weight for each rule, that is $h_i(z(t)) \geq 0$, $\sum_{i=1}^{r} h_i(z(t)) = 1$, and is given by:

$$
  h_i(z(t)) = \frac{w_i(t)}{\sum_{i=1}^{r} w_i(t)} \quad \text{where} \quad w_i(t) = \Pi_{j=1}^{r} M_{ij}(z_j(t)),
$$

$M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in $M_{ij}$.

The PDC scheme that stabilizes the Takagi-Sugeno fuzzy system was proposed by Wang et al. [8, 13] as a design framework for fuzzy control. The PDC controller is given by:

$$
u(t) = -\sum_{i=1}^{r} h_i(z(t)) F_i x(t).$$

(4)

The goal is to find appropriated $F_i$ gains that ensure the closed loop stability. In this sense and by substituting (4) in (2), we obtain the Takagi-Sugeno closed loop fuzzy system as follows:

$$
x(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) [A_i - B_i F_j] x(t)
$$

(5)

which can be rewritten as

$$
x(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) G_{ij} x(t) + 2 \sum_{i=1}^{r} \sum_{j<i}^{r} h_i(z(t)) h_j(z(t)) \left[ G_{ij} + G_{ji} \right] x(t)
$$

(6)

where $G_{ij} = A_i - B_i F_j$ and $G_{ii} = A_i - B_i F_i$.

The stabilization of a feedback system containing a state feedback fuzzy controller has been extensively considered. The objective is to select $F$ to stabilize the closed loop system. The stability conditions corresponding to a quadratic Lyapunov function were derived by Tanaka and Sugeno in [7]. They give sufficient conditions for stable fuzzy models based on Lyapunov approach. Due to their property of conservatism reduction, a fuzzy Lyapunov function is defined: $V(x(t)) = \sum_{i=1}^{r} h_i(z(t)) x^T(t) P x(t)$ [7, 15], for studying the stability and stabilization of a Takagi-Sugeno fuzzy system (2). In this context new non-quadratic stability conditions were proposed under two assumptions in [16].

III. FUZZY OBSERVER DESIGN

In practice if states are not available for measurement and feedback, an observer is needed. The objective is that the estimation error that is the difference between the system states and the observer states tends to zero, in other terms fuzzy observer is required to satisfy
the estimated state variables, designed via the PDC. The states vector estimated by the fuzzy observer that is following form:

Observer rule i :

IF \( z_i (t) \) is \( M_{a_i} \) and... \( z_p (t) \) is \( M_{a_p} \)

THEN \[
\hat{x}(t) = A_i \hat{x}(t) + B_i u(t) + K_i (y(t) - \hat{y}(t)) \] (7)

where the premise variables \( z(t) \) are independent from the estimated state variables, \( K_i, \ i = 1,..., r \) are the observation gain matrices and \( \hat{y}(t) \) is the estimated output. Then, the final outputs of the fuzzy observer are inferred as follows:

\[
\hat{x}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i \hat{x}(t) + B_i u(t)) \]

\[
\hat{y}(t) = \sum_{i=1}^{r} h_i(z(t))C_i \hat{x}(t) \] (8)

By substituting (3) and (9) into (8), we obtain:

\[
\hat{x}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i \hat{x}(t) + B_i u(t)) + \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t))h_j(z(t)) K_{ij} C_j (x(t) - \hat{x}(t)) \] (10)

that can be written as

\[
\hat{x}(t) = \sum_{i=1}^{r} h_i(z(t))h_j(z(t))[A_i - K_i C_j] \hat{x}(t) + B_i u(t) + K_i C_i x(t) \] (11)

The controller is also based on the estimate of the state, i.e., we have:

\[
u(t) = -\sum_{i=1}^{r} h_i(z(t)) F_i \hat{x}(t) \] (12)

Using (12) instead of (4) in (2) we obtain:

\[
x(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t))h_j(z(t))(A_i x(t) - B_i F_j \hat{x}(t)) \] (13)

Defining the estimation error as \( \tilde{x} = x - \hat{x} \), and subtracting (13) from (11), we obtain:

\[
\dot{\tilde{x}}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t))h_j(z(t)) [A_i - K_i C_j] \tilde{x}(t) \] (14)

The design of the fuzzy observer is to determine the local gains \( K_i \) using stability conditions proposed in [16], such that the estimation error tends to zero. Hence, the observer dynamics is stable if there exist positive definite matrices \( P_{a_1}, P_{a_2}, ..., P_{a_r} \) and matrices \( K_1, K_2, ..., K_r \) such that the following is satisfied:

\[
P_{a_i} > 0, \quad i = 1, 2, ..., r \]

\[
\sum_{i=1}^{r} P_{a_i} + \left( G_i^T P_{a_i} + P_{a_i} G_i \right) < 0, \quad i, j = 1, 2, ..., r \] (16)

\[
\left\{ \begin{array}{l}
\left[ G_{x} + G_{y} \right] \geq 0, \\
\forall i, j, \leq \frac{1}{2}, r \\
\left[ 1 \times T (0) \right] \\
\left[ x(0) \right] \geq 0, \text{for } i = 1, ..., r \\
\left[ \phi_i P_{a_i} W_{a_i} \right] \left[ \phi_i I \right] \geq 0, \forall i, j, \leq \frac{1}{2}, r 
\end{array} \right. \] (17)

\[
\left\{ \begin{array}{l}
\left[ 1 \times T (0) \right] \\
\left[ x(0) \right] \geq 0, \text{for } i = 1, ..., r \\
\left[ \phi_i P_{a_i} W_{a_i} \right] \left[ \phi_i I \right] \geq 0, \forall i, j, \leq \frac{1}{2}, r 
\end{array} \right. \] (18)

where \( G_x = A_x - K_x C_x \), \( G_y = A_y - K_y C_y \), and \( W_{a_i} = \xi_{a_i}(A_y - K_y C_y) \). These inequalities can be recast in terms of LMIs by the following changes of variables:

\[
P_{a_i} = X_{a_i}^{-1}, \forall i \in \{1, 2, ..., r\} \\
X_{a_i} = \alpha_i X_{a_i} \text{ s.t } \alpha_i = 1/\beta_i \forall i, j \in \{1, 2, ..., r\} \text{ and } i \neq j \\
K_i = \beta_i K_i \text{ s.t } \beta_i = 1/\beta_i \forall i, j \in \{1, 2, ..., r\} \text{ and } i \neq j \\
N_i = K_i C_i X_{a_i} \forall i \in \{1, 2, ..., r\} 
\]

The coefficients \( \alpha_i, \beta_i, \text{ and } \phi_i \) for \( i, j, \rho = 1, 2, ..., r \) and \( i \neq j \), can be chosen heuristically according to the considered application. \( \alpha_i \) and \( \beta_i \) must be different from 1 (for \( i = j \); \( \alpha_i = \beta_i = 1 \)). However, selection of \( \xi_{a_i} \) is obtained from \( h_i(z(t)) \). Subsequently, we will consider that the premises variables do not depend on the estimated states \( \hat{x}(t) \).

IV. SEPARATION PROPERTY OF OBSERVER/CONTROLLER

By augmenting the states of the system with the state estimation error, we obtain the following 2n dimensional state equations for the observer/controller closed-loop system:

\[
\begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) \begin{bmatrix} A_i - B_i F_j \\ 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \] (20)

\[
y = \left[ \sum_{i=1}^{r} h_i C_i \right] \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} \] (21)

To show that the whole system above is stable, we must show that the separation property holds. For this purpose we suggest to extend the separation property principle proposed by Ma et al. in their paper [26] to the non-quadratic design that we propose in [16]. We have to construct a comparison system \( \tilde{w} = Aw \), where \( A \) is function of \( \gamma_i \) and \( \tilde{\gamma}_i, \ i = 1, 2, 3, 4 \). Then using the vector comparison principle, we can obtain that the whole system is globally asymptotically stable. The
separation property is expressed by the following theorem:

**Theorem 1:** [26]: If there exist two scalar functions $V(x): R^n \rightarrow R$ and $\tilde{V}(\tilde{x}): R^n \rightarrow R$ and positive real numbers $\gamma_1, \gamma_2, \tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\gamma}_3$ and $\tilde{\gamma}_4$ such that:

\[
\gamma_1 \| F \|^2 \leq V(x) \leq \gamma_2 \| F \|^2, \quad \tilde{\gamma}_1 \| \tilde{F} \|^2 \leq \tilde{V}(\tilde{x}) \leq \tilde{\gamma}_2 \| \tilde{F} \|^2
\]  
\[\frac{\partial V(x)}{\partial x} \sum_{i=1}^{n} \mu_i \mu_j \left( A_i - B_i F_j \right) x \leq -\gamma_3 \| F \|^2,
\]  
\[
\frac{\partial \tilde{V}(\tilde{x})}{\partial \tilde{x}} \sum_{i=1}^{n} \mu_i \mu_j \left( A_i - K_i C_j \right) \tilde{x} \leq -\tilde{\gamma}_3 \| \tilde{F} \|^2
\]

\[
\left\| \frac{\partial V(x)}{\partial x} \right\| \leq \gamma_4 \| F \| \quad \text{and} \quad \left\| \frac{\partial \tilde{V}(\tilde{x})}{\partial \tilde{x}} \right\| \leq \tilde{\gamma}_4 \| \tilde{F} \|.
\]  
(22)

Then, the whole system is globally asymptotically stable.

Hence, this Theorem shows that the fuzzy controller and the fuzzy observer can be designed to be stable independently and the whole system that is fuzzy controller and fuzzy observer is still stable. This Theorem is extended to the case of non-quadratic stability conditions where

\[
V(x) = \sum_{i=1}^{n} \mu_i x^T P_i x \quad \text{and} \quad \tilde{V}(\tilde{x}) = \sum_{i=1}^{n} \mu_i \tilde{x}^T P_i \tilde{x}.
\]  
(23)

The principle of this method is to find the scalars $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \tilde{\gamma}_1, \tilde{\gamma}_2, \tilde{\gamma}_3$ and $\tilde{\gamma}_4$ that satisfy inequalities (22-24) and then to satisfy the following inequality:

\[
\left\| \frac{\partial V(x(t))}{\partial x} \right\| \leq -\gamma_3 \| x(t) \|^2 - \tilde{\gamma}_2 \| \tilde{x}(t) \|^2
\]

\[\frac{\partial \tilde{V}(\tilde{x}(t))}{\partial \tilde{x}} \sum_{i=1}^{n} \mu_i \mu_j \left( A_i - K_i C_j \right) \tilde{x} \leq -\tilde{\gamma}_3 \| \tilde{F} \|^2
\]

\[\left\| \frac{\partial V(x(t))}{\partial x} \right\| \leq A \| x(t) \|
\]  
(25)

where

\[
A = \begin{bmatrix}
\frac{\gamma_1}{\gamma_2} & \frac{\gamma_1}{\gamma_2} \\
0 & -\frac{\gamma_4}{\gamma_3}
\end{bmatrix}
\]  
(26)

is a stability matrix. Hence the construction of the comparison system $w = Aw$, which is obviously globally asymptotically stable and the use of the vector comparison principle, allow us to verify that the whole system (20) is globally asymptotically stable (the proof is given in [26]). In this paper, we consider the continuous case and all results are easily transposable to the discrete case. We also note that the separation property is not applicable for the case when $z(t)$ is replaced by $\tilde{z}(t)$.

**V. DESIGN EXAMPLE**

This part presents the design example that illustrates the effectiveness of the proposed controller-observer design with the separation property principle for the stability checking of the global system, i.e. fuzzy model, fuzzy controller and fuzzy observer. The inverted pendulum on a cart equations of motion are [6]:

\[
\dot{x}_1(t) = x_2(t),
\]

\[
\dot{x}_2(t) = g \frac{\sin(x_1(t)) - \omega \Delta x_3^2(t) \sin(2x_1(t))}{4l/3 - \omega \cos^2(x_1(t))} - \omega \frac{\cos(x_1(t))}{4l/3 - \omega \cos^2(x_1(t))} u(t)
\]  
(27)

where $x_1(t)$ denotes the angle (in radians) of the pendulum from the vertical and $x_2(t)$ is the angular velocity, $g = 9.8$ m/s$^2$ is the gravity constant, $m$ is the mass of the pendulum, $M$ is the mass of the car, $2l$ is the length of the pendulum, $u$ is the force applied to the cart (in Newton) and $\omega = 1/(m + M)$. For the simulations, the values of the parameters are $m = 2.0$ kg, $M = 8.0$ kg, $2l = 1.0$ m. The system (27) is modeled by the following two fuzzy rules:

Rule 1: IF $x_1(t)$ is about 0

\[\text{THEN } x(t) = A_1 x(t) + B_1 u(t), \quad y(t) = C_1 x(t),
\]

Rule 2: IF $x_1(t)$ is about $\pm \pi/2$ \{ $x_1 < \pi/2$ \}

\[\text{THEN } x(t) = A_2 x(t) + B_2 u(t), \quad y(t) = C_2 x(t),
\]

where

\[
A_1 = \begin{bmatrix}
0 & 1 \\
\frac{2g}{4l/3 - \omega \cos^2(x_1(t))} & 0
\end{bmatrix},
\]

\[
A_2 = \begin{bmatrix}
0 & 1 \\
\frac{2g}{4l/3 - \omega \cos^2(x_1(t))} & 0
\end{bmatrix},
\]

\[
C_1 = C_2 = \begin{bmatrix}
1 \\
0
\end{bmatrix},
\]

\[
B_1 = \begin{bmatrix}
0 \\
\frac{g}{4l/3 - \omega \cos^2(x_1(t))}
\end{bmatrix},
\]

\[
B_2 = \begin{bmatrix}
0 \\
-\frac{g}{4l/3 - \omega \cos^2(x_1(t))}
\end{bmatrix}, \beta = \cos(88^\circ),
\]

The control objective for this example is to balance the inverted pendulum for the approximate range $x_1 \in (-\pi/2, \pi/2)$ by using our fuzzy controller. The PDC control laws are as follows:

Rule 1: IF $x_1$ is about 0 THEN $u(t) = -F_1 \dot{x}_1(t)$

Rule 2: IF $x_1$ is about $\pm \pi/2$ THEN $u(t) = -F_2 \dot{x}_1(t)$

whereas the observer rules are:

Rule 1: IF $x_1$ is about 0

\[\text{THEN } \dot{x}_1(t) = A_1 \dot{x}_1(t) + B_1 u(t) + K_1 C_1 (x_1(t) - \hat{x}(t))
\]

Rule 2: IF $x_1$ is about $\pm \pi/2$

\[\text{THEN } \dot{x}_1(t) = A_2 \dot{x}_1(t) + B_2 u(t) + K_2 C_2 (x_1(t) - \hat{x}(t))
\]

Hence the control law that guarantees the stability of the fuzzy model and the fuzzy observer system is given by:

\[u(t) = -h_1(x_1(t)) F_1 \dot{x}_1(t) - h_2(x_1(t)) F_2 \dot{x}_1(t)
\]  
(28)

where $h_1$ and $h_2$ are the membership values of triangular form of rules 1 and 2, respectively. Applying our approach, the objective of balancing and stabilizing the pendulum and the estimation process are reached with success for different initial conditions of $x_1(0) \in (-\pi/2, \pi/2)$ and $x_1(0) \equiv 0$. We consider two cases:

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A. With pole placement

We choose the closed-loop eigenvalues $\begin{bmatrix} -3.0 & -5.0 \end{bmatrix}$ for $A_k - B_k F_1$ and $A_k - B_k F_2$, we then have:

$F_1 = \begin{bmatrix} -645.8824 & -160.0000 \\ 4.6525 & -1.5279 \end{bmatrix}$,

and the closed-loop eigenvalues $\begin{bmatrix} -50.0 & -60.0 \end{bmatrix}$ for $A_k - K_c C_1$ and $A_k - K_c C_2$,

and we have:

$K_1 = \begin{bmatrix} 110.0000 & 174.4694 \\ 321.5121 & -110.0000 \end{bmatrix}$.

Hence, for $\phi_1 = \phi_2 = 0.5$ and $\xi_{11} = -0.0637$, $\xi_{12} = 0.0637$, $\xi_{21} = -0.0637$, $\xi_{22} = 0.0637$ and for the initial condition $x(0) = \begin{bmatrix} \pi / 6.0 \end{bmatrix}$, we obtain the following results:

For the controller design:

$P_1 = 10^3 \begin{bmatrix} 3.0097 & 0.1270 \\ 0.1270 & 0.1262 \end{bmatrix} > 0$,

$P_2 = 10^3 \begin{bmatrix} 2.9974 & 0.1267 \\ 0.1267 & 0.1259 \end{bmatrix} > 0$.

For the observer design:

$P_{ov1} = 10^7 \begin{bmatrix} 7.3509 & -1.0523 \\ -1.0523 & 0.5042 \end{bmatrix} > 0$,

$P_{ov2} = 10^7 \begin{bmatrix} 7.3102 & -1.0432 \\ -1.0432 & 0.5000 \end{bmatrix} > 0$.

Fig. 1 and Fig. 2 show the closed loop behavior of the fuzzy controller and the fuzzy observer, for respectively, the inverted pendulum position, velocity and control force evolution of the closed loop system. The stability of the whole system (fuzzy controller - fuzzy observer - fuzzy model) is verified applying the vector comparison principle, having two scalar functions $V(x)$, $\hat{V}(\hat{x})$ and positive real numbers obtained from simulation and that satisfy the inequality (22-24), their values are:

$\gamma_1 = 4.6738 \times 10^7$, $\gamma_2 = 4.6039 \times 10^8$,

$\gamma_3 = 1.2411 \times 10^8$, $\gamma_4 = 4.3764 \times 10^8$,

$\gamma_5 = 1.3174 \times 10^8$, $\gamma_6 = 4.4097 \times 10^8$,

$\gamma_7 = 5.0038 \times 10^8$, $\gamma_8 = 4.3820 \times 10^8$.

Comparing our results with those obtained for the same example with a pole placement in [24], our results are very interesting, since on one side the stability design that depends on non-quadratic stability conditions is less conservative [16] and on the other side the separation property design is very flexible since it do not depends on the stability conditions but directly on the Lyapunov functions. In fact, the separation property proposed by [24] depends on the stability conditions, it requires to find a positive $\lambda$ such that the bloc diagonal matrix $\hat{P}$ satisfies the quadratic stability conditions of the augmented system and is given by $\hat{P} = \text{diag}[P_k, P_o]$ where $P$ and $P_o$ are respectively the positive definite matrices of the controller and the observer. Applying Jadababie approach, the following results (Fig. 3 and Fig. 4) are obtained for the same pole placement.

$\lambda > 0.0381$

$P = 10^3 \begin{bmatrix} 4.2519 & 0.1328 \\ 0.1328 & 0.1381 \end{bmatrix} > 0$,

$P_o = 10^3 \begin{bmatrix} 1.0315 & -0.1496 \\ -0.1496 & 0.0714 \end{bmatrix} > 0$.
Fig. 4: Inverted pendulum control evolution with pole placement: Jadbabaie approach

**B. Without pole placement**

For \( \phi_1 = \phi_2 = 1, \) \( \xi_{11} = -0.0064, \) \( \xi_{12} = 0.0064, \) \( \alpha_{21} = 0.0064, \) \( \alpha_{22} = 0.0064, \) \( \beta_{12} = 1, \) \( \beta_{21} = 1, \) we obtain the following: \( P_1, P_2, F_1, F_2, P_{01}, P_{02}, K_1 \) and \( K_2 \) for the initial condition \( x(0) = \begin{bmatrix} \pi/3 & 0 \end{bmatrix}^T \):

\[
P_1 = \begin{bmatrix} 0.6122 & 0.2216 \\ 0.2216 & 0.0878 \end{bmatrix} > 0,
F_1 = \begin{bmatrix} -937.3591 \\ -294.8718 \end{bmatrix},
F_2 = 10 \begin{bmatrix} -6.0636 \\ -2.0828 \end{bmatrix},
P_{01} = \begin{bmatrix} 0.1459 & -0.0034 \\ -0.0034 & 0.0095 \end{bmatrix} > 0,
P_{02} = \begin{bmatrix} 0.0425 & -0.0061 \\ -0.0061 & 0.0062 \end{bmatrix} > 0,
K_1 = \begin{bmatrix} 9.6602 \\ 24.7890 \end{bmatrix},
K_2 = \begin{bmatrix} 5.9401 \\ 12.6046 \end{bmatrix}
\]

Also very good results are obtained for the stability of the whole system which is checked applying the vector comparison principle, and the positive values are:

\( \gamma_1 = 0.0868, \) \( \gamma_2 = 2.0100 \times 10^4, \) \( \gamma_3 = 0.1475, \) \( \gamma_4 = 2.0327 \times 10^5, \) \( \tilde{\gamma}_1 = 1.6547 \times 10^{-6}, \) \( \tilde{\gamma}_2 = 1.9823 \times 10^5, \) \( \tilde{\gamma}_3 = 0.0053, \) \( \tilde{\gamma}_4 = 9.6191 \times 10^4. \)

According to these results, we can conclude that the proposed approach in this paper based on non quadratic stability conditions [16] and a vector comparison principle [26] presents very good performances (especially for case B) comparing to Jadbabaie approach [24] that is based on quadratic stability conditions, what makes it conservative. According to the control action of Fig. 2 and Fig. 4, we have a fast stabilization. Moreover, the proposed separation property of Jadbabaie becomes complex in the presence of several stability conditions what brings one to the Ma et al. separation property.
VI. CONCLUSION
In this paper a new fuzzy design procedure for fuzzy observer is discussed. The non-quadratic stability conditions also developed in [16] are used for the stabilization of Takagi-Sugeno fuzzy models; they are based on fuzzy Lyapunov functions and fuzzy state feedback laws. The controller and the observer are designed separately. The fuzzy controller guarantees the stabilization of the T-S fuzzy model, whereas the fuzzy observer guarantees that the estimation error for states converges to 0. However to check the stability of the whole system that comprises the fuzzy controller, the fuzzy observer and the fuzzy model, we applied a separation property based on a vector comparison principle. This principle conceived in the beginning for quadratic Lyapunov functions, is adapted in this paper to the case of non-quadratic Lyapunov functions, so as to obtain less conservative results. The design example allows us to assess the performances of the new proposed observer/controller design and to check the truth of the separation property and hence to prove the effectiveness of our proposal that is less conservative and a very flexible design.

REFERENCES

[27] K. Tanaka, T. Ikeda and H. O. Wang, "Fuzzy regulators and fuzzy observer: Relaxed stability conditions and LMI-


