Multi-objective supplier selection and order allocation under quantity discounts with fuzzy goals and fuzzy constraints

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Abstract: This paper investigates a multi-objective supplier selection and order allocation problem under quantity discounts in a fuzzy environment. Prior research on supplier selection and order allocation with quantity discounts mainly considered partial fuzziness of the decision problem; a situation where both the objectives of the decision maker and the constraints are fuzzy has not been studied up to now. This paper closes this gap by integrating both aspects into a single model. First, a combination of fuzzy preference programming and interval-based TOPSIS is proposed for evaluating suppliers. Secondly, based on the scores obtained in the first step, a fuzzy multi-objective linear programming model is developed. Subsequently, a new solution procedure for solving the fuzzy multi-objective linear programming model is presented. The procedure first transforms fuzzy constraints and coefficients into deterministic coefficients, and then three different fuzzy programming approaches, namely interactive fuzzy multi-objective linear programming, and the weighted additive as well as the weighted max-min method are implemented. Finally, the performance of each method is evaluated by computing the distance between each solution and the preferred solution.

Keywords: supplier selection; order allocation; quantity discount; fuzzy preference programming; FPP; interval-based TOPSIS; \( \alpha \)-cut approach; interactive fuzzy linear programming; i-FMOLP; weighted additive method; weighted max-min method.

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## 1 Introduction

In light of globalisation and the development of information technology, a well-designed supply chain management (SCM) system is regarded as an important tool to achieve competitive advantages (Choi et al., 2007). A well-managed supply chain (SC) enables organisations to focus on their core competencies, and it can provide benefits both to the company and to its customers (Souter, 2000). SCM, in this context, comprises both the management of an existing SC as well as efforts that aim on shaping the structure of the SC. The latter includes the selection of suppliers, who play a key role in improving the overall performance of the SC. The importance of suppliers makes it necessary to develop systematic and transparent approaches which help to select the best suppliers for the SC.
The supplier selection and order allocation problem (SSOAP) is typically a multi-criteria decision making (MCDM) problem, and in most cases relevant criteria are conflicting. Dickson (1966), for example, identified 23 different criteria for supplier selection in a survey that was carried out among purchasing managers in Canada and the USA. Weber et al. (1991) reviewed 74 articles on supplier selection and found that supplier selection involves multiple selection criteria, whose relative importance varies with the purchasing situation under study. Obviously, many criteria influence the SSOAP, and finding the best supplier(s) requires that a balance is found between conflicting tangible and intangible factors. One issue that makes the SSOAP difficult to solve in many cases is that multiple criteria have to be considered simultaneously, and that each supplier may have a different performance with respect to these criteria. The complexity of the SSOAP increases if suppliers offer discounts. The motivation for offering discounts is to encourage buyers to order larger quantities, which may result in operating advantages at the supplier (such as economies of scale or reduced transportation cost). Earlier works showed that both the buyer and the supplier can realise higher overall profits if discount schemes are used to set transfer prices (Wang et al., 2004). If discounts are offered, the optimisation problem becomes more complex, as the cost of a purchase now depends on the order quantity. The complexity of the planning problem is further increased if decision makers (DMs) do not have exact and complete information about decision criteria and constraints. In such a situation, the theory of fuzzy sets may be used to handle uncertainty.

In the past, several authors used a combination of MCDM and fuzzy programming approaches to support the SSOAP. A closer look at the literature reveals, however, that in most cases authors only considered fuzziness in either the evaluation or the order allocation phase. Only few works exist that assume that both phases are subjected to fuzzy conditions. These models, however, do either not include quantity discounts, or fuzzify only a part of the model. From the authors’ point of view, this is an incomplete representation of reality, since, in practice, buyers often face situations in which they have to make sourcing decisions under imprecise information and in the presence of discounts. This makes it necessary to develop a model that considers imprecise data in every process step of the SSOAP. To the best of the authors’ knowledge, this model is the first to simultaneously consider the SSOAP problem with quantity discounts in which the DM faces fully imprecise data.

The remainder of this paper is structured as follows: The next section gives an overview of related literature. This section is followed by Section 3 that defines the terminology used. Section 4 develops the problem under study along with the proposed model, and Section 5 illustrates the model in a numerical example. The last section presents a summary of the most important findings of this work and suggestions for future research.

2 Literature review

In the past, a variety of different methods were used to support the supplier selection decision [readers are referred to Deshmukh and Chaudhari (2011) for a comprehensive review of supplier selection methods]. Among these methods, mathematical programming (MP) was shown to be suitable to address the SSOAP. MP models can be subdivided into three groups:
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1. linear programming (LP)
2. mixed integer programming
3. goal programming (GP)/multi-objective goal programming (MOP).

Weber and Current (1993), for example, introduced a multi-objective approach that systematically tried to find a balance between multiple conflicting criteria. Karpak et al. (2001) proposed a GP model for the SSAOP and used three goals, which are cost, quality, and delivery reliability of purchased materials. Talluri and Narasimhan (2005) proposed a LP model to evaluate and select suppliers for a telecom company. Hong et al. (2005) developed a mixed-integer LP model for the SSAOP with the objective to maximise the revenue of the buyer. Their model also considered changes in the suppliers’ supply capabilities and the customer’s needs over time. The works considered above investigated the SSOAP in a deterministic situation with crisp goals and model constraints.

Some researches combined two or more methods to deal with the SSOAP. Ghodspour and O’Brien (1998), for example, integrated an analytical hierarchy process (AHP) approach and LP to evaluate suppliers and to assign optimal order quantities to each supplier. In their model, they considered both qualitative and quantitative factors with exact data. Lin (2009) developed an integrated fuzzy analytical network process (ANP) model that was combined with a multi-objective linear programming (MOLP) model for SSOAP. In the first phase of the model, the author used fuzzy preference programming (FPP) combined with ANP to model the uncertainty of the decision environment and interdependencies that exist between the selection criteria. In the second phase, a MOLP model that used three minimisation objectives – purchasing cost, late delivery rate and defect rate – and one maximisation objective – maximum overall value of the order quantity – was formulated, and the optimal order quantity for each supplier was determined. The model, however, considered fuzziness only in the evaluation process and not in the order allocation phase.

A closer look at the literature reveals that only a few of the works that developed methods for solving the SSOAP considered quantity discounts. In practical situations, however, buyers often face multiple potential sources of supply, which offer different price discount schemes. In such a situation, different discount schemes have to be considered in the model (Ebrahim et al., 2009). In this line of thought, Dahel (2003) developed a multi-objective mixed integer linear programming (MOMILP) approach to simultaneously determine the optimal number of suppliers and the optimal order quantities. Thereby, the author considered a multi-product, multi-supplier competitive sourcing environment, and the objective of the model was to optimise cost, delivery time and quality subject to the capacity constraints of the suppliers (Ebrahim et al., 2009). Wadhwa and Ravindran (2007) modelled the SSOAP as a multi-objective optimisation problem under quantity discounts in a multiple sourcing environment and included three objectives, namely minimising price, lead-time and the number of rejected items. Wang and Yang (2009) developed an integrated AHP-MOLP model in a single buyer-multiple supplier environment under a price discount scheme. Their intention was to measure the performance of the suppliers and to allocate the order quantity to the selected suppliers. To consider heterogeneous evaluation criteria, the authors proposed the fuzzy compromise programming solution, which is an efficient way to transform the MOP into a single-objective problem. Amid et al. (2009) formulated a multi-objective model that determined the optimal order quantities for each supplier under price breaks. The
problem included three objective functions: minimising net costs, minimising the number of rejected items and minimising item late deliveries, while satisfying capacity and demand requirement constraints. The model simultaneously dealt with unstructured information, imprecise input data and different weights for the evaluation criteria, and assumed that price breaks depend on the order quantities. For solving the fuzzy multi-objective programming (FMOP) model, a fuzzy weighted additive method was developed. Ebrahimi et al. (2009) presented an integrated AHP-MOLP approach for the SSAOP under price discounts, where suppliers are allowed to offer any of three discount schemes (which are all-unit, incremental and total business volume discount). In developing the model, the weighted sum of objective functions was used. Because the problem was found to be NP-hard, it was solved with the help of a scatter search algorithm. Recently, Tsai and Wang (2010) used a mixed integer programming approach to address the SSOAP in a multiple sourcing and multi-items scenario. Their model assumed that each supplier offers a discount scheme, whereby two types of quantity discounts, incremental and volume discounts, were considered. Table 1 contains an overview of works that developed integrated MP methods for the SSOAP. As can be seen, none of the existing works developed a model with quantity discount where both the supplier evaluation and order allocation decision are subjected to fuzzy information, which is a research gap this paper tries to close.

Table 1  The review of models published in SSOAP

<table>
<thead>
<tr>
<th>Papers</th>
<th>Used approach</th>
<th>Supplier evaluation</th>
<th>Order allocation</th>
<th>Fuzziness in supplier evaluation</th>
<th>Fuzziness in order allocation</th>
<th>Fuzziness in constraints</th>
<th>Quantity discount</th>
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<tbody>
<tr>
<td>Ghodsypour and O’Brien (1998)</td>
<td>AHP-LP</td>
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<td>Weber et al. (1998)</td>
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<td>Amid et al. (2006)</td>
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<td>Demirtas and Üstün (2008)</td>
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<td>Amid et al. (2009)</td>
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<td>Faez et al. (2009)</td>
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<td>Guneri et al. (2009)</td>
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<td>Kokangul and Susuz (2009)</td>
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Notes: A = fuzziness in the objective function, B = fuzziness in the objective function and constraints, and C = fuzziness in constraints.
Table 1 The review of models published in SSOAP (continued)

<table>
<thead>
<tr>
<th>Papers</th>
<th>Used approach</th>
<th>Supplier evaluation</th>
<th>Order allocation</th>
<th>Fuzziness in supplier evaluation</th>
<th>Fuzziness in order allocation</th>
<th>Quantity discount</th>
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</thead>
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<td>Lin (2009)</td>
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<td>Wang and Yang (2009)</td>
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<td>Sawik (2010)</td>
<td>MOMILP</td>
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<td>Tsai and Wang (2010)</td>
<td>MOLP</td>
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<tr>
<td>Amin et al. (2011)</td>
<td>SWOT-LP</td>
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<td>Amid et al. (2011)</td>
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<td>Glock (2011)</td>
<td>Non-linear optimisation</td>
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<td>Haleh and Hamidi (2011)</td>
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<tr>
<td>Mafakheri et al. (2011)</td>
<td>AHP-multi objective dynamic programming</td>
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<td>Xia and Wu (2007)</td>
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<td>Yücel and Güneri (2011)</td>
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</table>

Notes: A = fuzziness in the objective function, B = fuzziness in the objective function and constraints, and C = fuzziness in constraints

This paper develops an integrated fuzzy MCDM-MOLP approach under quantity discounts and multiple sourcing and explicitly assumes that both the objective function and the constraints are fuzzy. We proceed as follows: in a first step, an interval-based MCDM model is used for evaluating the network of suppliers, and then a fuzzy MOLP model is established to calculate the optimal order quantities for the selected suppliers. Before developing the proposed model, we define the following notation which will be used throughout this paper.

2.1 Notations

- $x_{ij}$ number of units assigned to supplier $i$ at price level $j$
- $TC_i$ transportation cost of supplier $i$
CC_i  closeness coefficients of supplier i
D  buyer’s demand
MC_i  maximum capacity of supplier i
R  maximum acceptable defect rate of the buyer
U_k  the upper bound of the k^{th} objective function
L_k  the lower bounds of the k^{th} objective function
t_i  delay time of supplier i
Q_{ij}  the j^{th} price level of supplier i
y_{ij} = \begin{cases} 1 & \text{if the } i^{th} \text{ supplier is selected at price level } j \\ 0, & \text{otherwise} \end{cases}
\alpha(i)  the number of the changes in the level of price for the i^{th} supplier
\beta  the auxiliary variable for the overall degree of DMs satisfaction with the specified multiple-objective values
p_{ij}  the unit price of supplier i at price level j
p  maximum acceptable price of the buyer
R_i  defect rate of supplier i.

3 Prerequisite mathematics

This section gives a brief overview of the terminology and basic concepts that will be used to develop the proposed model in Section 4 of this paper.

3.1 \( \alpha \)-cut

The \( \alpha \)-cut of a fuzzy set \( A \) of \( X \) is a crisp set denoted as \( A^\alpha \), and it is defined by a subset of all \( x \in X \), such that the values of their membership functions exceed or equal a real number \( \alpha \in [0, 1] \) as follows:

\[
A^\alpha = \{ x | \mu_A(x) \geq \alpha, \alpha \in [0,1], \forall x \in X \}
\]  

(1)

3.2 Triangular fuzzy numbers

Triangular fuzzy numbers are represented by \( \tilde{R} = (r_1, r_2, r_3) \), where \( r_i, i = 1, 2, 3 \) are crisp numbers with \( r_1 < r_2 < r_3 \). The membership function of a triangular fuzzy number can be described as follows:
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$\mu_k(x) = \begin{cases} 
0, & x \leq r_1, \\
\frac{x - r_1}{r_2 - r_1}, & r_1 \leq x \leq r_2, \\
\frac{r_2 - x}{r_2 - r_1}, & r_2 \leq x \leq r_3, \\
0, & x \geq r_3.
\end{cases}$

(2)

Hence, the $\alpha$-cut of $\tilde{R}$ can be calculated by using the following interval:

$(\tilde{R})^\alpha = \left[ (\tilde{R})^\alpha_L, (\tilde{R})^\alpha_U \right] = \left[ (r_2 - r_1)\alpha + r_1, r_3 - (r_3 - r_2)\alpha \right]$

(3)

### 3.3 Support set

The support set of a fuzzy set $A$ of $X$ is a crisp set characterised by $S(\tilde{A})$, and it is defined as:

$$S(\tilde{A}) = \{ x \in X | \mu_A(x) > 0 \}$$

(4)

### 3.4 Linear membership function

The linear membership functions of the fuzzy objective functions are defined by:

$$\mu_k(Z^k(x)) = \begin{cases} 
1, & \text{if } Z^k(x) \leq L_k, \\
\frac{U_k - Z^k(x)}{U_k - L_k}, & \text{if } L_k < Z^k(x) < U_k, \\
0, & \text{if } Z^k(x) \geq U_k.
\end{cases}$$

(5)

In a practical application, the linear membership functions can be determined by asking the DM to estimate the value interval $[L_k, U_k]$ for each objective function based on his/her experience. Figure 1 illustrates the linear membership function for equation (5).

**Figure 1** Linear membership function for minimising an objective function
4 The model

4.1 Fuzzy preference programming

Mikhailov (2000, 2002, 2003) developed the FPP method to derive priority vectors from a set of crisp or interval comparisons. Assume that the DMs formulate their subjective judgements about the comparison of criterion $C_i$ to $C_j$ as $\alpha_{ij} = (l_{ij}, u_{ij})$, where $l_{ij}$ and $u_{ij}$ are the lower and the upper bounds of the corresponding imprecise judgements. According to Mikhailov (2000), when the interval judgements are consistent, the priority vectors satisfy the following inequalities:

\[ l_{ij} \leq \frac{w_i}{w_j} u_{ij} \leq u_{ij}, \quad i = 1, 2, \ldots, n-1; \quad j = 2, 3, \ldots, n; \quad j > i. \]  

(6)

where $w_i$ and $w_j$ are weights of the $i^{th}$ and $j^{th}$ criterion, respectively. In contrast, when the DMs’ judgements are inconsistent, no priority vector satisfies all inequality constraints in (6) simultaneously (Mikhailov, 2000). Nevertheless, to support the decision, we have to find a good approximate solution that satisfies all judgements as good as possible. This solution can be expressed with the help of inequality (7), which indicates that a good enough solution vector has to be consistent with all judgements as much as possible.

\[ l_{ij} \leq \frac{w_i}{w_j} \leq u_{ij}, \quad i = 1, 2, \ldots, n-1; \quad j = 2, 3, \ldots, n; \quad j > i. \]  

(7)

Here, ‘$\leq$’ represents the statement “fuzzy less than or equal to”.

In order to find a solution, inequality (7) can be divided into a set of two single-side fuzzy constraints:

\[ w_i - w_j u_{ij} \leq 0, \]
\[ -w_i + w_j l_{ij} \leq 0, \quad i = 1, 2, \ldots, n-1; \quad j = 2, 3, \ldots, n; \quad j > i. \]  

(8)

The above $n(n-1)$ inequalities can be converted into matrix form as seen below:

\[ Rw \leq 0, \]  

(9)

where $R \in \mathbb{R}^{m \times n}$, $m = n(n-1)$.

Each row of inequality (9) represents a fuzzy linear constraint with the following membership function:

\[ \mu_k(R_k w) = \begin{cases} 1, & R_k w \leq 0, \\ 1 - \frac{R_k w}{d_k}, & 0 \leq R_k w \leq d_k, \\ 0, & R_k w \geq d_k. \end{cases} \]  

(10)
where \( d_k \) is a tolerance parameter that represents the permitted (approximate) satisfaction interval of the crisp inequality condition.

Two fundamental assumptions have to be made to guarantee that a solution to the prioritisation problem is found. The first requires the existence of a non-empty fuzzy feasible area \( \tilde{P} \) on the \((n - 1)\) - dimensional simplex \( Q^{n-1} \), with

\[
Q^{n-1} = \left\{ (w_1, w_2, \ldots, w_n) \mid w_i > 0, \sum_{i=1}^{n} w_i = 1 \right\}
\]

where the fuzzy feasible area \( \tilde{P} \) on the simplex \( Q^{n-1} \) is a fuzzy set described by the membership function:

\[
\mu_{\tilde{P}}(w) = \left\{ \min\left[ \mu_{R_i}(w), \ldots, \mu_{R_m}(w) \right] \mid w \in Q^{n-1} \right\}.
\]

The second assumption specifies a priority vector that is the best possible solution and maximises the DM’s degree of satisfaction, which is given by:

\[
\lambda = \max \left\{ \min\left[ \mu_{R_i}(w), \ldots, \mu_{R_m}(w) \right] \mid w \in Q^{n-1} \right\}.
\]

Finally, the maximisation problem can be transformed into a LP by using \( \lambda \) and the max-min operator proposed by Zimmerman (1976):

Maximise \( \lambda \),

Subject to \( d_k \lambda + R_k w \leq d_k \),

\[
\sum_{i=1}^{n} w_i = 1, \quad w_i > 0, \quad i = 1, 2, \ldots, n, \quad k = 1, 2, \ldots, m;
\]

Solving the above LP yields an optimal solution \((w^*, \lambda^*)\), where \( w^* \) represents the priority vector and \( \lambda^* \) the degree of satisfaction.

4.2 The TOPSIS method

Many traditional MCDM techniques for rating alternatives are based on the assumption that DMs have exact information about the alternatives. In many realistic situations, however, the DM’s judgements may be based on some degree of imprecision. One way to deal with imprecision is to define intervals with lower and upper bounds for relevant parameters. This paper uses the TOPSIS method combined with interval data to rate suppliers. The TOPSIS method with interval data was proposed by Jahanshahloo et al. (2006), and it works as follows:

Suppose that \( A_1, A_2, \ldots, A_m \) are \( m \) possible alternatives, \( C_1, C_2, \ldots, C_n \) are criteria for measuring the performance of the alternatives, \( x_{ij} \) is the rating of alternative \( A_i \) with respect to criterion \( C_j \), where \( x_{ij} \in [x_{ij}^L, x_{ij}^U] \).

The decision matrix with interval data can now be represented as in Table 2.
The steps of the TOPSIS method combined with interval data are given as follows:

Step 1  Construct the decision matrix as shown in Table 2.

$$\begin{array}{cccc}
C_1 & C_2 & \ldots & C_n \\
A_1 & \left[ x_{11}^L, x_{11}^U \right] & \left[ x_{12}^L, x_{12}^U \right] & \ldots & \left[ x_{1n}^L, x_{1n}^U \right] \\
A_2 & \left[ x_{21}^L, x_{21}^U \right] & \left[ x_{22}^L, x_{22}^U \right] & \ldots & \left[ x_{2n}^L, x_{2n}^U \right] \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_m & \left[ x_{m1}^L, x_{m1}^U \right] & \left[ x_{m2}^L, x_{m2}^U \right] & \ldots & \left[ x_{mn}^L, x_{mn}^U \right]
\end{array}$$

Step 2  Formulate the normalised fuzzy decision matrix as follows:

$$\begin{array}{cccc}
\left[ \bar{p}_{11}^L, \bar{p}_{11}^U \right] & \left[ \bar{p}_{12}^L, \bar{p}_{12}^U \right] & \ldots & \left[ \bar{p}_{1n}^L, \bar{p}_{1n}^U \right] \\
\left[ \bar{p}_{21}^L, \bar{p}_{21}^U \right] & \left[ \bar{p}_{22}^L, \bar{p}_{22}^U \right] & \ldots & \left[ \bar{p}_{2n}^L, \bar{p}_{2n}^U \right] \\
\vdots & \vdots & \ddots & \vdots \\
\left[ \bar{p}_{m1}^L, \bar{p}_{m1}^U \right] & \left[ \bar{p}_{m2}^L, \bar{p}_{m2}^U \right] & \ldots & \left[ \bar{p}_{mn}^L, \bar{p}_{mn}^U \right]
\end{array}$$

where

$$\begin{align}
\bar{p}_{ij}^L &= \frac{x_{ij}^L}{\sqrt{\sum_{j=1}^{m} (x_{ij}^L)^2 + (x_{ij}^U)^2}}, \quad j = 1, 2, \ldots, m; \quad i = 1, 2, \ldots, n, \\
\bar{p}_{ij}^U &= \frac{x_{ij}^U}{\sqrt{\sum_{j=1}^{m} (x_{ij}^L)^2 + (x_{ij}^U)^2}}, \quad j = 1, 2, \ldots, m; \quad i = 1, 2, \ldots, n,
\end{align}$$

Step 3  Calculate the weighted normalised fuzzy decision matrix as given below:

$$\begin{array}{cccc}
\left[ \bar{v}_{11}^L, \bar{v}_{11}^U \right] & \left[ \bar{v}_{12}^L, \bar{v}_{12}^U \right] & \ldots & \left[ \bar{v}_{1n}^L, \bar{v}_{1n}^U \right] \\
\left[ \bar{v}_{21}^L, \bar{v}_{21}^U \right] & \left[ \bar{v}_{22}^L, \bar{v}_{22}^U \right] & \ldots & \left[ \bar{v}_{2n}^L, \bar{v}_{2n}^U \right] \\
\vdots & \vdots & \ddots & \vdots \\
\left[ \bar{v}_{m1}^L, \bar{v}_{m1}^U \right] & \left[ \bar{v}_{m2}^L, \bar{v}_{m2}^U \right] & \ldots & \left[ \bar{v}_{mn}^L, \bar{v}_{mn}^U \right]
\end{array}$$

where

$$\begin{align}
\bar{v}_{ij}^L &= w_i \bar{p}_{ij}^L, \quad j = 1, 2, \ldots, m; \quad i = 1, 2, \ldots, n, \\
\bar{v}_{ij}^U &= w_i \bar{p}_{ij}^U, \quad j = 1, 2, \ldots, m; \quad i = 1, 2, \ldots, n,
\end{align}$$

And $w_i$ is the weight of the $i^{th}$ attribute or criterion with $\sum_{i=1}^{n} w_i = 1$.

After this stage, the steps of the interval TOPSIS method are exactly the same as those of the TOPSIS method of Hwang and Yoon (1981), such that it similarly reaches to a $CC_i$.
value for each supplier. As a result, suppliers can then be ranked. Therefore, we omit
describing the remaining steps to avoid repetition.

4.3 The fuzzy multi-objective order allocation model under quantity discounts

After obtaining the overall score of each supplier in the first stage, the second stage
allocates the order to the suppliers subject to the price discount offered by each supplier.
To formulate this model, the following assumptions are used:

4.3.1 Assumptions
1. Only one item is purchased from one supplier at a time.
2. The unit price is a function of the order quantity.
3. No minimum and/or maximum order quantities are specified by the suppliers.
4. The capacity constraint of each supplier is considered in calculating order quantities.
5. Shortages are not allowed.
6. The available data for decision making is imprecise and can be described by fuzzy
   numbers.

Based on the objectives of the purchasing policy, the fuzzy order allocation problem can
be modelled as a fuzzy multi-objective mixed integer programming model with a set of
policy constraints:

\[
\text{Min } \tilde{Z}_1 \equiv \sum_{i=1}^{n} \sum_{j=1}^{a(i)} P_{ij} x_{ij} \tag{21}
\]
\[
\text{Min } \tilde{Z}_2 \equiv \sum_{i=1}^{n} \sum_{j=1}^{a(i)} t_{ij} x_{ij} \tag{22}
\]
\[
\text{Min } \tilde{Z}_3 \equiv \sum_{i=1}^{n} \sum_{j=1}^{a(i)} R_{ij} x_{ij} \tag{23}
\]
\[
\text{Min } \tilde{Z}_4 \equiv \sum_{i=1}^{n} \sum_{j=1}^{a(i)} TC_{ij} x_{ij} \tag{24}
\]
\[
\text{Max } \tilde{Z}_5 \equiv \sum_{i=1}^{n} \sum_{j=1}^{a(i)} CC_{ij} x_{ij} \tag{25}
\]

subject to
\[
\sum_{i=1}^{n} \sum_{j=1}^{a(i)} x_{ij} \geq \{D_1, D_2, D_3\}, \tag{26}
\]
\[
\sum_{i=1}^{n} \sum_{j=1}^{a(i)} x_{ij} \leq \{MC_{1i}, MC_{2i}, MC_{3i}\}, \tag{27}
\]
\[
\sum_{i=1}^{n} \sum_{j=1}^{a(i)} (r_1, r_2, r_3) x_{ij} \leq R\{D_1, D_2, D_3\}, \tag{28}
\]
\[
\sum_{i=1}^{n} \sum_{j=1}^{a(i)} P_{ij} x_{ij} \leq P\{D_1, D_2, D_3\}, \tag{29}
\]
The objectives (21) to (24) are established to minimise purchasing cost, delay time, defect rate and transportation cost. The fifth objective, (25), maximises the total value of purchasing. Note that the delay time, $t_i$, and the defect rate of supplier $i$, $R_i$, are assumed to be triangular fuzzy numbers. Constraint (26) ensures that the total purchase quantity of the item meets the required quantity. Constraint (27) makes sure that the quantity ordered at each supplier does not exceed the supplier’s capacity, which is a triangular fuzzy number. Constraint (28) guarantees that the defect rate of the products purchased from each supplier does not exceed the maximum acceptable defect rate of the company. Constraint (29) requires that the purchasing cost does not exceed the company’s budget. Constraint (30) restricts values for the $Y_{ij}$-variables to integer values. Constraint (31) ensures that only one price break can be in effect per supplier. Constraint (32) restricts the order quantity to the quantity range that is valid for the respective purchase price of the supplier, and constraint (33) prohibits negative order quantities.

### 4.4 A new solution to the multi-objective supplier selection model with fuzzy goals and fuzzy constraints

This section develops a new solution for solving the fuzzy multi-objective supplier selection (MOSS) problem. The developed method consists of two steps: first, using the fuzzy mathematics and definitions described in Section 3, we transform the fuzzy multi-objective programme with fuzzy objectives and fuzzy constraints into a model with fuzzy objectives and crisp constraints. In the next step, we define membership functions for the objectives and then transform the model into an ordinary crisp LP programme.

The MOSS model with fuzzy coefficients is formulated as follows:

\[
\begin{align*}
\text{Minimise} & \quad \tilde{Z}_k(x_y) = \sum_{i=1}^{n} \sum_{j=1}^{a(i)} (C_k \times x_y) \\
\text{subject to} & \quad X \in X = \{x_y \in X \mid \tilde{A}_y \times x_y \leq \tilde{b}_y\} \\
\end{align*}
\]

where $C_k$ ($k = 1, 2, 3, \ldots, K$) is the coefficient of the objective function, $\tilde{A}_y$ is the coefficient of the constraint, and $\tilde{b}_y$ is the coefficient of the right hand side, whereby all are represented by fuzzy numbers. Suppose that $x_y$ is the solution of equation (34). Let $(\tilde{R})^\alpha$ be the $\alpha$-cut of a fuzzy number $\tilde{R}$ described by (see Pramanik and Kumar, 2008):

\[
(\tilde{R})^\alpha = \{r \in S(\tilde{R}) \mid \mu_{\tilde{R}}(Z^4(x)) \geq \alpha, \alpha \in [0,1]\}
\]
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where $S(\tilde{R})$ is the support of $\tilde{R}$. Let $(\tilde{R})^L_\alpha$ and $(\tilde{R})^U_\alpha$ be the lower and upper limits of the $\alpha$-cut such that:

$$(\tilde{R})^L_\alpha \leq r \leq (\tilde{R})^U_\alpha \quad r \in [(\tilde{R})^L_\alpha, (\tilde{R})^U_\alpha]$$

(36)

With the above definitions, we defuzzify the fuzzy inequality constraints by obtaining the closed interval of the $\alpha$-cut of the constraint coefficients as:

$$\sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} (\tilde{\alpha}_{ij})^L_\alpha x_{ij} \leq (\tilde{\beta}_{ij})^L_\alpha, \quad i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, \alpha(i)$$

(37)

$$\sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} (\tilde{\alpha}_{ij})^U_\alpha x_{ij} \geq (\tilde{\beta}_{ij})^U_\alpha, \quad i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, \alpha(i)$$

(38)

By applying the above formulation to equation (34), the MOSS model can be formulated as:

Minimise

$$\tilde{Z}_k(x) \equiv \sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} C_k x_{ij}$$

(39)

subject to:

$$\sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} (\tilde{\alpha}_{ij})^L_\alpha x_{ij} \leq (\tilde{\beta}_{ij})^L_\alpha, \quad i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, \alpha(i)$$

(40)

$$\sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} (\tilde{\alpha}_{ij})^U_\alpha x_{ij} \geq (\tilde{\beta}_{ij})^U_\alpha, \quad i = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, \alpha(i)$$

(41)

$$x_{ij} \geq 0$$

Finally, the FMOSS model can be transformed into a deterministic MOSS model as below:

Min $\tilde{Z}_1 = \sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} P_{ij} x_{ij}$

(42)

Min $\tilde{Z}_2 = \sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} t_{ij} x_{ij}$

(43)

Min $\tilde{Z}_3 = \sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} R_{ij} x_{ij}$

(44)

Min $\tilde{Z}_4 = \sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} TC_{ij} x_{ij}$

(45)

Max $\tilde{Z}_5 = \sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} CC_{ij} x_{ij}$

(46)

subject to

$$\sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} x_{ij} \geq (D_1 + (D_2 - D_1) \alpha),$$

(47)

$$\sum_{i=1}^{n} \sum_{j=1}^{\alpha(i)} x_{ij} \leq (M_3 C - (M_3 C - M_2 C)),$$

(48)
\[ \sum_{i=1}^{n} \sum_{j=1}^{n} a(i,j) x_{ij} \leq \left( RD_i - (RD_2 - RD_1) \alpha \right), \]  
(49)

\[ \sum_{i=1}^{n} \sum_{j=1}^{n} p(i,j) x_{ij} \leq \left( PD_i - (PD_2 - PD_1) \alpha \right), \]  
(50)

\[ Y_{ij} = \begin{cases} 1 & \text{if } x_{ij} > 0 \\ 0 & \text{if } x_{ij} = 0 \end{cases}, \quad \forall i,j, \]  
(51)

\[ \sum_{i=1}^{n} \sum_{j=1}^{n} a(i,j) Y_{ij} \leq 1, \]  
(52)

\[ Q_{i-1} Y_{ij} \leq x_{ij} \leq Q_{i} Y_{ij}, \]  
(53)

\[ x_{ij} \geq 0, \]  
(54)

Note that depending on different \( \alpha_i \)-values \( 0 \leq \alpha_i \leq 1 \), different compromise solutions can be obtained. After completing the first phase of the solution procedure to defuzzify the FMOSS model, we can solve the resulting multi-objective programme with fuzzy objective function, as will be described in the subsequent section.

4.5 Solution methodology

In the next three subsections, we employ three different approaches for solving the MOSS problem with fuzzy objective function to find the best solution to our model. In all the subsequent approaches, we use the FPP method where we need to specify the weights of the model’s objectives.

4.5.1 Interactive fuzzy multi-objective linear programming approach

Interactive fuzzy multi-objective linear programming (i-FMOLP) is an interactive method based on linear membership functions and the minimum operator of fuzzy decision making of Bellman and Zadeh (1970), which transforms the original FMOLP into an ordinary LP problem. By introducing the auxiliary variable \( \beta \), the crisp MOSS under quantity discounts can be transformed into an equivalent single-objective LP problem (see Liang, 2006):

\[ \max \beta \]

subject to

\[ \beta \leq \mu_k \left( Z^k (x) \right), \]  
(55)

\[ Ax \leq b, \]

\[ x \geq 0, \quad i = 1, 2, \ldots, n \]

The interactive solution procedure for the FMOSS model under quantity discounts can be described as follows (see Liang, 2006):

Step 1  Formulate the initial FMOSS model under quantity discounts.
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Step 2  Determine corresponding linear membership functions for all objective functions according to equation (5).

Step 3  Define the auxiliary variable $\beta$ and transform the FMOSS model under quantity discounts into an equivalent ordinary single-objective LP model using the minimum operator.

Step 4  Solve the LP problem and acquire initial compromise solutions.

Step 5  Execute an interactive decision process. If the DM is not satisfied with the initial compromise solution, the model has to be changed until a satisfactory solution is found (Wang and Yang, 2009).

4.5.2 Weighted additive model

The weighted additive model has frequently been used in multi-objective optimisation problems; the basic concept is to use a single utility function to express the overall preference of the DM to determine the relative importance of the criteria (Amid et al., 2009; Lai and Hwang, 1994). The model obtains a linear weighted utility function by multiplying each membership function of fuzzy goals and fuzzy constraints with their proportionate weights and by then aggregating the outcomes (see Amid et al., 2009).

Tiwari et al. (1987) proposed a weighted additive method to determine the priority of the fuzzy goals. Here, we modify the weighted additive model to consider the case where only the objective functions are fuzzy. The model is defined as follows:

$$\text{Max } \sum_{k=1}^{K} W_k \mu_k$$

subject to

$$\mu_k \leq \frac{Z_k^L(x) - L_k}{G_k - U_k} \left( \text{or } \mu_k \leq \frac{U_k - Z_k^U(x)}{U_k - G_k} \right), \quad (\text{For all objectives})$$

$$\mu_k \in [0,1],$$

$$\sum_{k=1}^{K} W_k = 1, \quad k = 1, 2, \ldots, K$$

$$x_i \geq 0, \quad k \in K$$

where $W_k$ ($k = 1, 2, \ldots, K$) are the weight coefficients that indicate the relative significance of the fuzzy goals. The following procedure is used for solving the FMOSS under quantity discounts:

Step 1  Formulate the initial FMOSS model under quantity discounts.

Step 2  Solve each objective function with the constraint separately and obtain lower and upper bounds for each objective function.

Step 3  By applying lower and upper bounds to the objective functions, compute linear membership functions for each objective functions as in equation (5).

Step 4  Calculate the coefficients of the criteria ($W_k$) for each objective function.
Step 5 Formulate a crisp FMOSS model under quantity discounts by using formula (56).

Step 6 Solve the multi-objective linear model and assign optimum order quantities to the suppliers.

4.5.3 Weighted max-min model

When the DM provides the weights of the objective functions, the achievement level ($\beta$) that results from setting the membership functions should be as close as possible to the objective weights to reflect the relative importance of the criteria. Lin (2004) proposed a weighted max-min model to solve this problem. This model is formulated as follows (Amid et al., 2011):

\[
\max \beta
\]

subject to

\[
W_k \beta \leq \mu_k \left( Z^k(x) \right), \quad k = 1, 2, \ldots, K \quad \text{(For all objectives)}
\]

\[
A_i \leq b, \quad i = 1, 2, \ldots, n
\]

\[
\beta \in [0, 1],
\]

\[
\sum_{k=1}^{K} W_k = 1, \quad k = 1, 2, \ldots, K
\]

\[
x_i \geq 0, \quad i = 1, 2, \ldots, n
\]

Lin (2004) defined new linear membership functions for each goal of the above model as follows:

\[
\mu_k \left( Z^k(x) \right) = \begin{cases} 
\sqrt{W_k}, & \text{if } Z^k(x) \leq L_k, \\
\frac{U_k - Z^k(x)}{w_k(U_k - L_k)}, & \text{if } L_k < Z^k(x) < U_k, \\
0, & \text{if } Z^k(x) \geq U_k.
\end{cases}
\]

This model finds the optimal solution within the possible area by obtaining the real achievement level for each objective function. The solution procedure of the weighted max-min model for solving the FMOSS model under quantity discounts can be described as follows:

Step 1 Establish the FMOSS model under quantity discounts.

Step 2 Solve the FMOSS model under quantity discounts as a single-objective problem by focusing on each objective separately.

Step 3 From the solution of step 2, insert the corresponding values for each objective into formula (57).

Step 5 Compute the weight of the criteria ($W_i$) for each objective function.
Step 6 Formulate the corresponding crisp model with the help of the weighted max-min method for FMOSS under quantity discounts according to equation (57).

Step 7 Calculate the optimal solution using the weighted max-min model.

5 Numerical example

To illustrate the proposed solution, we consider an example where an automotive supplier has to evaluate the suppliers of one of its products and determine the optimal allocation of its order quantity to each evaluated supplier. The model developed in this paper is employed to support this decision. Suppose that the DM has identified four potential suppliers, and that five main criteria have been defined for evaluating them: product quality ($C_1$), delivery performance ($C_2$), technological facilities ($C_3$), effort to establish a cooperation ($C_4$), and flexibility in responding to changes in demand ($C_5$). Figure 2 illustrates the hierarchy structure of the problem in the evaluation phase. The hierarchy structure includes goals, criteria, and alternatives. The goal for the selection of the best supplier(s) is on the highest level of the hierarchy, and the criteria and alternatives are on the second and third level.

Figure 2 Hierarchical structure of the numerical example (see online version for colours)

The proposed method is applied in the following to solve the problem described above. The procedure is summarised as follows:

5.1 First phase: supplier evaluation

Step 1 The weight of each criterion is calculated with the help of the FPP approach. The DM defines his/her subjective linguistic variables and the corresponding scales in terms of an interval as shown in Table 3 to calculate the relative importance (weight) of each criterion. This interval values are taken from Saaty’s 1 to 9 scales. The DM is then requested to fill out a questionnaire on the pairwise comparison of the five criteria. Table 4 shows the obtained comparison matrix. Using equation (8), each interval value is transformed into two linear inequalities; for the present example, 20 linear inequalities are extracted from the comparisons matrix. Finally, based on equation (14), a LP model is formulated to obtain the weights of each criterion. The results are shown in Table 5.
Step 2  The suppliers are ranked using the interval TOPSIS method. This main step is divided into the following sub-steps.

Step 2.1  The DM defines linguistic variables and their corresponding interval scales as shown in Table 6 to assess the performance of the four alternative suppliers with respect to each criterion. Table 7 shows the obtained ratings.

Step 2.2  Equations (16) and (17) are used to develop a normalised decision matrix, as is shown in Table 8.

Step 2.3  The weighted normalised decision matrix is found using equations (19) and (20). Table 9 shows the results for the present example.

Step 2.4  Fuzzy positive and negative ideal solutions are calculated with the help of equations (21) and (22). For the present example, these solutions equal:

\[ A^+ = \{0.021, 0.052, 0.031, 0.52, 0.126\} \]  \hspace{1cm} (59)

\[ A^- = \{0.010, 0.026, 0.003, 0.004, 0.010\} \]  \hspace{1cm} (60)

Step 2.5  The distances between each alternative and the fuzzy positive and negative ideal solutions are calculated by using equations (23) and (24). The results are shown in Tables 10 and 11.

Step 2.6  The closeness coefficient of each supplier is calculated, and suppliers are ranked. According to Table 12, the results are given as:

\[ S_1 > S_2 > S_4 > S_3 \]

where \( S_i > S_j \) means that supplier \( S_i \) is preferred to supplier \( S_j \).

<table>
<thead>
<tr>
<th>Table 3</th>
<th>Linguistic variables and corresponding scales defined by the DM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Linguistic variable</td>
<td>( VL )</td>
</tr>
<tr>
<td>Interval value</td>
<td>([1, 2])</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Matrix with pair wise comparisons</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>1</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>( M^{-1} )</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>( L^{-1} )</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>( ML^{-1} )</td>
</tr>
<tr>
<td>( C_5 )</td>
<td>( H^{-1} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 5</th>
<th>Obtained weights of the different evaluation criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_1 )</td>
<td>( W_2 )</td>
</tr>
<tr>
<td>0.086</td>
<td>0.216</td>
</tr>
</tbody>
</table>
Multi-objective supplier selection and order allocation

Table 6  Interval values for the rating of suppliers

<table>
<thead>
<tr>
<th>Linguistic rating</th>
<th>P</th>
<th>MP</th>
<th>F</th>
<th>MG</th>
<th>G</th>
<th>VG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interval value</td>
<td>[1, 2]</td>
<td>[2, 4]</td>
<td>[4, 5]</td>
<td>[5, 7]</td>
<td>[7, 8]</td>
<td>[8, 10]</td>
</tr>
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</table>

Table 7  Ratings of the four suppliers in light of the defined criteria

<table>
<thead>
<tr>
<th></th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₁</td>
<td>VG</td>
<td>G</td>
<td>P</td>
<td>MP</td>
<td>P</td>
</tr>
<tr>
<td>S₂</td>
<td>MG</td>
<td>MG</td>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>S₃</td>
<td>MG</td>
<td>VG</td>
<td>VG</td>
<td>F</td>
<td>VG</td>
</tr>
<tr>
<td>S₄</td>
<td>G</td>
<td>G</td>
<td>F</td>
<td>G</td>
<td>G</td>
</tr>
</tbody>
</table>

Table 8  Normalised decision matrix

<table>
<thead>
<tr>
<th></th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₁</td>
<td>[0.19, 0.24]</td>
<td>[0.17, 0.19]</td>
<td>[0.02, 0.1]</td>
<td>[0.05, 0.1]</td>
<td>[0.02, 0.05]</td>
</tr>
<tr>
<td>S₂</td>
<td>[0.12, 0.17]</td>
<td>[0.12, 0.17]</td>
<td>[0.1, 0.12]</td>
<td>[0.1, 0.12]</td>
<td>[0.1, 0.12]</td>
</tr>
<tr>
<td>S₃</td>
<td>[0.12, 0.17]</td>
<td>[0.19, 0.24]</td>
<td>[0.19, 0.24]</td>
<td>[0.1, 0.12]</td>
<td>[0.19, 0.24]</td>
</tr>
<tr>
<td>S₄</td>
<td>[0.17, 0.19]</td>
<td>[0.17, 0.19]</td>
<td>[0.1, 0.12]</td>
<td>[0.17, 0.19]</td>
<td>[0.17, 0.19]</td>
</tr>
</tbody>
</table>

Table 9  Weighted normalised decision matrix

<table>
<thead>
<tr>
<th></th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>S₁</td>
<td>[0.016, 0.021]</td>
<td>[0.037, 0.041]</td>
<td>[0.003, 0.013]</td>
<td>[0.022, 0.004]</td>
<td>[0.010, 0.026]</td>
</tr>
<tr>
<td>S₂</td>
<td>[0.01, 0.015]</td>
<td>[0.026, 0.037]</td>
<td>[0.013, 0.016]</td>
<td>[0.004, 0.005]</td>
<td>[0.052, 0.063]</td>
</tr>
<tr>
<td>S₃</td>
<td>[0.010, 0.015]</td>
<td>[0.041, 0.052]</td>
<td>[0.025, 0.031]</td>
<td>[0.004, 0.052]</td>
<td>[0.100, 0.126]</td>
</tr>
<tr>
<td>S₄</td>
<td>[0.015, 0.016]</td>
<td>[0.037, 0.041]</td>
<td>[0.013, 0.016]</td>
<td>[0.007, 0.008]</td>
<td>[0.089, 0.100]</td>
</tr>
</tbody>
</table>

Table 10  Distances between fuzzy positive ideal solution and the suppliers’ ratings

<table>
<thead>
<tr>
<th></th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>d(S₁, A⁺)</td>
<td>0.00025</td>
<td>0.00025</td>
<td>0.000784</td>
<td>0.248004</td>
<td>0.013456</td>
</tr>
<tr>
<td>d(S₂, A⁺)</td>
<td>0.000121</td>
<td>0.000676</td>
<td>0.000324</td>
<td>0.266256</td>
<td>0.005476</td>
</tr>
<tr>
<td>d(S₃, A⁺)</td>
<td>0.000121</td>
<td>0.000121</td>
<td>0.000036</td>
<td>0.266256</td>
<td>0.000676</td>
</tr>
<tr>
<td>d(S₄, A⁺)</td>
<td>0.000036</td>
<td>0.000225</td>
<td>0.000324</td>
<td>0.263169</td>
<td>0.001369</td>
</tr>
</tbody>
</table>

Table 11  Distances between fuzzy negative ideal solution and the suppliers’ ratings

<table>
<thead>
<tr>
<th></th>
<th>C₁</th>
<th>C₂</th>
<th>C₃</th>
<th>C₄</th>
<th>C₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>d(S₁, A⁻)</td>
<td>0.000121</td>
<td>0.000225</td>
<td>0.00001</td>
<td>0</td>
<td>0.000256</td>
</tr>
<tr>
<td>d(S₂, A⁻)</td>
<td>0.00025</td>
<td>0.000121</td>
<td>0.000169</td>
<td>0.000001</td>
<td>0.005476</td>
</tr>
<tr>
<td>d(S₃, A⁻)</td>
<td>0.00025</td>
<td>0.000676</td>
<td>0.000784</td>
<td>0.002304</td>
<td>0.000676</td>
</tr>
<tr>
<td>d(S₄, A⁻)</td>
<td>0.000036</td>
<td>0.000225</td>
<td>0.000169</td>
<td>0.000016</td>
<td>0.001369</td>
</tr>
</tbody>
</table>
Table 12  Closeness coefficient of each supplier

<table>
<thead>
<tr>
<th>Supplier</th>
<th>$d_i^n$</th>
<th>$d_i^+$</th>
<th>$d_i^n + d_i^+$</th>
<th>CC$_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>0.026</td>
<td>0.512</td>
<td>0.538</td>
<td>0.951</td>
</tr>
<tr>
<td>$S_2$</td>
<td>0.056</td>
<td>0.522</td>
<td>0.578</td>
<td>0.903</td>
</tr>
<tr>
<td>$S_3$</td>
<td>0.372</td>
<td>0.517</td>
<td>0.889</td>
<td>0.581</td>
</tr>
<tr>
<td>$S_4$</td>
<td>0.092</td>
<td>0.514</td>
<td>0.606</td>
<td>0.848</td>
</tr>
</tbody>
</table>

Table 13  Model parameters for the suppliers

<table>
<thead>
<tr>
<th>Supplier</th>
<th>Quantity level</th>
<th>Price</th>
<th>% of late delivery</th>
<th>Defective rate (%)</th>
<th>Capacity constraint (pcs)</th>
<th>Transportation cost per pcs($)</th>
<th>Defect rate of supplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_1$</td>
<td>$Q &lt; 2,400$</td>
<td>10</td>
<td>6</td>
<td>2</td>
<td>(9,800, 10,300, 11,400)</td>
<td>1</td>
<td>(2, 3, 4)</td>
</tr>
<tr>
<td></td>
<td>2,400 $\leq$ $Q &lt; 4,800$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Q \geq 4,800$</td>
<td>9.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_2$</td>
<td>$Q &lt; 1,800$</td>
<td>11</td>
<td>3</td>
<td>3</td>
<td>(8,200, 9,300, 10,400)</td>
<td>1</td>
<td>(3, 7, 9)</td>
</tr>
<tr>
<td></td>
<td>1,800 $\leq$ $Q &lt; 6,220$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Q \geq 4,800$</td>
<td>10.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_3$</td>
<td>$Q &lt; 3,400$</td>
<td>10.7</td>
<td>8</td>
<td>1</td>
<td>(11,100, 11,900, 12,300)</td>
<td>0.25</td>
<td>(1, 4, 5)</td>
</tr>
<tr>
<td></td>
<td>3,400 $\leq$ $Q &lt; 7,200$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Q \geq 7,200$</td>
<td>10.2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$S_4$</td>
<td>$Q &lt; 1,200$</td>
<td>10.6</td>
<td>9</td>
<td>5</td>
<td>(7,300, 8,700, 9,400)</td>
<td>0.75</td>
<td>(3, 5, 8)</td>
</tr>
<tr>
<td></td>
<td>1,200 $\leq$ $Q &lt; 3,400$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$Q \geq 3,400$</td>
<td>9.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.2  The second phase: order allocation

To allocate the order quantity to the evaluated suppliers, a MP model with five objectives is formulated. The scores for the suppliers that were obtained in phase 1 are used as coefficients in this model. The criteria for the allocation phase are purchasing cost, delay time, defect rate, transportation cost, and total value of purchasing. In this step, it is assumed that some input data from suppliers are not known precisely, but can be described with the help of fuzzy numbers. Table 13 summarises the essential information for each supplier based on the objectives and constraints provided in Section 4.3. Moreover, the maximum acceptable defect rate and the maximum acceptable price are 4% and $11$, respectively.

Based on the data given in Table 13 and by using equations (21) to (33), the fuzzy order allocation model of the numerical example is formulated. Then, by using the $\alpha$-cut approach described in Section 4.4.1 (for $\alpha = 0.5$), the model is transformed into the following fuzzy MP problem:
Min $\tilde{Z}_1 \equiv 10x_{11} + 9.5x_{12} + 8.7x_{13} + 11x_{21} + 10.2x_{22} + 9.7x_{23} + 10.7x_{31} + 10.2x_{32} + 9.8x_{33} + 10.6x_{41} + 9.3x_{42} + 9.1x_{43}$

Min $\tilde{Z}_2 \equiv 6(x_{11} + x_{12} + x_{13}) + 3(x_{21} + x_{22} + x_{23}) + 8(x_{31} + x_{32} + x_{33}) + 9(x_{41} + x_{42} + x_{43})$

Min $\tilde{Z}_3 \equiv 2(x_{11} + x_{12} + x_{13}) + 3(x_{21} + x_{22} + x_{23}) + 1(x_{31} + x_{32} + x_{33}) + 5(x_{41} + x_{42} + x_{43})$

Min $\tilde{Z}_4 \equiv (x_{11} + x_{12} + x_{13}) + (x_{21} + x_{22} + x_{23}) + 0.25(x_{31} + x_{32} + x_{33}) + 0.75(x_{41} + x_{42} + x_{43})$

Min $\tilde{Z}_5 \equiv 0.951(x_{11} + x_{12} + x_{13}) + 0.903(x_{21} + x_{22} + x_{23}) + 0.581(x_{31} + x_{32} + x_{33}) + 0.848(x_{41} + x_{42} + x_{43})$

Subject to

$(x_{11} + x_{12} + x_{13}) + (x_{21} + x_{22} + x_{23}) + (x_{31} + x_{32} + x_{33}) + (x_{41} + x_{42} + x_{43}) \geq 18,200,$

$(x_{11} + x_{12} + x_{13}) \leq 10,850,$

$(x_{21} + x_{22} + x_{23}) \leq 9,850,$

$(x_{31} + x_{32} + x_{33}) \leq 12,100,$

$(x_{41} + x_{42} + x_{43}) \leq 9,050,$

$2.5(x_{11} + x_{12} + x_{13}) + 5(x_{21} + x_{22} + x_{23}) + 2.5(x_{31} + x_{32} + x_{33}) + 4(x_{41} + x_{42} + x_{43}) \leq 110,320,$

$10x_{11} + 9.5x_{12} + 8.7x_{13} + 11x_{21} + 10.2x_{22} + 9.7x_{23} + 10.7x_{31} + 10.2x_{32} + 9.8x_{33} + 10.6x_{41} + 9.3x_{42} + 9.1x_{43} \leq 303,380,$

$(Y_{11} + Y_{12} + Y_{13}) + (Y_{21} + Y_{22} + Y_{23}) + (Y_{31} + Y_{32} + Y_{33}) + (Y_{41} + Y_{42} + Y_{43}) \leq 1,$

$0 \leq x_{11} \leq 2,400, 2,400 \leq x_{12} \leq 4,800, 4,800 \leq x_{13},$

$0 \leq x_{21} \leq 1,800, 1,800 \leq x_{22} \leq 6,220, 6,220 \leq x_{23},$

$0 \leq x_{31} \leq 3,400, 3,400 \leq x_{32} \leq 7,200, 7,200 \leq x_{33},$

$0 \leq x_{41} \leq 1,200, 1,200 \leq x_{42} \leq 3,400, 3,400 \leq x_{43},$
In the remaining sections of the paper, we abbreviate all of the above constraints as $\alpha X \leq \gamma, X \geq 0$ for those constraints that contain $X$ variables, and $\delta Y \leq \theta, Y = \text{integer}$ for those constraints that contain $Y$ variables. By solving each goal separately with the constraint, the lower bounds of the problem, $Z_i$ for $i = 1, 2, 3, 4, 5$, are $257,774, 136,380, 111,020, 16,145, 21,107$, respectively, and the upper bounds, $Z_i$ for $i = 1, 2, 3, 4, 5$, are $340,507, 215,870, 177,505, 28,332, 28,735$, respectively. The linear membership function for each objective function can now be obtained using equation (5), which yields:

$$
\mu_1(Z_i(x)) = \begin{cases} 
1, & \text{if } Z_i(x) \leq 257,774, \\
\frac{340,507 - Z_i(x)}{82,733}, & \text{if } 257,774 < Z_i(x) < 340,507, \\
0, & \text{if } Z_i(x) \geq 340,507.
\end{cases}
$$

(61)

$$
\mu_2(Z_2(x)) = \begin{cases} 
1, & \text{if } Z_2(x) \leq 136,380, \\
\frac{215,870 - Z_2(x)}{79,490}, & \text{if } 136,380 < Z_2(x) < 215,870, \\
0, & \text{if } Z_2(x) \geq 215,870.
\end{cases}
$$

(62)

$$
\mu_3(Z_3(x)) = \begin{cases} 
1, & \text{if } Z_3(x) \leq 111,020, \\
\frac{177,505 - Z_3(x)}{66,485}, & \text{if } 111,020 < Z_3(x) < 177,505, \\
0, & \text{if } Z_3(x) \geq 177,505.
\end{cases}
$$

(63)

$$
\mu_4(Z_4(x)) = \begin{cases} 
1, & \text{if } Z_4(x) \leq 16,145, \\
\frac{28,332 - Z_4(x)}{12,187}, & \text{if } 16,145 < Z_4(x) < 28,332, \\
0, & \text{if } Z_4(x) \geq 28,332.
\end{cases}
$$

(64)

$$
\mu_5(Z_5(x)) = \begin{cases} 
1, & \text{if } Z_5(x) \geq 28,735, \\
\frac{Z_5(x) - 21,107}{7,628}, & \text{if } 21,107 < Z_5(x) < 28,735, \\
0, & \text{if } Z_5(x) \leq 21,107.
\end{cases}
$$

(65)

Now, by using the three solution approaches which were introduced in Sections 4.5.1 to 4.5.3, we calculate the compromise solution for the model. To solve the model with the i-FMOLP method, following the algorithm in Section 4.5.1 and using the linear membership functions defined in equations (61) to (65), the FMOSS model under quantity discounts is transformed into an equivalent ordinary single objective LP model. The result is given as:

$$
\max \beta
$$
Multi-objective supplier selection and order allocation

Subject to:

$$\beta \leq \frac{340,507 - Z_1}{82,733}$$, $$\beta \leq \frac{215,870 - Z_2}{79,490}$$, $$\beta \leq \frac{177,505 - Z_3}{66,485}$$,

$$\beta \leq \frac{28,332 - Z_4}{12,187}$$, $$\beta \leq \frac{Z_5 - 21,107}{7,628}$$,

$$\alpha X \leq \gamma$$,

$$\delta Y \leq \theta$$,

$$0 \leq \beta \leq 1, X \geq 0, Y = \text{integer}.$$ 

For solving this model, LINGO (Ver. 8.0) was used. Table 14 presents the optimal solution for the model, which led to an overall DM satisfaction value of 0.48548.

Table 14  Optimal solution for the supplier selection problem under quantity discounts using the i-FMOLP method

<table>
<thead>
<tr>
<th>The number of units purchased from the i&lt;sup&gt;th&lt;/sup&gt; supplier at price level j</th>
<th>Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$x_{ij}$</td>
</tr>
<tr>
<td></td>
<td>$x_{i1} = 695.7449, x_{i2} = 2,400, x_{i3} = 4,800, x_{i4} = 0, x_{i5} = 1,800, x_{i6} = 6,220,$</td>
</tr>
<tr>
<td></td>
<td>$x_{i3} = 0, x_{i7} = 3,400, x_{i8} = 7,200, x_{i9} = 0, x_{i10} = 1,200, x_{i11} = 3,400$</td>
</tr>
<tr>
<td>Objective values</td>
<td>$Z_1 = 297,551, Z_2 = 169,196, Z_3 = 143,836, Z_4 = 22,016, Z_5 = 24,811, \beta = 0.48548$</td>
</tr>
</tbody>
</table>

To solve the problem with the weighted additive approach, the weights of the fuzzy goals are calculated with the help of the FPP technique as $w_1 = 0.473$, $w_2 = 0.25$, $w_3 = 0.139$, $w_4 = 0.083$ and $w_5 = 0.055$. These weights represent the DM’s preferences for cost ($Z_1$), delivery time ($Z_2$), defect rate ($Z_3$), transportation cost ($Z_4$) and total purchased value of each supplier ($Z_5$), respectively. By using these weights and the solution method described in Section 4.5.2 and the linear membership functions obtained in equations (61) to (65), the crisp formulation of the model can be written as:

$$\text{Max } f(\mu) = 0.473\mu_{Z_1} + 0.25\mu_{Z_2} + 0.139\mu_{Z_3} + 0.083\mu_{Z_4} + 0.055\mu_{Z_5}$$

subject to

$$\mu_{Z_1} \leq \frac{340,507 - Z_1}{82,733}$$, $$\mu_{Z_2} \leq \frac{215,870 - Z_2}{79,490}$$, $$\mu_{Z_3} \leq \frac{177,505 - Z_3}{66,485}$$,

$$\mu_{Z_4} \leq \frac{28,332 - Z_4}{12,187}$$, $$\mu_{Z_5} \leq \frac{Z_5 - 21,107}{7,628}$$,

$$\alpha X \leq \gamma$$,

$$\delta Y \leq \theta,$$
The fuzzy programming model presented above was solved using LINGO (Ver. 8.0). The results are shown in Table 15.

Table 15 Optimal solution for the supplier selection problem under quantity discounts using the weighted additive method

| Solutions |  
|-----------|---|
| \( x_{ij} \) | \( x_{i1} = 0, x_{i2} = 2,400, x_{i3} = 4,800, x_{i4} = 0, x_{i2} = 1,800, x_{i3} = 6,220, \) \( x_{i3} = 0, x_{i3} = 3,400, x_{i3} = 7,200, x_{i4} = 0, x_{i3} = 1,200, x_{i3} = 3,400 \) |
| Objective values | \( Z_1 = 290,594, Z_2 = 168,500, Z_3 = 143,140, Z_4 = 21,320, Z_3 = 24,149, f(\mu) = 0.57388 \) |

Finally, using the same weights as were used for the weighted additive method, the weighted max-min method results in the following mathematical programme.

\[
\max \beta
\]

Subject to:

\[
0.473\beta \leq \frac{340,507 - Z_1}{82,733}, \quad 0.25\beta \leq \frac{215,870 - Z_2}{79,490}, \quad 0.139\beta \leq \frac{177,505 - Z_3}{66,485},
\]

\[
0.083\beta \leq \frac{28,332 - Z_4}{12,187}, \quad 0.055\beta \leq \frac{Z_5 - 21,107}{7,628},
\]

\( \alpha X \leq \gamma, \)

\( \delta Y \leq \theta, \)

\( 0 \leq \beta \leq 1, X \geq 0, Y = \text{integer}. \)

Again, LINGO was used to find the compromise solution, which is shown in Table 16.

Table 16 Optimal solution for the supplier selection problem under quantity discounts using the weighted max-min method

| Solutions |  
|-----------|---|
| \( x_{ij} \) | \( x_{i1} = 0, x_{i2} = 2,400, x_{i3} = 4,800, x_{i4} = 0, x_{i2} = 1,800, x_{i3} = 6,220, \) \( x_{i3} = 0, x_{i3} = 3,400, x_{i3} = 7,200, x_{i4} = 0, x_{i3} = 1,200, x_{i3} = 3,400 \) |
| Objective values | \( Z_1 = 290,594, Z_2 = 168,500, Z_3 = 143,140, Z_4 = 21,320, Z_3 = 24,149, \beta = 1.0 \) |
5.4 Performance analysis

To evaluate the performance of the proposed approaches, we compare the solutions of our numerical example that were obtained with different methods (see Table 17). To measure the degree of closeness of the three solutions to the desired solution, we specify the following family of distance functions (see Abd El-Wahed and Lee, 2006):

\[
D_p(\lambda, K) = \left[ \sum_{k=1}^{K} \lambda_k^p (1 - d_k)^p \right]^{1/p}
\]  

where \(d_k\) is the degree of closeness of the preferred compromise solution vector \(X^*\) to the optimal solution vector with respect to the \(k^{th}\) objective function. \(\lambda = (\lambda_1, \lambda_2, ..., \lambda_K)\) is the vector of objectives aspiration levels. The power \(p\) represents a distance parameter \(1 < p < \infty\). Assuming that \(\sum_{k=1}^{K} \lambda_k = 1\), we formulate \(D_p(\lambda, K)\) with the values \(p = \{1, 2, \infty\}\) as follows (see Abd El-Wahed and Lee, 2006):

\[
D_1(\lambda, K) = 1 - \sum_{k=1}^{K} \lambda_k d_k
\]  \(\text{(the Manhattan distance)}\)  

\[
D_2(\lambda, K) = \left[ \sum_{k=1}^{K} \lambda_k^2 (1 - d_k)^2 \right]^{1/2}
\]  \(\text{(the Euclidean distance)}\)  

\[
D_\infty(\lambda, K) = \max_k \{\lambda_k (1 - d_k)\}
\]  \(\text{(the Tchebycheff distance)}\)  

In a minimisation problem, \(d_k\) takes on the form \(d_k = (\text{the optimal solution of } Z_k) / (\text{the preferred compromise solution } Z_k)\). In a maximisation problem, \(d_k\) is computed as \(d_k = (\text{the preferred compromise solution } Z_k) / (\text{the optimal solution of } Z_k)\). Here, we assume that \(\lambda_i = 0.2, i = 1, 2, ..., 5\). Table 17 summarises the results of the three approaches and the desired solution.

<table>
<thead>
<tr>
<th></th>
<th>(1) i-FMOLP</th>
<th>(2) Weighted additive</th>
<th>(3) Weighted max-min</th>
<th>Desired solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Z_1)</td>
<td>297,551</td>
<td>290,594</td>
<td>290,594</td>
<td>257,774</td>
</tr>
<tr>
<td>(Z_2)</td>
<td>169,196</td>
<td>168,500</td>
<td>168,500</td>
<td>136,380</td>
</tr>
<tr>
<td>(Z_3)</td>
<td>143,836</td>
<td>143,140</td>
<td>143,140</td>
<td>111,020</td>
</tr>
<tr>
<td>(Z_4)</td>
<td>22,016</td>
<td>21,320</td>
<td>21,320</td>
<td>16,145</td>
</tr>
<tr>
<td>(Z_5)</td>
<td>24,811</td>
<td>24,149</td>
<td>24,149</td>
<td>28,735</td>
</tr>
<tr>
<td>(D_1)</td>
<td>0.1918</td>
<td>0.186</td>
<td>0.186</td>
<td>-</td>
</tr>
<tr>
<td>(D_2)</td>
<td>0.0888</td>
<td>0.0857</td>
<td>0.0857</td>
<td>-</td>
</tr>
<tr>
<td>(D_\infty)</td>
<td>0.0534</td>
<td>0.0485</td>
<td>0.0485</td>
<td>-</td>
</tr>
</tbody>
</table>

With the results given in Table 17, we can compare the degree of closeness of the three employed approaches to the desired solution. As can be seen, the preferred compromise solution of the weighted additive and weighted max-min approaches outperformed the i-FMOLP approach for all distance functions \(D_1, D_2\) and \(D_\infty\). In addition, the overall satisfaction degree of the DM, \(\beta\), of the weighted max-min approach is higher than the one obtained for the weighted additive approach. Hence, for the present example, the
weighted max-min approach is the method that should be preferred for solving the FMOSS model under quantity discounts. A closer look at Table 17 also reveals that the i-FMOLP method has the highest distance to the ideal solution, as compared to the other two methods. This implies that the i-FMOLP method has still potential to be further improved to reduce its distance to the ideal solution.

6 Summary and conclusions

Supplier selection under quantity discounts is an important and complex decision problem for many firms. The problem becomes more complex if the available information is imprecise. Prior research on supplier selection under quantity discount mainly considered partial fuzziness of the decision problem; a situation where both the objectives of the DM and the constraints are fuzzy has not been studied so far. In this paper, a two-phase model, consisting of an evaluation and an allocation process, was developed to provide decision support for this problem. In the first phase, a combination of FPP method and interval-based fuzzy TOPSIS for evaluating a set of suppliers was proposed. In the second phase, a fuzzy multi-objective model that considered the discounts offered by the suppliers as well as several fuzzy constraints and goals was formulated to determine the optimal order quantity for each supplier. To solve the FMOLP model, a new solution approach was developed to convert the fuzzy multi-objective programme with fuzzy objective function and fuzzy constraints into a model with fuzzy objective function and crisp constraints. Subsequently, three different solution approaches were proposed, namely the i-FMOLP, the weighted additive approach and the weighted max-min approach. The advantage of the proposed method is that it can easily be used to solve a mathematical programme with fuzziness in both the objective function and the constraints. The proposed method also provides a systematic framework that helps to obtain a satisfactory solution. Moreover, the interactive and non-interactive solution methodologies presented in this paper yield an efficient compromise solution and serve the overall satisfaction of the DM with the determined goal values in the FMOSS problem. Finally, the three methods were compared with regard to their distance from the ideal solution.

Our numerical example showed that the weighted additive method and the weighted max-min method provided better results than the i-FMOLP method. In addition, the weighted max-min method was observed to be better than the weighted additive method, as it led to a higher satisfaction degree for the DM than the weighted additive method.

The model and the solution approach developed in this paper can be applied to various management decision problems where DMs face multiple imprecise decision parameters. Our model could be extended in various directions. For example, instead of a quantity discount, other types of discounts (e.g., volume discounts) and their impact on the quantities assigned to the suppliers could be investigated. It may also be interesting for future works to replace the linear properties of the model by non-linear ones. For example, considering non-linear cost components at the suppliers and applying non-linear membership functions to the order allocation problem could be an instant extension of this work. These, and other possible extensions, are reserved for future research.
Multi-objective supplier selection and order allocation

References


