Rate Allocation for Satellite Systems with Correlated Channels Based on a Stackelberg Game

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Abstract— In this paper, we consider the problem of capacity allocation for fixed broadband (Ka band) satellite networks with a large coverage and correlated atmospheric channel conditions both in time and space. The transmitted bit rate adapts to the channel by varying the coding rate and the constellation. The network is operated by a single service provider. The model of interest is the downlink where the provider allocates capacity to users with Quality of Service (QoS) guarantees. We first analyze the optimal single-objective optimum allocation by which users’ utilities are maximized. We find a closed form for both the proportionally fair and opportunistic allocations. However, these allocations do not show any clear benefit for a profit-seeking service provider because both the efficiency of the network and the user satisfaction with the service cannot be controlled as it mostly depends on the random nature of the correlation properties of the channel. We then propose a Stackelberg game which provides an equilibrium that allows us to prove that 1) the proportionally fair allocation corresponds to a flat-rate pricing model (as used in today’s satellite systems) and 2) Pareto improving policies can be obtained by using differential pricing. From the simulation results, we show that the bit rate allocation based on the Stackelberg formulation yields a network efficiency which is in between the one achieved by the proportionally fair and the opportunistic allocations (as expected from the Pareto improvement), with the advantage that now the efficiency is under full control of the system designer. We also show that the higher the differential pricing the higher the network efficiency. As a general conclusion, both the efficiency of the network and the satisfaction of the users improve with our proposed scheme compared to current capacity allocations. Furthermore, the spatial nature of the atmospheric attenuation induces a high degree of channel correlation within the coverage. In other words, a large number of users can simultaneously be affected by either good or bad channel conditions for a relatively long period of time (in the order of seconds or even minutes) [1].

The introduction of adaptive transmission techniques such as Adaptive Coding and Modulation (ACM) helps counteract the channel slow fading by providing a sufficiently large dynamic range (e.g., 21 dB in Digital Video Broadcasting Second Generation, DVB-S2 [2]). Such a dynamic range poses a challenge on the capacity allocation. The support of unicast or multicast multimedia transmission over these networks shall, therefore, rely on powerful Adaptive Call Admission Control (ACAC) schemes. However, the challenge of allocating the time-location variant capacity plays a critical role in the overall network operation towards efficient operation. The similar problem has been addressed by terrestrial wired and wireless networks designers.

In wired networks, the links’ capacities are fixed and thus the overall capacity dynamics depend on the level of congestion. In this case, the problem can be treated at transport layer as an end-to-end congestion control problem to determine the equilibrium properties of distributed mechanisms such as rate control and active queue management [3],[4]. This problem has been solved for elastic traffic, treated as an economic problem. Fair and efficient allocation of resources to users is formulated as a maximization of utilities (also called social welfare). Moreover, a distributed solution is straightforward by formulating the dual problem where shadow prices correspond to congestion signals from the links, in fact, TCP congestion control has been proved to be a primal-dual algorithm maximizing certain utility functions [5].

When the network is wireless, the dynamics are related to link capacity drop rather than congestion and thus the problem has been primarily addressed at the physical and access layers. In particular, we have reviewed the solutions on the literature related to cellular wireless networks, which have a star topology with two levels of network elements: Base stations controlling cells and distributed users. This is also the case of our satellite network where the two levels of network elements are: gateways controlling beams and distributed users. The main differences are two. First, there is a scale factor both in space and time due to the bigger dimensions of satellite networks. Second, the nature of the
channel is totally different both for the fixed and the mobile scenario. Furthermore, in our specific case of fixed terminals and Ka band, a key difference is also the presence of channel correlation in time and space both within the beams and among the beams.

The study of adaptive capacity allocation for cellular networks has been mostly centered on maximizing one single-objective utility functions rooted in economics [6]-[10]. While [6]-[8], [16] use utility functions that model user’s satisfaction, [10]-[12] focus on modeling resource pricing to find the optimal allocation. Recently, multi-objective functions have also been proposed such as in [12], [13]. In these works, a two-level sequential game-theoretic formulation is proposed. This alternative formulation allows serially assessing the best different responses of the players and eventually finding the equilibrium.

Since our system model entails substantial differences with terrestrial models, we apply both approaches. We first apply the social welfare maximization approach by using a slightly modified version of the commonly used utility functions that better model user’s satisfaction in a satellite scenario. We find out that the optimal allocation shows an unclear benefit for a profit-seeking satellite service provider and among the beams.

Furthermore, in our specific case of fixed terminals the channel is totally different both for the fixed and the mobile scenario. Furthermore, in our specific case of fixed terminals and Ka band, a key difference is also the presence of channel correlation in time and space both within the beams and among the beams.

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The paper is structured as follows: Section II introduces the broadband satellite system model including channel and QoS assumptions. In Section III, both the social optimum capacity allocation and the opportunistic allocation are derived analytically from convex formulations. A Stackelberg game formulation is then presented in Section IV. Section V shows the numerical results that illustrate the theoretical findings. Finally, Section VI presents the conclusions.

II. SYSTEM MODEL

We assume the downlink of a large-scale broadband satellite communications system. The coverage region is attained through multiple spot beams covering large areas by making use of frequency-reuse as in cellular systems. Transmission to each beam is based on time division and power is assumed to be optimally allocated to the beams according to the per-beam traffic requirements.

Due to orthogonality in time, at any given time slot only one user is supported in each beam which is interfered by the co-channel transmissions. The spectral efficiency of the i-th user within the coverage of a specific beam is

\[
\eta_i = \frac{R_i}{B_i} = \log \left( 1 + \frac{G_i H_i P_i}{\sigma_i^2 + \sum_{j: i \neq j} G_j H_j P_j} \right),
\]

where \( R_i \) and \( \sigma_i^2 \) is the i-th user maximum bit rate and receiver noise power respectively, \( B_i \) is the total bandwidth of the beam, \( P_i \) is the downlink transmitted power towards the i-th beam, \( G_i \) is the satellite antenna gain from the j-th beam to the i-th user, \( H_i \) is the i-th user channel power gain and \( H_j \) is the channel power gain from the j-th beam to the i-th user. There are \( J \) co-channel beams. In the downlink, desired and interfering signals are transmitted from virtually the same point: the satellite antenna and hence \( H_{ij} = H_{ji} \).

We consider adaptive transmission by means of a total number of \( M \) possible transmission modes available to the transmitter (mode meaning combination of coding rate and constellation). This means that within the coverage, users will have a spectral efficiency \( \eta_i \in \{ \eta_1, \eta_2, ..., \eta_M \} \). Note that this assumption corresponds to a given granularity, \( \Delta_p \), of the Signal-to-Interference plus Interference Ratio (SINR) throughout the coverage, i.e. \( SINR_p = SINR_{p-1} + \Delta_p \), for \( p = 1, 2, ..., M \).

We assume the channel conditions are time and location variant and, hence, the optimization is performed within a time window, \( W \), during which the channel can be considered spatially quasi-static. This time window can range from milliseconds in a terrestrial setting but up to a few minutes in a satellite fixed scenario transmitting in Ka band. During the time window, the active users within the coverage are grouped according to their channel condition into \( m \) groups, each group described by the spectral efficiency they can afford. Up to \( M \) groups can be simultaneously active within the coverage (i.e., the same number as the number of different spectral efficiencies). In a normal scenario, \( m \leq M \). Fig. 1 shows a simple example of \( m = 3 \) in one of the beams of a multibeam coverage.

Under uniform demographic density, a reasonable assumption is that every group has the same number of users. The more general case of non-uniform demographic density assumes that \( n_i \) active users are transmitted with the i-th spectral efficiency, \( i = 1, 2, ..., m \), \( m \leq M \), and \( \sum_{i=1}^{m} n_i = N \), where \( N \) is the total number of users within a beam.
As for the QoS model, we assume $Q$ levels of differentiated quality, each level will be characterized by a minimum guaranteed bit rate allocated. Further, we assume that $L$ out of $Q$ levels are lossless while losses are allowed in the other $Q-L$ levels. All Internet traffic entering the satellite network is assigned to the lossy $Q-L$ levels. It is apparent that these assumptions correspond to a DiffServ QoS model for elastic traffic and external traffic and an IntServ QoS model approach for internal traffic [18]. In order to attain this, we assume a pre-distribution of the instantaneous capacity among the $Q$ levels. We further assume that drop in capacity can be absorbed by the system by transferring capacity from the lossy levels to the lossless levels and stability is secured by an admission control algorithm.

Our model is fully in line with modern design of adaptive satellite systems and its details are out of the scope of this paper. We simply assume here that during the optimization time window, $W$, the reserved allocated capacity for the downlink(s) of interest is given in terms of an available symbol rate, $S$. The allocation can be per-formed either for only one beam or for a set of co-channel beams. We do not take into account upper layer adaptive mechanisms such as rate adaptation.

III. CENTRALIZED SINGLE-OBJECTIVE OPTIMIZATION

We analyze here the bit rate allocation among the users of destination beam(s) knowing the current pre-allocated capacity in symbol rate, $S$. This symbol rate corresponds to a variable bit rate that should be distributed during the time window $W$ among the target users, which are grouped into $m$, $m \leq M$ groups. Each group supports a minimum spectral efficiency $\eta_i$ and has a total number of users, $n_i, i=1,2, ..., m$, $m \leq M$. The problem is to find the optimal centralized capacity allocation algorithm to distribute the current available capacity among the target users.

We modify the $i$-th user utility function presented in [5] and assume full dissatisfaction until the minimum required bit rate is allocated, $\rho_c$. When more capacity than the minimum required is allocated, the increase of satisfaction is logarithmic. The utility function for the $i$-th group of users is then given by

$$U(R_i) = \begin{cases} 0 & 0 \leq R_i < \rho_i, \; i = 1,2, ..., m \\ \rho_i \log(R_i/\rho_i) & R_i \geq \rho_i, \end{cases}$$

where $R_i$ is the bit rate to be allocated to the $i$-th group of users. Note that the minimum guaranteed bit rate will depend on the QoS. Without loss of generality, we assume here that all the users within a group belong to the same QoS level.

A. Symmetrical Case

First we focus on the symmetric case, in which the number of users that belongs to each spectral efficiency is the same. The optimal centralized rate allocation is given by

$$\max_k \sum_i \rho_i \log \left( \frac{R_i}{\rho_i} \right)$$

s.t. $\sum_i \frac{R_i}{\eta_i} \leq S; \; R_i \geq \rho_i; \; i = 1,2, ..., m.$

This formulation implicitly states that the optimal rate will be equally distributed among users within each group. Note that the constraint can be expressed in the space of bit rates or in the space of symbol rates. After applying Lagrangian dual problem we obtain the following optimal allocation

$$R_i^* = \rho_i + \left[ \rho_i \frac{\eta_i}{\lambda} - \rho_i \right], \; i = 1,2, ..., m,$$

where $\lambda$ is the Lagrangian multiplier and should be computed iteratively so as the overall available capacity constraint is fulfilled with equality. Note that it is quite intuitive to see that when $\eta_i$ is small (bad channel conditions, low bit rate) the allocated rate will be set to the minimum. Instead, users with $\eta_i$ large (good channel conditions) will get more than the minimum. In this paper, we will assume that the reserved capacity (see Section II) guarantees the minimum, which is a realistic assumption in satellite system dimensioning. Therefore, the allocation of the remaining capacity can be obtained as

$$\max_k \sum_i \log(R_i)$$

s.t. $\sum_i \frac{R_i}{\eta_i} \leq S; \; R_i \geq 0; \; i = 1,2, ..., m.$

Note that since we assume the system dimensioning guarantees the minimum required requests, we optimize now over the remaining capacity, $S$, subject to positive allocation. The allocation in this case results

$$R_i^* = \eta_i \frac{S_i}{m}, \; i = 1,2, ..., m,$$

which is clearly opportunistic. Note that since the channel is correlated the opportunism will not only depend on the spectral efficiencies but also on the number of users within each group as shown herein below.

In order to enforce fairness among the correlated channels we introduce a weighted optimization as follows
the throughput is given by:

$$\max_i \sum w_i \log(R_i)$$

subject to:

$$\sum_i R_i \leq S, \quad R_i > 0, \quad i = 1,2,\ldots,m,$$

where $w_i$ is the positive weight the service provider chooses to give different priorities to the different physical layers. If we apply Lagrange multipliers we again obtain an analytical expression for the optimal allocation as follows:

$$R_i^* = \frac{w_i}{\sum_j w_j} \eta_i S_i, \quad i = 1,2,\ldots,m, \quad R_i > 0.$$  \hspace{1cm} (8)

With this policy, fairness at bit rate level can be tuned by appropriately selecting $w_i$. If we choose $w_i = 1/\eta_i$, the allocation becomes proportionally fair since it is the condition that corresponds to the variational inequality, i.e. $F^*\left(R_i^*, R_i^* - R_i^* \right) = 0$, $i = 1,2,\ldots,m$. This optimal value can be expressed as:

$$R_i^* = \frac{m}{\sum_{j=1}^m \frac{1}{\eta_j}} S_i, \quad i = 1,2,\ldots,m.$$  \hspace{1cm} (9)

It can be observed that the allocated bit rate is proportional to the harmonic mean of the spectral efficiencies. This is a typical result in other fields such as physics when different speeds are involved. In these cases, a measure of average speed is usually given in terms of the harmonic mean of all the speeds. In our problem, the spectral efficiencies are actually different speeds of transmission rates. Let us denote this optimal value as $R_i^{*pf}$.

### B. Non-Symmetrical Case

In the case of a non-symmetric distribution of users within the coverage, the optimal bit rate allocated to each user can be easily derived from the results above.

For the case of opportunistic allocation, we have that the optimal user throughput $r_i^*$ is:

$$r_i^* = \frac{\eta_i}{\eta_i} S_i, \quad i = 1,2,\ldots,m,$$

which means that the allocation becomes no longer opportunistic in terms of channel conditions (as it is the case in uncorrelated terrestrial channels) but in terms of number of users with the same spectral efficiency. Namely, we have that:

$$\frac{r_i^*}{r_j^*} = \frac{\eta_i n_i}{\eta_j n_j}, \quad i,j = 1,2,\ldots,m, \quad i \neq j.$$  \hspace{1cm} (11)

For the case of proportionally fair allocation, we have that the throughput is given by:

$$r_i^{pfr} = \frac{R_i^{pfr}}{n_i}, \text{ for } i = 1,2,\ldots,N.$$  \hspace{1cm} (12)

As a result of the above analysis, it is clear that the two centralized optimal solutions are not good options from the service provider’s profit-seeking perspective for the following reasons. In the opportunistic approach, the number of users using each spectral efficiency is a random variable that by no means can be controlled by the service provider and consequently the efficiency of the channel use cannot be controlled by the service provider. In the proportionally fair approach, a strictly fair allocation produces the subsequent system throughput decrease.

In the next section, we show that a formulation of the problem as a multi-objective function with differential pricing gives the service provider the means to depart from these two typical approaches and efficiently improve the overall throughput with a joint Pareto improvement respect to the proportionally fair that is independent of the correlation characteristics of the channel.

### IV. CENTRALIZED MULTI-OBJECTIVE OPTIMIZATION

Based on the previous analysis, we propose now a multi-objective function to design an allocation mechanism that results in an equilibrium with desirable properties for both the service provider and the users. Conversely to usual assumption of marginal costs, we consider a differential pricing approach [15], by which users are modeled with their different perception for the same amount of resources. In a satellite system, this is a quite reasonable assumption as satellite users are of different types within the same network. For example, in a typical commonly-defined “professional” user, the traffic is usually aggregated and symmetric. In the case of the commonly-defined “consumer” user, the traffic is bursty and very asymmetric. The problem is then to find an optimal global price that maximizes both the users’ and service provider interests.

We formulate the problem as a Stackelberg game that shall be solved at the central control station of the satellite network, where it is realistic to assume availability of the information characterizing the type of all users. Hence, a central capacity manager will be able to perform the Stackelberg game assuring that the total instantaneous capacity is used. We assume that the groups of users (grouped according to their spectral efficiencies) are the followers and the leader is the service provider.

#### A. The Followers’ Problem

Let us define “marginal willingness to pay” as the willingness to pay for an incremental unit of capacity and “marginal cost” as the cost of providing an incremental unit of capacity. If marginal willingness to pay exceeds marginal cost, there is an incentive for the service provider to allocate capacity at some price between the willingness-to-pay and the marginal cost of provision, thereby, increasing its profits [15].

Therefore, based on this differentiation basis, let us assume that the followers (groups of users) can be modeled with a maximum price that they are willing to pay, $p_i$, which includes the effect of the minimum requested bit rate. Note that this is equivalent to understand $p_i$ as the different perception that different user have for the allocated resource. Note also that this modeling is totally independent of the channel conditions and consequently also independent of the
correlation characteristics of the channel.

As a final interesting remark, we can see this introduction of differential pricing as an artificial way to introduce uncorrelation among target users, hence allowing decoupling the users’ utilities from the correlated characteristics of the channel.

The service provider is interested in knowing the response to its proposed price, \( P \), and hence the optimization to be performed is the following

\[
\max \left\{ p_i \log_2 \left( R_i \right) - PR_i \right\} \quad (12)
\]

where the utility is still of the same type as the one assumed in the previous optimizations. However, now we also introduce the effect of a global price since the final objective is that both service provider and final user are satisfied with the rate allocation.

Note that now the formulation is in terms of the users, not of groups of users using the same spectral efficiency since by introducing differentiated pricing we decouple both the service provider profit and user satisfaction from the channel conditions. Note also that this formulation of the problem is not the dual of the centralized formulation due to the introduction of a pricing scheme that is independent of the channel conditions. The optimal allocated bit rate admits a closed form and is

\[
R_i = \frac{P_i}{P}, \quad P > 0, \quad i = 1, 2, \ldots, N. \quad (13)
\]

The process would be repeated any time the channel conditions vary so the total available capacity is updated. The overall process would entail two steps. First, update channel feedback from the users and prices within the current optimization window. If the conditions have not changed, keep the current bit rate allocation, otherwise, obtain the optimal bit rates to allocate to each user according to the Stackleberg equilibrium allocation given by \( P^* \) and

\[
R_i^* = \frac{P_i}{P_i^*}.
\]

C. Stackelberg equilibrium and joint Pareto improvement

From the previous formulations and solutions, it is possible to extract the two following propositions:

**Proposition 1:** The Stackelberg trivial solution of all users having the same willingness to pay (null differential price) corresponds to an optimal global price, \( P^*_g \), that results in the same capacity allocation policy as the social welfare policy, the proportionally fair allocation, \( R^*_i \).

**Proof:** Both the proportionally fair solution and the Stackelberg followers’ solution are the unique solutions of concave functions subject to the same convex set of constraints. Since both optimal solutions happen to be constants (see (8) and (11)), the global optimal price \( P^*_g \) yields the same allocation as the proportionally fair solution.

**Proposition 2:** \( P^* > P^*_g \) yields a capacity allocation policy that provides a joint Pareto improvement with respect to the proportionally fair solution and is independent of the channel conditions and user locations.

**Proof:** The total aggregate user utility for the optimal global price is

\[
TU(P^*|P) = \sum_{i=1}^{N} p_i \log_2 \left( \frac{P_i}{P_i^*} \right) - \sum_{i=1}^{N} p_i, \quad \text{and the total aggregate revenue is } TR(P^*) = \sum_{i=1}^{N} p_i.
\]

A Pareto improvement respect to the proportionally fair solution is achieved whenever at least one player is better off without making any other player worse off. Hence, Pareto improvement can be ensured by checking that the aggregate utility second derivative is non-negative. We easily can obtain that

\[
\frac{\partial^2 TU(p_i)}{\partial p_i^2} = \frac{1}{p_i^2} \quad \forall i \text{ and } \frac{\partial^2 TR(p_i)}{\partial p_i^2} = 0.
\]

Note that a Pareto optimal policy would be obtained if both aggregate utilities present the same slope.

Therefore, we can conclude that our proposed rate allocation based on a Stackelberg game provides an equilibrium that allows us to prove that 1) the proportionally fair allocation corresponds to a flat-rate pricing model (as used in today’s
satellite systems) and 2) Pareto improving policies can be obtained by using differential pricing. In the following section, we comparatively analyze the rate allocations given by the different optimization in order to draw overall conclusions.

V. NUMERICAL RESULTS

We have set up a simulation framework in order to obtain numerical results where we simulate the channel conditions, from which we derive the corresponding spectral efficiencies, which are constant during the optimization time interval \( W \). As for the correlation properties of the channel, we simulate them by assuming different percentage of active users distributed between the spectral efficiencies.

In the first set of simulations, we assume a given time window \( W \), during which the channel conditions remain constant providing a remaining available capacity for allocation of \( S = 0.5 \times 10^5 \) bauds. We assume \( N = 100 \) active users within the beam coverage of interest. We assume two correlated groups of users as follows: 50% of the users have spectral efficiency \( \eta_1 = 1 \) and 50% of the users have spectral efficiency \( \eta_2 = 3 \). Fig. 2 shows the evolution of the aggregated utilities of both service provider and users as a function of increasing differential pricing. Note that a price difference of 0 corresponds to the proportionally fair solution. The difference in slope makes the allocation jointly Pareto improving, but not Pareto optimal.

It is interesting to observe that the slope of the aggregated revenue increases as the channel conditions are less correlated while the slope of the aggregated users' utility is not affected. This is due to our formulation of the users' utility, totally independent of the channel conditions.

Figure 2. Aggregated Utilities

Figure 3. Comparison of rates allocations. 50% of users with \( \eta_1 = 1 \) and 50% of users with \( \eta_2 = 2 \)
In the second set of simulations, we want to visualize the bit rate allocation obtained with differential pricing compared to the single-objective optimal centralized solutions: proportionally fair and opportunistic. In this case, we assume 50% of users with $\eta_1=1$ and 50% of user with $\eta_1=2$. Fig.3 shows the obtained bit rate allocations. It can be observed that the Stackelberg-based allocation becomes uncorrelated with the channel conditions. Of course, the higher the differential pricing, the better. However, in a realistic system implementation this would mean a system providing services to a wide variety of terminals and users. This “diversity gain” provides the satellite system designer with an additional degree of freedom to design an adaptive system with correlated channels.

In the third set of simulations, we want to quantify the system sum-rate that the Stackelberg solution gets compared to the single-objective optimal solutions. Fig. 4 to 6 show the sum-rate increase with linear increase of available capacity. We observe that the Stackelberg solution is a middle solution between the opportunistic and the proportionally fair solutions due to the diversity introduced by the differential pricing. Moreover, we can observe that the sum-rate in all cases will of course depend on both the range of spectral efficiencies and distribution over the coverage area. As a consequence of this, the third figure shows an interesting result: as the distribution of users approach a correlated distribution of users where all users undergo the same channel conditions, the Stackelberg solution approaches the opportunistic.

![Figure 4. Rate allocations with spectral efficiencies (1, 3) distributed (50%,50%).](image)

![Figure 5. Rate allocations with spectral efficiencies (1, 3) distributed (80%,20%).](image)

![Figure 6. Rate allocations with spectral efficiencies (1, 3) distributed (20%,80%).](image)

### VI. CONCLUSIONS

In this paper we have presented a model for the downlink of a multibeam satellite system where the provider allocates capacity to users with Quality of Service (QoS) guarantees. We have assumed the system operates at Ka band resulting in highly correlated channel conditions within and among beams. Therefore, we have tackled the problem of capacity allocation of this system for a large coverage area and correlated atmospheric channel conditions both in time and space. We first analyze the optimal single-objective optimum allocation and show that these allocations do not show any clear benefit for a profit-seeking service provider because
both the efficiency of the network and the user satisfaction cannot be controlled. This is due to the fact that the optimal allocation depends on the random nature of the correlation properties of the channel. We then propose a Stackelberg game which provides a multi-objective optimization framework that yields an equilibrium that allows us to prove that 1) the proportionally fair allocation corresponds to a flat-rate pricing model (as used in today’s satellite systems) and 2) Pareto improving allocation policies independent of the channel can be obtained with differential pricing among users.

From the simulation results, we show that the bit rate allocation based on the Stackelberg formulation yields a network efficiency which is in between the one achieved by the proportionally fair and the opportunistic allocations (as expected from the Pareto improvement), with two advantages. First, now the efficiency is under full control of the system designer. Second, the higher the differential pricing the higher the network efficiency, which approaches the opportunistic allocation as the channel becomes totally correlated, which is an optimal solution. As a general conclusion, both the efficiency of the network and the satisfaction of the users improve with our proposed scheme compared to current designs and therefore a differentiated pricing can be considered a relevant design tool for this type of systems.

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