Abstract— In ultra wideband (UWB) system, multiband orthogonal frequency division multiplexing (MB-OFDM) has been proposed to provide high-speed transmission for short-range wireless links. In this paper, 2 dimensional (2D) single parity check product code (SPCPC) is investigated in MB-OFDM UWB communication for Rayleigh fading channel and additive white Gaussian noise (AWGN) channel in order to improve the error performance. The encoder and parallel decoder of the 2D-SPCPC are explained. It is shown that the proposed model may provide significantly better error performance compared to the conventional MB-OFDM at low SNR, but with complexity tradeoff.

Keywords—MB-OFDM; turbo product code; ultra wideband

I. INTRODUCTION

The UWB communication system has become an emerging technology that offers great satisfaction for future short and medium range wireless communication networks. It also has a potential for providing high data rates at the same time has the robustness in multpath fading environment. Any wireless communication system with spectral occupancy more than 500 MHz or fractional bandwidth higher than 0.2 can be considered as UWB system. In 2002, Federal Communications Commission (FCC) regulation allowed UWB to be operated at 3.1 to 10.6 GHz band without license requirement. The approval comes with limited power signal in order to allow coexistence with traditional and protected wireless services to ensure a minor interference [1]. In order to provide a secure transmission, the power spectrum is embedded into the noise floor.

There are two techniques that have been proposed in UWB technology which are impulse radio (IR) and multiband orthogonal frequency division multiplexing (MB-OFDM). MB-OFDM UWB transmits several lower rate streams using different carriers with minimum frequency band (above 500MHz) instead of using the entire UWB frequency band.

In MB-OFDM UWB communication, the error correcting code proposed in IEEE 802.13a is using convolutional codes. However, turbo codes have more robust performance compare to convolutional code as it close to the Shannon’s channel capacity theorem in AWGN channel. In turbo codes, it can be categorized into convolutional turbo code (CTC) and turbo product code (TPC) according to the type of constituent encoder [2]. CTC is built from a parallel concatenation of Recursive Systematic Convolutional (RSC) encoders separated by a pseudo random interleaver while TPC consist of the product of two systematic block codes separated by uniform interleaver [2].

The performance of turbo code is much affected by different parameters such as component codes, block size, interleaver design and weight spectrum [3], [4]. Component codes of TPC are chosen to give specific code parameters for instance code rate, error correcting capability and codeword length. Generally, the linear block code family that are used as a component codes are Reed-Solomon (RS) codes [5], Bose-Chaudri-Hochquenghem (BCH) codes [6], Hamming codes [7] and single parity check (SPC) product codes [8]. The main challenge in TPC application is to reduce the overhead as low as possible which is able to sustain the quality of transmission. SPC product code (SPCPC) provides the fewest parity bits with the highest code rate among the other codes [12].

In multidimensional SPCPC, maximum a posteriori (MAP) decoder was shown to have very good error performance [9]. In [10], the performance improvement depends on the dimensionality of the SPCPC. However, as the dimensionality increased, the code rate is decreased. Moreover, the block length is not flexible. In order to overcome the flexibility of the block length, [11] proposed the use of parallel decoder. Later, [12] proposed multiple serial and parallel concatenated SPC. It was found that parallel decoder can give better error performance at low and moderate signal to noise ratio (SNR) but with complexity trade-off.

In the literature of MB-OFDM UWB, the application of turbo codes is very limited. For examples, concatenated Reed Solomon-convolutional codes (RS-CC) was explored in [13] under UWB channel and the performance is compared to convolutional coding and turbo coding. Even though the proposed coding outperforms turbo code and convolutional code, it occurs only at high SNR. In [14], the performance of turbo codes and repeat-accumulate (RA) codes as well as bit loading algorithm was studied. It is shown that using turbo coding, the power efficiency can be improves up to 5dB depending on data rates. Another type of turbo codes is low density parity check (LDPC) codes. It was studied in [15] using quasi-cyclic LDPC and the performance is compared with...
convolutional codes (CC) in MB-OFDM UWB system. Although it performs better than CC, the complexity of the decoder make it insufficient to be implemented. In this paper, SPCPC is adapted to MB-OFDM UWB communication as forward error control in AWGN and Rayleigh channel. Two dimensional (2D) product codes are applied to the system and parallel decoder is used.

The paper is organized as follows. The single parity check product code including the parallel decoder is described in Section II. In Section III, 2D-SPCPC in MB-OFDM UWB system is proposed. The simulation results are presented in Section IV and finally, conclusions are given in Section V.

II. SINGLE PARITY CHECK PRODUCT CODE (SPCPC)

Consider parallel SPCPC which have the same length in every dimension, the $i$-th component code can be defined as

$$ (n_i, k_i, \delta_i) = (kD + 1, kD, 2) $$

where $n_i$, $k_i$, $\delta_i$ (i=1,2…$D$) and $D$ stand for codeword length, number of information bits, minimum Hamming distance and dimension, respectively. For a 2D code, it consists of data block, parity checks on the rows, parity checks on the columns and parity on parity checks. Therefore the code rate can be given as

$$ R = \left( \frac{n - 1}{n} \right)^D $$

In this paper, the data is encoded using dimensional based reading order (DBRO) to obtain several distinct of the codeword sequences [18]. As an example of 2D-SPCPC, the encoding is performed by generating the even parity check bit for every rows and columns of the block code as illustrated in Figure 1. This code consists of the data block, the parity checks on row and column also parity on parity check bits for $n_1 = n_2 = n = 6$. The first and second possible codeword sequences, $X_1$ and $X_2$ in Fig. 1 (b) are obtained from DBRO algorithm which is given as

$$ X_i = \chi(e_1, e_2) \mid e_i = 1 + (l + n_i - 1) \mod n_i; e_2 = \left\lceil \frac{l}{n_i} \right\rceil $$

where the $l$-th bit of the 2D-SPCPC codeword sequence is the bit at $(e_1, e_2)$ in two dimensional coordinate of codeword block $\chi$ for $l = 1, 2, ..., N$. $N$ is the length of the codeword sequence and $\left\lceil \cdot \right\rceil$ is ceil function that defined the closest integer towards infinity

A. Parallel Decoder

The proposed parallel decoder structure in this paper is based on [12] with an extension of using weighting extrinsic information as illustrated in Fig. 2. In order to avoid the decoder to converge slowly, a non-zero weighting constant at the first iteration will be applied [2].

$$ \Lambda(b) = \log \left( \frac{P(b = 1 | Y)}{P(b = 0 | Y)} \right) $$

where $b$ represents the transmitted data bits. Equation (4) can be simplified as (5) since the second term is ignored for the assumption that all bits are equally likely. This given the soft channel output LLCR or channel output metric, $\Lambda(b)$ as

$$ \Lambda(b) = \log \left( \frac{P(Y | b = 1)}{P(Y | b = 0)} \right) $$

Figure 1. 2D-SPCPC Codeword, (a) two dimensions codeword block, (b) corresponding two possible codeword sequences

Figure 2. 2D-SPCPC parallel decoder with weighting extrinsic information
\[ \Lambda_i(b) = \log_2 \left( \frac{p(Y|b=1)}{p(Y|b=0)} \right) \]

\[ = \log_2 \left( \frac{1}{\sigma \sqrt{2\pi}} \exp \left( \frac{-(Y - HS_i)^2}{2\sigma^2} \right) \right) \]

\[ = \frac{1}{2\sigma^2} \left[ (Y - HS_i)^2 - (Y - HS_0)^2 \right] \]

where \( S_i \) is the hypothesis representation of \( b=1 \) and \( S_0 \) is the hypothesis representation of \( b=0 \). The \( \Lambda \) bits consist of LLR data bits that are passed to all component decoders and LLR parity bits are passed to the corresponding decoder. The decoding starts by computing extrinsic information \( \Lambda \) for the \( i \)-th data bit \( b_i \) using log likelihood algebra which is given as

\[ \Lambda_i(b_i) = 2(-1)^{b_i} \arctan \left( \frac{\tanh(\Lambda(p))}{2} \right) \prod_{j \neq i} \tanh(\Lambda(b_j)) \]

where \( \Lambda(p) \) is the LLR for the parity bit and \( \Lambda(b_j) \) is the LLR for the \( j \)-th data bit. The soft detected bits, \( \Lambda_i(b_i) \) is computed as

\[ \Lambda_i(b_i) = \Lambda_i(b_i) + \Lambda(b_i) + \Lambda_i(b_i) \]

The soft detected bits from all component decoder are summed up. Since the soft-detected bits have yielded from decoder, the first decoding iteration has been done. The received bits are obtained by applying hard-decision detector to the soft detected bit. For the next iteration, the extrinsic information from all component decoders is fed back to the input of all component decoders. The extrinsic information is used as a priori probability of detected bit. The decoding process will be terminated until a defined iteration.

### III. 2D-SPCPC IN MB-OFDM SYSTEM

In this paper, we investigate the performance of 2D-SPCPC in MB-OFDM UWB. Fig. 3 shows the block diagram of SPCPC MB-OFDM transceiver. The detail of SPCPC block is illustrated in Fig. 4. The scramble information bits are divided into a data frame with length \( K \) for \( K = \prod_{d=1}^D k_d \) and \( k_d \) are the length of component encoder input at dimension \( d \)-th. In this case, a data block size is \( k_1 \times k_2 \). The 2D data block is encoded with identical SPC component codes of \( (k_2, k_2, 2) \) and the resulting codeword is \( n_1 \times n_2 \). Using (3), the possible codeword sequence is selected and fed to the quadrature phase shift keying (QPSK) modulator.

\[ y = h \cdot s + z \]  

where vectors \( y \) and \( s \) denote received and transmitted symbol, respectively; \( h \) is a Rayleigh fading channel; and \( z \) is a complex Gaussian random vector with distribution \( z \sim \mathcal{CN}(0, \sigma^2) \).

### IV. SIMULATION RESULT

The simulation results for the proposed SPCPC MB-OFDM are presented. The parameters uses for UWB are based on the proposed ECMA standard [17] and the simulation parameters are listed in Table 1. MB-OFDM UWB uses convolutional coding as forward error control (FEC) and different code rate is achieved by puncturing the \( R=1/3 \) for \( K=7 \). However, in this paper, we only concentrate on \( R=1/3 \).

For SPCPC encoder, the component code \((111_2, 110_2, 2)\) is applied for code rate 0.98. The reason of choosing this component codes is to achieve the minimum of 30 OFDM
symbol transmissions as required by ECMA standard. The SPCPC parallel decoder shows that the data is converge at iteration equal to 4 [12], thus in this simulation, we decode the data up to 5 iteration.

![Table 1. Simulation Parameters](image)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Conv. coding</th>
<th>2D-SPCPC</th>
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</thead>
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<tr>
<td>FFT Size</td>
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<td>128</td>
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<tr>
<td>Code rate</td>
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<td>0.98</td>
</tr>
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<td>Decoding</td>
<td>Viterbi</td>
<td>MAP</td>
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<td>Modulation</td>
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<tr>
<td>No. of guard subcarriers</td>
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<td>10</td>
</tr>
<tr>
<td>No. of samples</td>
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<td>165</td>
</tr>
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</table>

Monte Carlo simulation is considered in this work. It is also assumed that channel estimation is perfect and the receiver is able to decode TFC well. In order to have a fair comparison, both systems used the raised cosine filter and least mean square (LMS) equalizer with training sequence to further improve the performance.

Fig. 5 compare the bit error rate performance for 12k bits transmission between conventional MB-OFDM and 2D-SPCPC MB-OFDM for code rate 0.33 and 0.98, respectively in AWGN and Rayleigh channel. We can realize that 2D-SPCPC performs better than convolutional codes. In AWGN channel for example, at least 2.2 dB enhancements can be achieved at BER $10^{-3}$ and nearly 4.3 dB in Rayleigh fading channel at the same BER. It needs to be noted that, we analyzed the system in one-tap Rayleigh fading channel with 130 Hz Doppler shift. Even though 2D-SPCPC has better error performance, the computational time is increased compare to conventional MB-OFDM.

![Figure 5. Performance of conventional MB-OFDM and 2D-SPCPC MB-OFDM in AWGN and Rayleigh fading channel.](image)

V. Conclusion

This paper has conducted a study of 2D-SPCPC encoder based on DBRO algorithm and its parallel decoder. The proposed codes is then applied into MB-OFDM UWB system and simulated in AWGN and Rayleigh fading channel. The simulation results show significant improvement in BER performance compared to conventional MB-OFDM UWB that used convolutional coding as FEC. In the future, the proposed code will be considered the channel estimation and simulate in IEEE 802.15.3a UWB channel for more realistic application.

REFERENCES
