FULL-DUPLEX FAST ESTIMATION OF ECHO AND CHANNEL RESPONSES IN THE PRESENCE OF FREQUENCY OFFSETS IN BOTH FAR ECHO AND FAR SIGNAL

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ABSTRACT

In a previous publication [1] we proposed a full-duplex fast training procedure for simultaneously estimating echo and channel responses. It reduces the tap-setting time to half of that required by the traditional half-duplex fast training schemes. However, its estimation accuracy may be degraded by the frequency offsets in both far echo and far signal that are caused by the analog carrier network. In this paper, we extend the previous work and develop a new algorithm that can compensate for the frequency offsets in both far echo and far signal. The performance of the new method is analyzed in terms of mean-square error. Simulation results are presented to confirm the analysis.

1. INTRODUCTION

The problem of reducing the initialization time of the echo cancellers and the channel equalizers in the modems is extremely important for high-speed full-duplex data transmission over two-wire lines. A wide variety of approaches to this problem have been proposed and evaluated [2], [3], although most of them were operated in the half-duplex transmission mode during the startup period. In our recent publication [1] we proposed a full-duplex fast training procedure for simultaneously estimating the echo and channel responses. Its novelty was that the echo cancellers and the channel equalizers at both ends can be trained simultaneously, rather than separately. It reduces the tap-setting time to half of that required by the traditional half-duplex fast training schemes. However, its estimation accuracy may be degraded by the frequency offsets in both far echo and far signal that are caused by up- and down-frequency shifts in the analog carrier network where the modulators and the demodulators are not exactly matched.

When passing through an analog carrier system, the data signal is first modulated to the frequencies suitable for transmission over the network, and then demodulated to the baseband signal at the end of transmission. Since the modulator and the demodulator, which are distant located, are not synchronized in frequency, the demodulated baseband signal may be distorted by the frequency offset (difference) between them. This frequency offset can go as high as 7 hertz [4], [5] and is commonly present in the far signal. The far echo is the interfering signal that loops back to the modem through the carrier system. Since it traverses the carrier system twice and the same oscillator is often used for the modulator and the demodulator co-located, it is to be expected that frequency offset will be less prevalent in the far echo than in the far signal. However, frequency offsets with values of 1 hertz or less have been observed in the far echoes on the international calls [6].

In this paper, we will extend the previous work [1] and develop a new algorithm that can compensate for the frequency offsets in both far echo and far signal. In the mathematical modeling of the received samples' vector, we discover that these frequency offsets have the effects of premultiplying the near-end and far-end data matrices by the diagonal phase rotation matrices associated with the far echo and the far signal, respectively. By constructing the appropriate estimation matrix, the far-signal component embedded in the received samples' vector can be effectively canceled when estimating the echo response. After obtaining the estimate of the echo response, the echo component embedded in the received samples' vector can be canceled by its synthesized counterpart. As a result, we obtain the estimate of the far-signal vector. The channel response can thus be calculated based on this far-signal estimate by using the appropriate pseudoinverse matrix.

2. ESTIMATION OF THE ECHO RESPONSE

By denoting the near echo, the far echo, the remote signal, and the channel noise as \( e(n) \), \( e_r(t) \), \( s(t) \), and \( v(t) \), respectively, the symbol rate \((1/T)\) sampled signal received at one end can be expressed as

\[
x(n) = e(n) + e_r(n) + s(n) + v(n),
\]

where we assume that the transmitters and receivers at both ends have exactly the same symbol rates, and the echo path and the transmission channel have finite complex impulse responses. Let \((a(n))\) and \((b(n))\) denote the data sequences transmitted by end A and end B, respectively, \((g(l))\) and \((h(i))\) denote the sampled impulse responses of the echo path and the channel, respectively, \(N1, N2, N3, \) and \(M\) denote the spans of the near echo, the far echo, and the channel response, respectively, \(N1\) and \(n\)
denote the transmission delays of the far echo and the far signal, respectively, \( f_1 \) and \( f_2 \) denote the frequency offsets in far echo and far signal, respectively. Each item in (1) can then be written as

\[
x(n) = \sum_{i=1}^{N1} a(n-i)g(i) + \sum_{i=1}^{N3} a(n-N2-1)g(N2+1)a(n)
\]

\[
+ \sum_{i=1}^{N} b(n-n'-i)h(i)\beta(n) + v(n),
\]

where \( a(n) = \exp(j2\pi n f_1 T) \) and \( \beta(n) = \exp(j2\pi n f_2 T) \) are the phase rotation components introduced by the frequency offsets \( f_1 \) and \( f_2 \), respectively.

In a manner similar to (1), the training data signals \( \{a(n)\} \) and \( \{b(n)\} \) are designed to be the two mutually orthogonal periodic sequences with the following requirements: \( a(1+n) = a(n) \) and \( b(n+L) = b(n) \), where the half period \( L \) is between \( N1+N3 \) and \( 2(N1+N3)-1 \) and can be calculated using the recursive algorithm developed in [3]. It is easy to verify that these two sequences have zero cross correlations for any integer \( i \) and \( j \).

As discussed in [1], we need 2L received samples \( x = [x(1) x(2) \cdots x(2L)]^T \) to estimate the echo and the channel responses, where the superscript "\(^T\)" designates the transpose of a vector or a matrix. From (2), \( X \) can be expressed in a matrix form as

\[
X = A_nG_n + \Gamma A_rG_r + \Phi B(M,n')H + V.
\]

where

\[
G_n = [g(1) \; g(2) \; \cdots \; g(N1)]^T,
\]

\[
G_r = [g(N2+1) \; g(N2+2) \; \cdots \; g(N2+N3)]^T,
\]

\[
H = [h(1) \; h(2) \; \cdots \; h(M)]^T,
\]

\[
A_n = \begin{bmatrix}
a(0) & a(-1) & \cdots & a(1-N1) \\
a(1) & a(0) & \cdots & a(2-N1) \\
\vdots & \vdots & \ddots & \vdots \\
a(2L-1) & a(2L-2) & \cdots & a(2L-N1)
\end{bmatrix}
\]

\[
A_r = \begin{bmatrix}
a(-N2) & a(-N2-1) & \cdots & a(1-N2-N3) \\
a(1-N2) & a(-N2) & \cdots & a(2-N2-N3) \\
\vdots & \vdots & \ddots & \vdots \\
a(2L-N2-1) & a(2L-N2-2) & \cdots & a(2L-N2-N3)
\end{bmatrix}
\]

\[
B(M,n') = \begin{bmatrix}
b(-n') & b(-n'-1) & \cdots & b(-n'-M+1) \\
b(-n'+1) & b(-n') & \cdots & b(-n'-M+2) \\
\vdots & \vdots & \ddots & \vdots \\
b(-n'+2L-1) & b(-n'+2L-2) & \cdots & b(-n'-M+2L)
\end{bmatrix}
\]

From (3), it can be seen that the frequency offsets in both far echo and far signal have the effects of premultiplying the data matrices \( A_n \) and \( B(M,n') \) by the phase rotation matrices \( \Gamma \) and \( \Phi \), respectively.

In order to cancel the far-signal component \( S = \Phi B(M,n')H \) embedded in the received samples' vector \( X \) when estimating the echo path response \( G \), we define a far-signal canceling matrix as

\[
C = \Phi^H A_n A_r.
\]

It follows that \( \Phi^H \Phi B(M,n) = [A_n A_r]H \Phi B(M,n) = 0 \) for any \( n \) and \( N \), where the superscript "\(^H\)" designates the complex-conjugate transpose of a vector or a matrix. Let \( A \) denote \( [A_n \; A_r] \). The pseudo-inverse of \( A \) is then defined as \( A^* = (C^H)\Phi^H \). Since the near-end transmitted data \( \{a(n)\} \) are exactly known by the canceller, the estimate of \( G \) can easily be obtained from

\[
\hat{G} = A^*X = A^*A^*G + A^*\Phi B(M,n')H = G + A^*V.
\]

3. ESTIMATION OF THE CHANNEL RESPONSE

In this section, we develop the channel estimation procedure. Let \( n' \) denote the estimate of the time delay \( n' \) and \( d \) denote the maximum absolute difference between \( n'' \) and \( n' \), that is, \( |n'-n''| \leq d \) or equivalently \( 0 \leq n'-n'' \leq 2d \). Here we assume that \( 2d+L-M \). By inserting appropriate number of dummy zeros, the far-signal vector \( S \) embedded in \( X \) can be written as

\[
S = \Phi B(M,n')H = \Phi B_{2L}(n')\begin{bmatrix}
H \\
0_{2L-M}
\end{bmatrix},
\]

where
and \( \Omega L = \mathbf{n} \) is a null vector having all of its elements zero. The subscript "2L-H" denotes its dimension. Since \( \{b(n)\} \) is a 2L-periodic sequence, it follows that \( \mathbf{B}_L(n') \) is a circular matrix and can be expressed as

\[
\mathbf{B}_L(n') = \mathbf{B}_L(0) \Omega^{n'} = \mathbf{B}_L(0) \Omega^{(n'-d) + (d+n'-n')},
\]

where \( n=n'-n \) is the estimation error of the far-signal transmission delay, and \( \Omega \) is a square permutation matrix

\[
\Omega = \begin{bmatrix}
0 & 0 & \cdots & 0 & 1 \\
1 & 0 & \cdots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & \cdots & 0 & 1
\end{bmatrix},
\]

Its order is assumed to be conformable to the matrix or vector with which it is associated. Let \( M'=M+2d \). Substituting (7) into (6) yields

\[
\mathbf{S} = \Phi \mathbf{B}(2L, n', d) \Omega^{d+n} \mathbf{H} = \Phi \mathbf{B}(M', n', d) \mathbf{H}',
\]

where

\[
\mathbf{H}' = \Omega^{d+n} \mathbf{H} = \begin{bmatrix}
\mathbf{H} \\
0_{2L-N}
\end{bmatrix},
\]

and \( \mathbf{B}(M', \mathbf{n}', \mathbf{d}) \) is a 2L x M' matrix whose element on row \( i \) and column \( j \) is \( b(d+i-n'-j) \), \( 1 \leq i \leq 2L; 1 \leq j \leq M' \). In the second equality of (10) we used the condition \( 0 \leq d+n'-n \leq 2d-2L-M \). We call \( \mathbf{H}' \) as the expansion vector of \( \mathbf{H} \).

Since the estimate of \( \mathbf{G} \) has already been obtained in (5), the replica of the echo component embedded in \( \mathbf{X} \) can be synthesized by

\[
\hat{\mathbf{G}} = \mathbf{AG} + \mathbf{AA}' \mathbf{V}.
\]

The estimate of far-signal vector can then be calculated from

\[
\hat{\mathbf{S}} = \mathbf{X} - \hat{\mathbf{G}} = \mathbf{X} - \mathbf{AG} - \mathbf{AA}' \mathbf{V} = \mathbf{S} - \mathbf{AA}' \mathbf{V}.
\]

Substituting (9) into (12), we have

\[
\hat{\mathbf{S}} = \Phi (\mathbf{B}(M', \mathbf{n}', \mathbf{d}) \mathbf{H}') - \mathbf{AA}' \mathbf{V}.
\]

Let \( \mathbf{B}(n') = \Phi (\mathbf{B}(M', \mathbf{n}', \mathbf{d}) \mathbf{H}') \). The pseudoinverse of \( \mathbf{B}(n') \) is then defined as \( \mathbf{B}(n')^{-1} = \mathbf{B}(n')' \mathbf{B}(n')^{-1} \mathbf{B}(n') \). The optimum solution to \( \mathbf{H}' \) can thus be obtained from the estimate of \( \mathbf{S} \) by using \( \mathbf{B}(n') \):

\[
\mathbf{H}' = \mathbf{B}(n')' \mathbf{S} = \mathbf{B}(n')' \mathbf{S} - \mathbf{B}(n')' \mathbf{AA}' \mathbf{V} = \mathbf{B}(n')' \mathbf{B}(n') \mathbf{H}' + \mathbf{D}(n') \mathbf{V} = \mathbf{H}' + \mathbf{D}(n') \mathbf{V},
\]

where \( \mathbf{D}(n') \) is defined as \( \mathbf{D}(n') = \mathbf{B}(n')' \mathbf{AA}' \). The most possible estimate of \( \mathbf{H} \) is then determined from \( M \) consecutive elements of \( \mathbf{H}' \) that give the maximum energy. To be more specific, let \( \mathbf{h}(i) \) denote the \( i \)th element of \( \mathbf{H}' \). For each time index \( n \) from 0 to 2d, a sum of \( M \) successive squared values is calculated by

\[
\epsilon(n) = \sum_{k=1}^{M} |\mathbf{h}(k+n)|^2, \text{ } 0 \leq n \leq 2d.
\]

Then the maximum value of \( \epsilon(n) \) is searched over \( n \) from 0 to 2d. Let \( \mathbf{n} \) denote the particular value of \( n \) which maximizes \( \epsilon(n) \). We may choose

\[
\hat{\mathbf{h}}(\mathbf{n}+1) \mathbf{h}(\mathbf{n}+2) \cdots \mathbf{h}(\mathbf{n}+M)\mathbf{T}
\]

as the optimum choice of \( \mathbf{H} \). From (10) and (14), the estimate of \( \mathbf{H} \) can be expressed as

\[
\hat{\mathbf{H}} = \mathbf{H} + \mathbf{D}_\mathbf{A}(\mathbf{n}) \mathbf{V},
\]

where \( \mathbf{D}_\mathbf{A}(\mathbf{n}) \) is defined as a \( M \times 2L \) submatrix of \( \mathbf{D}(\mathbf{n}) \). Its element on row \( i \) and column \( j \) is the element of \( \mathbf{D}(\mathbf{n}) \) on row \( \mathbf{d}+\mathbf{i} \) and column \( j \).

Before ending this section, it is worth mentioning here that the frequency offsets, as well as the far-signal and the far-echo delays, can be estimated during the process of far-echo detection by using the methods discussed in [2], [3].

4. THE ANALYTICAL AND SIMULATION RESULTS

Suppose that \( \mathbf{v}(n) \) is a white channel noise with zero mean and constant variance \( \mathbf{v}^2 \). By using (5) and (16), the mean-square error (MSE) of the estimator due to channel noise can be easily calculated from

\[
\epsilon = \mathbf{E}(|\hat{\mathbf{G}} - \mathbf{G}|^2) + \mathbf{E}(|\hat{\mathbf{H}} - \mathbf{H}|^2) = \mathbf{E}(|\mathbf{A}' \mathbf{V}|^2) + \mathbf{E}(|\mathbf{D}_\mathbf{A}(n') \mathbf{V}|^2)
\]

\[
= \sigma^2 (\text{Trace} \mathbf{A}' \mathbf{A}) + L^{-1} \sum_{n'=1}^{L} \text{Trace} \mathbf{D}_\mathbf{A}(n') \mathbf{D}_\mathbf{A}(n'),
\]

where \( \mathbf{| \mathbf{A} |} \) stands for the modulus of a vector, \( \text{Trace} \) stands for the sum of the diagonal elements of a matrix, the time delay estimate \( n' \) modulo L is assumed to have a uniform distribution between 1 and L. The normalized MSE is then defined as

\[
\epsilon/\sigma^2 = \text{Trace} \mathbf{A}' \mathbf{A} + L^{-1} \sum_{n'=1}^{L} \text{Trace} \mathbf{D}_\mathbf{A}(n') \mathbf{D}_\mathbf{A}(n'),
\]

The above formula shows that the normalized MSE does not depend on the shapes of the echo and channel responses to be estimated, the ratio of
the far-signal power to the echo power, and the channel noise power. It depends on the training data sequences, their half period, the sum of the estimated coefficients, and the frequency offsets in both far signal and far echo.

As an example, the channel response used in our simulation is assumed to be the response of the cosine roll-off passband filter with its center frequency at 3/(4T)

\[ h(t) = \frac{\sin(\pi t/T)\cos(0.5\pi t/T)\cos(1.5\pi t/T)}{(\pi t/T)(1 - t^2/T^2)} \exp(-j\pi/4) \]  

(19)

where 1/T is the symbol rate, and the roll-off coefficient is assumed to be 0.5. The training sequences \( \{a(n)\} \) and \( \{b(n)\} \) are chosen as two "white" polyphase sequences:

\[ a(n) = \exp\{j\pi(n+k)\pi/L\}, \]

\[ b(n) = a(n)\exp\{j\pi n/L\}, \quad k = L \mod 2 \]  

(20)

Fig.1 shows the normalized MSE as a function of the frequency offset in far signal for the far-signal-to-channel-noise ratio of 40 dB, where the frequency offset in far echo is assumed to be 1 hertz. We use \( M=16, N=32, \) and \( L=48 \) in our simulation. The simulation results are averaged over \( L \) different far-signal delays ranging from 1 to \( L \). In the curves, the analytical and simulation results are indicated by "\( \Delta \)" and "\( x \), respectively. As can be observed, the normalized MSE is slightly increased with the increase of the frequency offset in far signal. However, even in the worst case of \( f_1=1 \) hertz and \( f_2=7 \) hertz, the estimation error is approximately 1 dB lower than that of the channel noise.

5. CONCLUSIONS

In this paper, a new full-duplex fast training algorithm has been developed to co-estimate the echo and channel responses in the presence of frequency offsets in both far signal and far echo that are caused by the analog carrier network. The phase rotation components introduced by these frequency offsets have been considered in designing the appropriate estimation matrices to compensate for their effects. When estimating the echo response, the far-signal component can be effectively canceled by the estimation matrix. When estimating the channel response, the echo component can be canceled by its synthesized counterpart. As a result, the joint estimation problem has been de-coupled into the process of estimating the echo response and channel response sequentially. The estimation performance has been analyzed and simulated in terms of mean-square error. The results show that the normalized MSE does not depend on the shapes of the echo and channel responses to be estimated, the ratio of the far-signal power to the echo power, and the channel noise power. It depends on the training data sequences, their half period, the sum of the estimated coefficients, and the frequency offsets in both far signal and far echo. The normalized MSE is slightly increased with the increase of the frequency offset in far signal. However, even in the worst case of far-echo and far-signal frequency offsets equal to 1 hertz and 7 hertz, respectively, the estimation error based on the complex samples is approximately 1 dB lower than that of the channel noise.

REFERENCES


