Asymptotically Optimal Power Allocation for Energy Harvesting Communication Networks

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Abstract

For a general energy harvesting (EH) communication network, i.e., a network where the nodes generate their transmit power through EH, we derive the asymptotically optimal online power allocation solution which optimizes a general utility function when the number of transmit time slots, $N$, and the battery capacities of the EH nodes, $B_{\text{max}}$, satisfy $N \to \infty$ and $B_{\text{max}} \to \infty$. The considered family of utility functions is general enough to include the most important performance measures in communication theory such as the ergodic information rate, outage probability, average bit error probability, and average signal-to-noise ratio. The proposed power allocation solution is very simple. Namely, the asymptotically optimal power allocation for the EH network is identical to the optimal power allocation for an equivalent non-EH network whose nodes have infinite energy available and employ the same average transmit power as the corresponding nodes in the EH network. Furthermore, the maximum average performance of a general EH network, employing the proposed asymptotically optimal power allocation, converges to the maximum average performance of an equivalent non-EH network, when $N \to \infty$ and $B_{\text{max}} \to \infty$. Although the proposed solution is asymptotic in nature, it is applicable to EH systems transmitting in a large but finite number of time slots and having a battery capacity much larger than the average harvested power and/or the maximum average transmit power.

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I. Introduction

Energy harvesting (EH) transmitters collect random amounts of energy and store them in their batteries. For this purpose, several techniques for harvesting energy from various renewable sources, such as pressure, motion, solar, etc., have been proposed, see [1], [2], [3] and references therein. Using the stored harvested energy, EH transmitters send codewords to their designated receivers. To this end, the codewords’ powers have to be adapted to the random amounts of harvested energy and also to the quality of the channels between the EH transmitters and their designated receivers, which may be time-varying due to fading.

In the literature, there are two approaches for solving the EH power allocation problem. The aim of the first approach is to obtain the optimal online solution. Online solutions require only causal energy and channel state information, therefore, they are feasible in practice. However, for finite numbers of transmitted codewords, the optimal online solution can often not be computed even for simple communication channels, such as the point-to-point channel. This is due to the fact that computing the optimal online solution typically involves dynamic programing [4], [5]. However, the computational complexity of dynamic programing, even for the simple point-to-point channel, grows exponentially with the number of transmitted codewords, and therefore, cannot be computed even for small-to-moderate numbers of codewords. As a result, the second approach whose objective it is to obtain the optimal offline solution is often adopted in the literature [4], [5]. However, offline solutions require non-causal energy and channel state information, therefore, they are not feasible in practice. Nevertheless, offline solutions still may serve as performance upper bounds for the performance of any online solution.

In the literature, in general, the optimal offline solution is investigated assuming a specific system model, e.g., the point-to-point channel, the broadcast channel, etc., and a specific performance measure, most often the achievable information rate [4]-[17] and rarely other performance measures such as the outage probability [18]. Hence, the derived offline solutions, and the framework for deriving the offline solutions, are usually applicable to the specific considered system model and the specific considered performance measure only, and cannot be easily generalized to different system models and/or different performance measures. For example, the optimal offline power allocation which maximizes the achievable information rate has been investigated for the point-to-point channel in [4]-[7], for the broadcast channel in [8]-[11], for the multiple-access network in [12], [13], and for the EH relay network in [14]-[17]. The
outage probability for the EH point-to-point channel has been investigated in [18]. The above references make different assumptions about the battery capacities and the numbers of transmitted codewords, namely, they assume finite and/or infinite battery capacities and finite and/or infinite numbers of transmitted codewords. Furthermore, they all provide optimal offline solutions. In the cases where optimal online solution are provided, they are based on dynamic programing and thus, can not be computed even for small-to-moderate numbers of transmitted codewords.

Optimal online solutions which maximize an average performance measure, such as the ergodic information rate, the outage probability, the average bit error probability, and the average signal-to-noise ratio, for a general EH network are not known. An exception is the EH point-to-point additive white Gaussian noise (AWGN) channel without fading where the optimal online solution, which achieves the channel capacity, was found in [6], under the assumption of infinitely many transmitted codewords and infinite battery size. However, the framework developed for deriving the optimal online solution in [6] is only applicable to the specific considered case, and cannot be generalized to a different performance measure and/or different system model. Motivated by this, the objective of this paper is to develop a framework for obtaining the optimal online power allocation solution which maximizes some average performance measure of a general EH communication network. In particular, we present a framework for optimal online power allocation in a general EH communication network which optimizes a predefined utility function. The admissible utility functions are general enough to include the most important average performance measures in communication theory, including the ergodic information rate, outage probability, average bit error probability, and average signal-to-noise ratio. The developed framework is asymptotic and holds when the number of transmit time slots, $N$, and the battery capacity, $B_{\text{max}}$, at each EH node in the network are infinite. Based on the developed framework the optimal online solution for a general EH network with a general utility function is relatively easy to obtain. Namely, the optimal online power allocation for the EH network is given by the optimal online power allocation for an equivalent non-EH network where each node has infinite available energy, under the constraint that each node in the non-EH network employs the same average transmit power as the corresponding node in the EH network. As a consequence, the maximum average performance of an EH network converges to the maximum average performance of an equivalent non-EH network, as $N \to \infty$ and $B_{\text{max}} \to \infty$. Hence, the EH network suffers no average performance loss compared to the equivalent non-EH network. The
reason for this convergence in average performance is that when \( N \to \infty \) and \( B_{\text{max}} \to \infty \) hold, the optimal power allocation for an EH network becomes independent of the instantaneous harvested energies and only depends on the average harvested energy, thereby transforming the EH network into a non-EH network in terms of average performance. Hence, in the asymptotic case, instead of finding the optimal power allocation for the EH network, it is sufficient to find the optimal power allocation for the equivalent non-EH network and apply it to the EH network. The practical value of the developed framework is that it gives an average performance upper bound for online power allocation in general EH networks with finite \( N \) and \( B_{\text{max}} \). Furthermore, in practice, the proposed online solution is applicable to EH networks transmitting in a large but finite numbers of time slots and having nodes with battery capacities much larger than their average harvested powers and/or the maximum average transmit powers.

In order to gradually introduce the proposed framework for general EH networks, we first present the framework for the point-to-point EH system in Section II. Then, we generalize the framework to the broadcast and the multiple-access EH networks in Section III. Finally, in Section IV, we introduce the framework for the general EH network. In Section V, we illustrate the applicability of the developed framework through numerical examples, and Section VI concludes the paper.

II. THE POINT-TO-POINT EH SYSTEM

In the following, we consider the point-to-point EH communication system and formulate the power allocation problem. Then, we define an equivalent point-to-point non-EH system which differs from the EH system only in the energy available for transmission of the codewords. For both systems, the transmission time is divided into slots of equal length and each codeword spans one time slot. Furthermore, the number of transmit time slots, \( N \), satisfies \( N \to \infty \). We note that all of the assumptions and definitions that we introduce for the EH transmitter in the point-to-point EH system are also valid for the individual EH transmitters in the more complex EH networks considered in Sections III and IV.

A. Point-to-Point EH System Model

We consider an EH transmitter which harvests random amounts of energy in each time slot and stores them in its battery. It uses the energy stored in its battery to transmit codewords to a receiver. Let the maximum capacity of the battery, denoted by \( B_{\text{max}} \), be unlimited, i.e.,
Let $B_{\text{max}} \rightarrow \infty$ holds. Let $B(i)$ denote the amount of power\(^1\) available in the battery at the end of time slot $i$. Let the amount of harvested power that is added to the battery storage in time slot $i$ be denoted by $P_{\text{in}}(i)$. We assume that $P_{\text{in}}(i)$ is a stationary and ergodic random process with average $\bar{P}_{\text{in}}$ given by

$$
\bar{P}_{\text{in}} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} P_{\text{in}}(i) = E\{P_{\text{in}}(i)\}, \quad (1)
$$

where $E\{\cdot\}$ denotes expectation.

In an EH system, there may be a difference between the desired amount of power that we want to extract from the battery in time slot $i$ and the actual amount of power that can be extracted from the battery in time slot $i$. Let $P_d(i)$ denote the desired amount of power that the EH transmitter wants to extract from the battery in time slot $i$ in order to use it for the transmission of the $i$-th codeword. On the other hand, let $P_{\text{out}}(i)$ denote the actual amount of power extracted from the battery in time slot $i$ and used for transmission of the $i$-th codeword. The relation between $P_d(i)$ and $P_{\text{out}}(i)$ is given by

$$
P_{\text{out}}(i) = \min\{B(i - 1), P_d(i)\}, \quad (2)
$$

i.e., the power extracted from the battery at time slot $i$, $P_{\text{out}}(i)$, is limited by the desired amount of power that the EH transmitter wants to extract from the battery, $P_d(i)$, and the amount of power stored in the battery at the end of the previous time slot, $B(i - 1)$. Obviously, $P_{\text{out}}(i) \leq P_d(i)$, $\forall i$, always holds. Considering the harvested power, $P_{\text{in}}(i)$, and the extracted power, $P_{\text{out}}(i)$, the amount of power in the battery at the end of time slot $i$ is given by

$$
B(i) = B(i - 1) + P_{\text{in}}(i) - P_{\text{out}}(i). \quad (3)
$$

Since $P_{\text{in}}(i)$ is stationary and ergodic, and as a result of the law of conservation of flow in the battery, $P_{\text{out}}(i)$ is also a stationary and ergodic random process with mean $\bar{P}_{\text{out}}$ given by

$$
\bar{P}_{\text{out}} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} P_{\text{out}}(i) = E\{P_{\text{out}}(i)\}
$$

$$
= \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \min\{B(i - 1), P_d(i)\} = E\{\min\{B(i - 1), P_d(i)\}\}. \quad (4)
$$

\(^1\)In this paper, we adopt the normalized energy unit Joule-per-second. As a result, we use the terms "energy" and "power" interchangeably.
On the other hand, the average desired power that the EH transmitter wants to extract from the battery, denoted by $P_d$, is given by $P_d = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} P_d(i)/N$. In communication systems, there usually exists an upper limit on the average transmit power, which for the EH transmitter we denote by $\bar{P}_{lim}$. Then, for an EH transmitter $\bar{P}_{out} \leq \bar{P}_{lim}$ has to hold. We note that, in terms of powers, the desired powers $P_d(i), \forall i$, are the only parameters with a degree of freedom in the EH system. We now define a general utility function which we wish to optimize.

**Definition 1:** The utility function, denoted by $U(i)$, is associated with the $i$-th codeword. It is a predefined function that measures some desired quality of the $i$-th codeword. We assume that $U(i)$ has the following properties.

1. $U(i)$ depends on the transmit power $P_{out}(i)$ and to emphasize this dependence we use the notation $U(P_{out}(i))$.
2. $U(P_{out}(i))$ is either a monotonically increasing or a monotonically decreasing function of $P_{out}(i), \forall i$, i.e., either $U(P_{out}(i) + \epsilon) \geq U(P_{out}(i))$ or $U(P_{out}(i) + \epsilon) \leq U(P_{out}(i))$ holds $\forall i$ and $\epsilon \geq 0$.
3. $U(P_{out}(i))$ is finite for finite $P_{out}(i)$, i.e., $U(P_{out}(i)) < \infty$ for $P_{out}(i) < \infty$.
4. $U(P_{out}(i))$ is a stationary and ergodic random process.

Exploiting the ergodicity of $U(P_{out}(i))$, we can obtain the average utility function, $\bar{U}$, as

$$\bar{U} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} U(P_{out}(i)) = E\{U(P_{out}(i))\}. \quad (5)$$

**Remark 1:** Given the above properties of the utility function $U(P_{out}(i))$, valid utility functions include the information rate of the $i$-th codeword, the average SNR of the $i$-th codeword at the receiver, the outage probability of the $i$-th codeword, and the symbol (bit) error probability of the $i$-th symbol (bit) for uncoded transmission. Hence, our definition of $U(P_{out}(i))$ includes the most important performance measures in communication theory.

In the following, for simplicity of notation, we write $U(i)$ instead of $U(P_{out}(i))$. Furthermore, we assume that $U(i)$ is a monotonically increasing function of $P_{out}(i), \forall i$. However, the developed framework holds also if $U(i)$ is a monotonically decreasing function of $P_{out}(i)$, cf. Remark 2.

For the point-to-point EH system, given a limit on the average transmit power, $\bar{P}_{lim}$, we want to devise an optimal power allocation strategy that maximizes the average utility function $\bar{U}$, i.e., we want to determine the optimal desired powers $P_d(i), \forall i$, which produce a corresponding
$P_{\text{out}}(i), \forall i$, such that $P_{\text{out}} \leq \bar{P}_{\text{lim}}$ holds and the average utility function $\bar{U}$ is maximized. We define this rigorously in the following maximization problem for $N \to \infty$

$$\begin{align*}
\text{Maximize:} & \quad \frac{1}{N} \sum_{i=1}^{N} U(i) \\
\text{Subject to:} & \quad C_1: P_{\text{out}}(i) = \min\{B(i-1), P_d(i)\} \\
& \quad C_2: \bar{P}_{\text{out}} \leq \bar{P}_{\text{lim}} \\
& \quad C_3: \text{Optional constraints on } P_d(i) \\
& \quad C_4: B(i) = B(i-1) + P_{\text{in}}(i) - P_{\text{out}}(i),
\end{align*}$$

(6)

where the $P_{\text{in}}(i)$ are known causally at the EH transmitter. More precisely, the amount of harvested power during the $i$-th time slot is revealed at the EH transmitter at the end of the $i$-th time slot. Furthermore, $C_3$ represents optional constraints on $P_d(i)$, if any. For example, $C_3$ may constrain $P_d(i)$ to be constant for all time slots, or not to exceed some upper limit, or to be zero in certain time slots. Thereby, we assume that the constraints on $P_d(i), \forall i$, if they exist, are such that they allow $\bar{P}_d = \bar{P}_{\text{lim}}$ to be achievable. This assumption simplifies the presentation of the paper and does not restrict the generality of the developed framework. In particular, if $\bar{P}_d = \bar{P}_{\text{lim}}$ is not achievable, then instead of $\bar{P}_{\text{lim}}$ in (6) we can introduce another upper limit on the average transmit power, denoted by $\bar{P}_{\text{lim,new}}$, which is the maximum possible $P_d$ allowed by $C_3$. Then, $\bar{P}_d = \bar{P}_{\text{lim,new}}$ is achievable and our framework holds$^2$.

Remark 2: If $U(i)$ is a monotonically decreasing function of $P_{\text{out}}(i), \forall i$, then, in the optimization problem, the maximization of $\bar{U}$ has to be replaced with a minimization.

Our objective is to solve (6), i.e., to obtain the desired powers $P_d(i), \forall i$, which produce transmit powers $P_{\text{out}}(i), \forall i$, that maximize the average utility function $\bar{U}$ and guarantee $\bar{P}_{\text{out}} \leq \bar{P}_{\text{lim}}$. To this end, we introduce a non-EH communication system having infinite energy available for transmission of the codewords that is equivalent to the EH system. This non-EH system is defined in the following.

B. Equivalent Point-to-Point non-EH System

The equivalent point-to-point non-EH system is identical to the EH system, defined in Section II-A, but with the following two differences. First, the non-EH system has infinite power

$^2$We note that the assumption that $\bar{P}_d = \bar{P}_{\text{lim}}$ is achievable will be used for all individual EH transmitters considered in this paper.
available for the transmission of its codewords, and second, the upper limit on its average transmit power is $P_{\text{lim,non-EH}}$. It will be shown that if $P_{\text{lim,non-EH}}$ is appropriately adjusted, the EH and non-EH system become equivalent in terms of maximum average performance, cf. Theorems 1 and 2. As a result of the infinite power available, in the non-EH system, any desired power, $P_d(i)$, can be provided and therefore the transmit power of each codeword, $P_{\text{out}}(i)$, is identical to the desired power, i.e., $P_{\text{out}}(i) = P_d(i), \forall i$, holds. This is the fundamental difference between an EH and a non-EH system since, contrary to a non-EH system, in an EH system not every desired power $P_d(i)$ can be provided by the power supply (i.e., battery) and therefore the transmit power of the $i$-th codeword is given by (2).

For the equivalent non-EH system, the aim is again to maximize the average utility function, given an upper limit on the average transmit power. However, since in this case $P_d(i) = P_{\text{out}}(i)$, $\forall i$, the maximization problem, for $N \to \infty$, is given by

$$\begin{align*}
\text{Maximize} : & \quad \frac{1}{N} \sum_{i=1}^{N} U(i) \\
\text{Subject to} : & \quad C1 : P_{\text{out}}(i) = P_d(i) \\
& \quad C2 : \bar{P}_{\text{out}} \leq \bar{P}_{\text{lim,non-EH}} \\
& \quad C3 : \text{Optional constraints on } P_d(i),
\end{align*}$$

(7)

where $U(i), \forall i$, and C3 are as in (6). The optimization problem in (7) is the conventional power allocation problem for a conventional (non-EH) point-to-point communication system, and therefore it has been solved in the literature for many different utility functions, e.g., [19], [20].

In the following, we provide the framework for solving (6). In particular, we show that, for $N \to \infty$ and $B_{\text{max}} \to \infty$, the EH optimization problem in (6) becomes identical to the non-EH optimization problem in (7), if $P_{\text{lim,non-EH}}$ in (7) is appropriately adjusted. Therefore, the optimized average performance of the EH system becomes identical to the optimized average performance of the equivalent non-EH system with adjusted $P_{\text{lim,non-EH}}$. As a result, the solution of the non-EH optimization problem is also the solution of the EH optimization problem. In other words, instead of solving the EH optimization problem in (6), one only needs to solve the non-EH optimization problem in (7) and apply the solution in the EH system. This claim is proved in the next subsection.
C. Asymptotically Optimal Power Allocation for the Point-to-Point EH System

In order to derive the solution of the EH optimization problem in (6), we model the battery of the EH transmitter as a queue with average arriving and departing powers. The average arriving power into the queue is the average harvested power, whereas the average departing power out of the queue is the average transmit power drained from the battery. In an EH transmitter, the battery (i.e., the queue), depending on the values of $\bar{P}_{\text{in}}$ and $\bar{P}_{\text{lim}}$, is either in the absorbing state or in the non-absorbing state. The absorbing and non-absorbing states are formally defined in the following definition.

**Definition 2:** A battery is said to be an absorbing battery if, on average, the accumulated power in the battery grows with each time slot, i.e., when $E\{B(i+1)\} > E\{B(i)\}$ and $\lim_{i \to \infty} E\{B(i)\} = \infty$ hold. In this case, a fraction of the average harvested power is trapped in the battery and can never be extracted from it. On the other hand, a battery is said to be a non-absorbing battery if the accumulated power in the battery does not grow with time, i.e., when $E\{B(i+1)\} \leq E\{B(i)\}$ holds. In this case, all the harvested energy that enters the battery eventually leaves the battery.

Depending on the values of $\bar{P}_{\text{in}}$ and $\bar{P}_{\text{lim}}$, the state of the battery of the EH transmitter is given in the following lemma.

**Lemma 1:** The battery of an EH transmitter is absorbing and non-absorbing when $\bar{P}_{\text{lim}} < \bar{P}_{\text{in}}$ and $\bar{P}_{\text{lim}} \geq \bar{P}_{\text{in}}$ hold, respectively.

**Proof:** See Appendix A.

Before introducing the solution to (6), we provide another useful lemma.

**Lemma 2:** $\bar{U}$ is maximized when $\bar{P}_{\text{out}}$ reaches its maximum possible value, given by $\bar{P}_{\text{out}} = \bar{P}_{\text{lim}}$ and $\bar{P}_{\text{out}} = \bar{P}_{\text{in}}$ for an absorbing and non-absorbing battery, respectively.

**Proof:** Note that $\bar{P}_{\text{lim}}$ is the ultimate limit for $\bar{P}_{\text{out}}$. Hence, an absorbing battery has this limit. On the other hand, due to the law of conservation of flow in the battery, $\bar{P}_{\text{out}}$ cannot be larger than $\bar{P}_{\text{in}}$. Hence, this is the upper limit for $\bar{P}_{\text{out}}$ in a non-absorbing battery. On the other hand, since $U(i), \forall i$, is a monotonically increasing function of $P_{\text{out}}(i), \forall i$, it is straightforward to show that $\bar{U}$ achieves its maximum value when $\bar{P}_{\text{out}}$ achieves its maximum value.

Now we are ready to provide the solution to (6) for an absorbing battery in the following theorem.

**Theorem 1:** If the battery is absorbing, the number of codewords transmitted with power
$P_{\text{out}}(i) = B(i - 1)$ is negligible compared to the number of codewords transmitted with power $P_{\text{out}}(i) = P_d(i)$. As a result, only the codewords transmitted with power $P_{\text{out}}(i) = P_d(i)$ contribute to the average utility function $\bar{U}$, whereas the codewords transmitted with power $P_{\text{out}}(i) = B(i - 1)$ have negligible contribution to the average utility function $\bar{U}$. Therefore, practically all codewords are transmitted with power $P_{\text{out}}(i) = P_d(i)$. As a result, in (6), C1 is transformed to $P_{\text{out}}(i) = P_d(i)$ and C4 is not necessary. Consequently, (6) can be written equivalently as (7) by setting $\bar{P}_{\text{lim,non-EH}} = \bar{P}_{\text{lim}}$. Hence, the solution to the non-EH maximization problem in (7) with $\bar{P}_{\text{lim,non-EH}} = \bar{P}_{\text{lim}}$ is the solution to the considered EH maximization problem in (6) for an absorbing battery.

Proof: Please refer to Appendix B.

We now turn to the case when the battery is non-absorbing. Interestingly, this case also has a very simple solution, which is provided in the following theorem.

Theorem 2: If the battery is non-absorbing, the solution to the maximization problem in (6) is obtained when the average of the desired powers is chosen such that it satisfies the following equality

$$\bar{P}_d = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} P_d(i) = \bar{P}_\text{in} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} P_{\text{in}}(i).$$ (8)

When (8) holds, $\bar{P}_{\text{out}} = \bar{P}_\text{in}$ and therefore the average transmit power $\bar{P}_{\text{out}}$ reaches its maximal value, the average harvested power $\bar{P}_\text{in}$. Furthermore, when (8) holds, the number of codewords transmitted with power $P_{\text{out}}(i) = B(i - 1)$ is negligible compared to the number of codewords transmitted with power $P_{\text{out}}(i) = P_d(i)$. As a result, only the codewords transmitted with power $P_{\text{out}}(i) = P_d(i)$ contribute to the average utility function $\bar{U}$, whereas the codewords transmitted with power $P_{\text{out}}(i) = B(i - 1)$ have negligible contribution to the average utility function $\bar{U}$. Therefore, practically all codewords are transmitted with power $P_{\text{out}}(i) = P_d(i)$. As a result, in (6), C1 is transformed to $P_{\text{out}}(i) = P_d(i)$, C4 is not necessary, and C2 is always satisfied since $\bar{P}_{\text{out}} = \bar{P}_\text{in} \leq \bar{P}_{\text{lim}}$ holds. Consequently, (6) can be written equivalently as (7) by setting $\bar{P}_{\text{lim,non-EH}} = \bar{P}_\text{in}$. Hence, for the non-absorbing battery case the solution to the non-EH maximization problem in (7) with $\bar{P}_{\text{lim,non-EH}} = \bar{P}_\text{in}$ is the solution to the considered EH maximization problem in (6).

By ”practically all”, we mean all except a negligible fraction.
Proof: Please refer to Appendix C.

Remark 3: We have proved that, for $N \to \infty$ and $B_{\text{max}} \to \infty$, instead of solving the EH optimization problem, we can always solve the corresponding non-EH optimization problem with appropriately adjusted $\bar{P}_{\text{lim,non-EH}}$ and use the solution for power allocation in the EH system. This means that the EH system achieves the same performance as the equivalent non-EH system with appropriately chosen $\bar{P}_{\text{lim,non-EH}}$. Furthermore, an interesting consequence arising from this approach is that the asymptotically optimal online power allocation for the EH system requires only knowledge of the average harvested power $\bar{P}_{\text{in}}$ and does not need any causal or noncausal knowledge of $P_{\text{in}}(i), \forall i$, i.e., any additional knowledge would not increase $\bar{U}$.

In [6], the authors proved that the maximum average information rate of a point-to-point EH system operating over an AWGN channel without fading, converges to the maximum average information rate of the equivalent point-to-point non-EH system with average transmit power $\bar{P}_{\text{in}}$, when $N \to \infty$ and $B_{\text{max}} \to \infty$. Now, Theorem 2 confirm this convergence. Furthermore, it provides a deeper understanding of why this convergence happens, not only for the maximum average information rate, but also for other average performance measures, and not only for non-fading channels, but also for fading channels.

D. Examples for Power Allocation in Point-to-Point EH Systems

In this subsection, we illustrate the application of the proposed framework. To this end, we consider a point-to-point EH system that operates over a slow time-continuous fading channel with AWGN having unit variance. For this system, let the square of the fading gain in the $i$-th time slot be denoted by $\gamma(i)$. Furthermore, we assume that the average power harvested by the EH transmitter is $\bar{P}_{\text{in}}$. In the following, we consider two examples with different utility functions $U(i)$.

Example 1: For the first example, $U(i)$ is the outage indicator for the $i$-th codeword, which

\footnote{We note that the manner in which the maximum average information rate of the point-to-point EH system in [6] is achieved is different from the approach laid out in Theorem 2. Namely, in [6], the EH transmitter harvests energy without transmitting in the first $K$ time slots and then harvests energy and transmits in the following $N$ time slots, where $K \to \infty$, $N \to \infty$, and $K/N \to 0$ hold.}
is given by
\[ U(i) = \begin{cases} 
1 & \text{if } \log_2(1 + P_{\text{out}}(i)\gamma(i)) < R_0 \\
0 & \text{if } \log_2(1 + P_{\text{out}}(i)\gamma(i)) \geq R_0,
\end{cases} \] (9)
i.e., the value of \( U(i) \) indicates whether or not an outage occurs when the EH transmitter transmits a codeword with a fixed information rate \( R_0 \). Hence, \( \bar{U} \) represents the fraction of codewords received in outage, or in other words, the outage probability of a codeword.

Since utility function \( U(i) \) is a monotonically decreasing function of \( P_{\text{out}}(i) \), Remark 2 applies and the maximization in (6) has to be replaced by a minimization. Furthermore, for this example, we impose a constraint on \( P_d(i) \). Namely, \( P_d(i) \) can take any value but has to be constant for all time slots. Moreover, \( \bar{P}_{\text{lim}} \geq \bar{P}_{\text{in}} \) is assumed. Using Lemma 1, we see that the battery is non-absorbing and therefore Theorem 2 is applicable. Hence, we have to solve the non-EH problem in (7) with \( \bar{P}_{\text{lim,non-EH}} = \bar{P}_{\text{in}} \) and \( P_d(i) \) being constant \( \forall i \). The solution to this non-EH problem is straightforward and given by \( P_d(i) = \bar{P}_{\text{in}}, \forall i \), which, according to Theorem 2, is also the solution to the considered EH problem in (6). Hence, the output power for the \( i \)-th codeword in the EH system is \( P_{\text{out}}(i) = \min\{B(i - 1), \bar{P}_{\text{in}}\} \).

Example 2: For the second example, let \( U(i) \) represent the maximum information rate of the \( i \)-th codeword. To underline the generality of the proposed framework, we consider now an example that accounts for power amplifier inefficiency [21], [22]. In particular, we model the "transmit" power\(^5\) for the \( i \)-th codeword as [22]
\[ P_{\text{out}}(i) = \varepsilon P^T_{\text{out}}(i) + P_C, \] (10)
where \( P^T_{\text{out}}(i) \) is the actual transmit power of the \( i \)-th codeword and \( \varepsilon \geq 1 \) is a constant which accounts for the inefficiency of the power amplifier. For example, if \( \varepsilon = 5 \), then 5 Watts are consumed in the power amplifier and have to be drawn from the battery for every 1 Watt of power radiated in the RF, which results in a power efficiency of \( 1/\varepsilon = 1/5 = 20\% \). The power that is not radiated is dissipated as heat in the power amplifier [22]. Furthermore, \( P_C \geq 0 \) is the static circuit power consumption of the transmitter device electronics such as mixers, filters, digital-to-analog converters, and is independent of the actual transmitted power \( P^T_{\text{out}}(i) \). Hence, \( U(i) \) is given by
\[ U(i) = \log_2 \left(1 + P^T_{\text{out}}(i)\gamma(i)\right) = \log_2 \left(1 + \frac{1}{\varepsilon}(P_{\text{out}}(i) - P_C)^+\gamma(i)\right), \] (11)
\(^5\)In this case, \( P_{\text{out}}(i) \) is actually the power drawn from the battery and consumed for the transmission of the \( i \)-th codeword.
where \((x)^+ = \max\{0, x\}\). Given \(U(i)\), \(\bar{U}\) is the maximal average information rate that the EH transmitter can transmit to the receiver. For this example, we do not impose any additional constraints on \(P_d(i)\) and assume \(\bar{P}_{\text{lim}} \geq \bar{P}_{\text{in}}\). Hence, according to Lemma 1, the EH transmitter has a non-absorbing battery, and therefore Theorem 2 is applicable. According to Theorem 2, the optimal power allocation for this EH system can be found by solving the non-EH optimization problem in (7) with \(\bar{P}_{\text{lim, non-EH}} = \bar{P}_{\text{in}}\). To this end, we follow the approach in [19] (where \(\varepsilon = 1\) and \(P_C = 0\) was assumed) and obtain the solution to the non-EH optimization problem in (7) as

\[
P_d(i) = \begin{cases} 
P_C + (1/\lambda - 1/\gamma(i))/\varepsilon, & \text{if } \gamma(i) > \lambda \\
0, & \text{if } \gamma(i) \leq \lambda,
\end{cases}
\]

where \(\lambda\) is found as the solution to \(E\{P_d(i)\} = \bar{P}_{\text{in}}\). Moreover, according to Theorem 2, the desired power of the non-EH system in (12) is also the desired power for the EH system. As a result, the power spent by the EH system for the transmission of the \(i\)-th codeword is given by

\[
P_{\text{out}}(i) = \begin{cases} 
\min\{B(i - 1) , P_C + (1/\lambda - 1/\gamma(i))/\varepsilon\}, & \text{if } \gamma(i) > \lambda \\
0, & \text{if } \gamma(i) \leq \lambda.
\end{cases}
\]

Numerical results for both considered examples are provided in Section V.

III. THE BROADCAST AND MULTIPLE-ACCESS EH NETWORKS

In the following, we generalize the framework developed for the point-to-point EH channel to the broadcast (point-to-multipoint) and the multiple-access (multipoint-to-point) EH networks. Thereby, we show that the maximum average performance of the broadcast and multiple-access EH networks converge to the maximum average performance of their equivalent broadcast and multiple-access non-EH networks, respectively.

A. The Broadcast EH Network

Let us assume a single EH transmitter transmitting to \(M\) receiving nodes (receivers). We note that we do not assume any restriction on how the \(M\) receiving nodes mutually cooperate or on the correlation between the channels to the \(M\) receiving nodes. In each time slot, the EH transmitter extracts power from its battery and uses it to transmit codewords to each of the receivers. Let the transmit power of the codeword transmitted to the \(k\)-th receiver in the \(i\)-th time slot be denoted
by $P_{\text{out},k}(i)^6$. Without loss of generality, we assume that in each time slot the EH transmitter extracts power from the battery in the following predefined sequential manner. In each time slot, the transmitter first extracts the power for the codeword transmitted to the first receiver, then, from the leftover power in the battery it extracts the power for the codeword transmitted to the second receiver, and so on until it extracts the power for the codeword transmitted to the $M$-th receiver from the leftover power in its battery. Let $P_{d,k}(i)$ denote the desired transmit power for the codeword to the $k$-th receiver in the $i$-th time slot. Then, $P_{\text{out},k}(i)$ is given by

$$
P_{\text{out},k}(i) = \begin{cases} 
P_{d,k}(i), & \text{if } B(i - 1) \geq \sum_{j=1}^{k} P_{d,j} \\
B(i - 1) - \sum_{j=1}^{k-1} P_{\text{out},j}, & \text{otherwise}, \end{cases}$$

where $B(i)$ is the amount of power in the battery of the EH transmitter in the $i$-th time slot and is given by

$$
B(i) = B(i - 1) + P_{\text{in}}(i) - \sum_{k=1}^{M} P_{\text{out},k}(i). \tag{14}
$$

Here, $P_{\text{in}}(i)$ is the amount of power harvested in the $i$-the time slot. The total transmit power and the desired total transmit power of the EH transmitter in time slot $i$, denoted by $P_{\text{out}}(i)$ and $P_{d}(i)$, respectively, are given by

$$
P_{\text{out}}(i) = \sum_{k=1}^{M} P_{\text{out},k}(i). \tag{15}
$$

and

$$
P_{d}(i) = \sum_{k=1}^{M} P_{d,k}(i). \tag{16}
$$

Hence, the average transmit power and the average desired power of the EH transmitter, denoted by $\bar{P}_{\text{out}}$ and $\bar{P}_{d}$, respectively, are given by

$$
\bar{P}_{\text{out}} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} P_{\text{out}}(i) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{M} P_{\text{out},k}(i) \tag{17}
$$

$$
\bar{P}_{d} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} P_{d}(i) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \sum_{k=1}^{M} P_{d,k}(i). \tag{18}
$$

Using $P_{\text{in}}(i)$, the average harvested power, $\bar{P}_{\text{in}}$, can be found by (1). Now, let $\bar{P}_{\text{lim}}$ denote the upper limit on the average transmit power for all channels. Then, the average transmit power

\footnote{If the same codeword is transmitted to more than one receiver, then, in terms of transmit power, these receivers can be merged into a single equivalent receiver.}
must satisfy $\bar{P}_{\text{out}} \leq \bar{P}_{\text{lim}}$. In the following, we introduce the utility function of the broadcast network which is a multivariate version of the utility function of the point-to-point system.

Let $U(i)$ denote the utility function of the broadcast network in the $i$-th time slot. The utility function $U(i)$ is now associated with all $M$ codewords transmitted in the $i$-th time slot, and, similar to the point-to-point case, it measures some desired quality of the codewords transmitted in the $i$-th time slot. Let $U(i)$ be a function of all $M$ transmit powers $P_{\text{out},k}(i)$, $k = 1, \ldots, M$. We formally express the dependence of the utility function on the $M$ transmit powers as $U(P_{\text{out},1}(i), P_{\text{out},2}(i), \ldots, P_{\text{out},M}(i))$. For simplicity of presentation, we write $U(i)$ instead of $U(P_{\text{out},1}(i), P_{\text{out},2}(i), \ldots, P_{\text{out},M}(i))$. We assume that the properties of $U(i)$ as a function of an individual transmit power $P_{\text{out},k}(i)$, $\forall k = 1, \ldots, M$, are as outlined in Definition 1. Given $U(i)$, for simplicity of presentation, we write $U(i)$ instead of $U(P_{\text{out},1}(i), P_{\text{out},2}(i), \ldots, P_{\text{out},M}(i)).$

We assume that the properties of $U(i)$ as a function of an individual transmit power $P_{\text{out},k}(i)$, $\forall k = 1, \ldots, M$, are as outlined in Definition 1. Given $U(i)$, for simplicity of presentation, we write $U(i)$ instead of $U(P_{\text{out},1}(i), P_{\text{out},2}(i), \ldots, P_{\text{out},M}(i))$. We assume that the properties of $U(i)$ as a function of an individual transmit power $P_{\text{out},k}(i)$, $\forall k = 1, \ldots, M$, are as outlined in Definition 1. Given $U(i)$, for simplicity of presentation, we write $U(i)$ instead of $U(P_{\text{out},1}(i), P_{\text{out},2}(i), \ldots, P_{\text{out},M}(i))$. We assume that the properties of $U(i)$ as a function of an individual transmit power $P_{\text{out},k}(i)$, $\forall k = 1, \ldots, M$, are as outlined in Definition 1. Given $U(i)$, for simplicity of presentation, we write $U(i)$ instead of $U(P_{\text{out},1}(i), P_{\text{out},2}(i), \ldots, P_{\text{out},M}(i)).$

We formally express the dependence of the utility function on the $M$ transmit powers as $U(P_{\text{out},1}(i), P_{\text{out},2}(i), \ldots, P_{\text{out},M}(i))$. For simplicity of presentation, we write $U(i)$ instead of $U(P_{\text{out},1}(i), P_{\text{out},2}(i), \ldots, P_{\text{out},M}(i))$. We assume that the properties of $U(i)$ as a function of an individual transmit power $P_{\text{out},k}(i)$, $\forall k = 1, \ldots, M$, are as outlined in Definition 1. Given $U(i)$, for simplicity of presentation, we write $U(i)$ instead of $U(P_{\text{out},1}(i), P_{\text{out},2}(i), \ldots, P_{\text{out},M}(i))$. We assume that the properties of $U(i)$ as a function of an individual transmit power $P_{\text{out},k}(i)$, $\forall k = 1, \ldots, M$, are as outlined in Definition 1. Given $U(i)$, for simplicity of presentation, we write $U(i)$ instead of $U(P_{\text{out},1}(i), P_{\text{out},2}(i), \ldots, P_{\text{out},M}(i))$. We assume that the properties of $U(i)$ as a function of an individual transmit power $P_{\text{out},k}(i)$, $\forall k = 1, \ldots, M$, are as outlined in Definition 1. Given $U(i)$, for simplicity of presentation, we write $U(i)$ instead of $U(P_{\text{out},1}(i), P_{\text{out},2}(i), \ldots, P_{\text{out},M}(i))$. We assume that the properties of $U(i)$ as a function of an individual transmit power $P_{\text{out},k}(i)$, $\forall k = 1, \ldots, M$, are as outlined in Definition 1. Given $U(i)$, for simplicity of presentation, we write $U(i)$ instead of $U(P_{\text{out},1}(i), P_{\text{out},2}(i), \ldots, P_{\text{out},M}(i)).$

Based on $U(i)$, we introduce now the power allocation problem for the broadcast EH network.

For the broadcast EH network, given a limit on the average transmit power, $\bar{P}_{\text{lim}}$, we wish to determine the optimal desired powers $P_{\text{d},k}(i)$, $\forall i, k$, which produce the corresponding $P_{\text{out},k}(i)$, $\forall i, k$, such that $\bar{P}_{\text{out}} \leq \bar{P}_{\text{lim}}$ holds and the average utility function $\bar{U}$ is maximized. We define this rigorously in the following maximization problem for $N \to \infty$

Maximize : $\frac{1}{N} \sum_{i=1}^{N} U(i)$

Subject to : C1 : $P_{\text{out},k}(i) = \begin{cases} P_{\text{d},k}(i), & \text{if } B(i - 1) \geq \sum_{j=1}^{k} P_{\text{d},j} \\ B(i - 1) - \sum_{j=1}^{k-1} P_{\text{out},j}, & \text{otherwise} \end{cases}$

C2 : $\bar{P}_{\text{out}} \leq \bar{P}_{\text{lim}}$

C3 : Optional constraints on $P_{\text{d},k}(i)$

C4 : $B(i) = B(i - 1) + P_{\text{in}}(i) - \sum_{k=1}^{M} P_{\text{out},k}(i)$

where $P_{\text{in}}(i)$ is known causally at the EH transmitter as explained for the point-to-point EH channel. On the other hand, for the equivalent non-EH broadcast network, the non-EH transmitter can satisfy any desired powers, thus $P_{\text{out},k}(i) = P_{\text{d},k}(i)$, $\forall i, k$. Therefore, maximizing the average utility function, $\bar{U}$, for the equivalent broadcast non-EH network has the following form for $N \to \infty$

Maximize : $\frac{1}{N} \sum_{i=1}^{N} U(i)$

Subject to : C1 : $P_{\text{out},k}(i) = P_{\text{d},k}(i)$

C2 : $P_{\text{out}} \leq P_{\text{lim},\text{non-EH}}$

C3 : Optional constraints on $P_{\text{d},k}(i)$,
where \( U(i), \forall i \), and C3 are the same as in (19).

Depending on the values of \( \bar{P}_{\text{in}} \) and \( \bar{P}_{\text{lim}} \), the battery can be absorbing or non-absorbing, cf. Lemma 1. For both an absorbing and a non-absorbing battery, the solution to (19) is provided in the following theorem.

**Theorem 3:** If the EH transmitter in the broadcast network has an absorbing and a non-absorbing battery, the maximization problem in (19) can be written equivalently as (20) by setting \( \bar{P}_{\text{lim,non-EH}} \) in (20) to \( \bar{P}_{\text{lim,non-EH}} = \bar{P}_{\text{in}} \) and to \( \bar{P}_{\text{lim,non-EH}} = \bar{P}_{\text{lim}} \), respectively. Hence, the solution to the non-EH maximization problem in (20) with appropriately chosen \( \bar{P}_{\text{lim,non-EH}} = \bar{P}_{\text{in}} \) is the solution to the EH maximization problem in (19).

**Proof:** Please refer to Appendix D.

Next, we consider the multiple-access EH network.

### B. The Multiple-Access EH Network

The multiple-access EH network is comprised of \( M \) EH transmitters transmitting to a single receiving node (receiver). Similar to the broadcast EH network, we do not impose any restriction on how the \( M \) transmitting nodes cooperate or on the correlation between the transmitting channels. However, we impose one practical constraint by assuming that the harvested energy from one EH transmitter cannot be transferred to another EH transmitter. In each time slot, each EH transmitter extracts power from its battery and uses it to transmit a codeword to the receiver. Let the transmit power of the codeword transmitted by the \( k \)-th EH transmitter in the \( i \)-th time slot be denoted by \( P_{\text{out},k}(i) \). Let \( P_{\text{d},k}(i) \) denote the desired power that the \( k \)-th EH transmitter wants to extract from its battery in the \( i \)-th time slot. Then, \( P_{\text{out},k}(i) \) and \( P_{\text{d},k}(i) \) are related by
\[
P_{\text{out},k}(i) = \min\{B_k(i-1), P_{\text{d},k}(i)\},
\]
where \( B_k(i) \) is the amount of power in the battery of the \( k \)-th EH transmitter in the \( i \)-th time slot and is given by
\[
B_k(i) = B_k(i-1) + P_{\text{in},k}(i) - P_{\text{out},k}(i).
\]
Here, \( P_{\text{in},k}(i) \) is the harvested power at the \( k \)-th EH transmitter in the \( i \)-th time slot. For the \( k \)-th EH transmitter, the average transmit power, denoted by \( \bar{P}_{\text{out},k} \), the average desired power, denoted by \( \bar{P}_{\text{d},k} \), and the average harvested power, denoted by \( \bar{P}_{\text{in},k} \), are given by
\[
\bar{P}_{\alpha,k} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} P_{\alpha,k}(i), \quad \alpha \in \{\text{out}, \text{d}, \text{in}\}.
\]
Furthermore, each EH transmitter imposes an upper limit on the average transmit power, which for the $k$-th EH transmitter is denoted by $P_{\text{lim},k}$. Thus, $P_{\text{out},k} \leq P_{\text{lim},k}$ has to hold.

Similar to the broadcast EH network, the utility function of the multiple-access EH network in the $i$-th time slot, $U(i)$, depends on all $P_{\text{out},k}(i)$, $k = 1, \ldots, M$, and, as a function of any individual $P_{\text{out},k}(i)$, has the properties laid out in Definition 1.

For the multiple-access EH network, given a limit on the average transmit power of each EH transmitter, $\bar{P}_{\text{lim},k}$, we wish to determine the optimal desired powers $P_{d,k}(i)$, $\forall i, k$, which produce the corresponding $P_{\text{out},k}(i)$, $\forall i, k$, such that $P_{\text{out},k} \leq \bar{P}_{\text{lim},k}$, $\forall k$, holds, and the average utility function $\bar{U}$ is maximized. This leads to the following maximization problem for $N \to \infty$

$$\begin{align*}
\text{Maximize:} & \quad \frac{1}{N} \sum_{i=1}^{N} U(i) \\
\text{Subject to:} & \quad C1 : P_{\text{out},k}(i) = \min\{B_k(i-1), P_{d,k}(i)\} \\
& \quad C2 : P_{\text{out},k} \leq \bar{P}_{\text{lim},k} \\
& \quad C3 : \text{Optional constraints on } P_{d,k}(i) \\
& \quad C4 : B_k(i) = B_k(i-1) + P_{\text{in},k}(i) - P_{\text{out},k}(i),
\end{align*}$$

where $P_{\text{in},k}(i)$ is known causally only at the $k$-th EH transmitter. On the other hand, for the equivalent non-EH multiple-access network, since $P_{\text{out},k}(i) = P_{d,k}(i)$, $\forall k$, $i$, we have the following optimization problem for $N \to \infty$

$$\begin{align*}
\text{Maximize:} & \quad \frac{1}{N} \sum_{i=1}^{N} U(i) \\
\text{Subject to:} & \quad C1 : P_{\text{out},k}(i) = P_{d,k}(i) \\
& \quad C2 : P_{\text{out},k} \leq \bar{P}_{\text{lim,non-EH},k} \\
& \quad C3 : \text{Optional constraints on } P_{d,k}(i),
\end{align*}$$

where $U(i)$, $\forall i$, and C3 are the same as in (24).

Depending on the values of $\bar{P}_{\text{in},k}$ and $\bar{P}_{\text{lim},k}$, as explained in Lemma 1, the battery of the $k$-th EH transmitter can be absorbing or non-absorbing. Let us consider the general case in which a fraction of the $M$ nodes have an absorbing battery and the rest have a non-absorbing battery. Then, let us put the indices of the nodes with absorbing battery into set $\mathcal{A}$ and the indices of the nodes with non-absorbing battery into set $\bar{\mathcal{A}}$, where $|\mathcal{A}| + |\bar{\mathcal{A}}| = M$, with $|\cdot|$ denoting the cardinality of a set. Using sets $\mathcal{A}$ and $\bar{\mathcal{A}}$ we now characterize the solution of (24).

**Theorem 4:** Maximization problem (24) can be transformed to maximization problem (25) by setting $P_{\text{lim,non-EH},k}$ in (25) to $P_{\text{lim,non-EH},k} = P_{\text{lim},k}$ for $k \in \mathcal{A}$ and to $P_{\text{lim,non-EH},k} = P_{\text{in},k}$ for
\( k \in \hat{A} \). Hence, the solution to the non-EH maximization problem in (25) with properly adjusted 
\( P_{\text{lim, non-EH,k}} \) is also the solution to the EH maximization problem in (24).

Proof: Please see Appendix E.

In the following, we present examples for power allocation in the broadcast and multiple-access EH networks.

C. Examples for Power Allocation in Broadcast and Multiple-Access EH Networks

We consider broadcast and multiple-access EH networks with \( M \) receiving and \( M \) transmitting nodes, respectively, operating over time-continuous fading channels with AWGN having unit variance. We first consider an example for optimal power allocation in the broadcast EH network.

Example 3: In the broadcast EH network, the square of the fading gain of the channel from the EH transmitter to the \( k \)-th receiver, \( k = 1, ..., M \), in the \( i \)-th time slot is denoted by \( \gamma_k(i) \). We assume that the average power harvested by the EH transmitter in the broadcast network is \( \bar{P}_{\text{in}} \), \( \bar{P}_{\text{lim}} \geq \bar{P}_{\text{in}} \), and there is no constraint on \( P_{d}(i), \forall i \).

In this example, the utility function \( U(i) \) is the maximum sum rate of the \( M \) receivers. Therefore, the utility function, \( U(i) \), is given by [23]

\[
U(i) = \log_2 \left( 1 + \sum_{k=1}^{M} P_{\text{out,k}(i)} \gamma_k(i) \right).
\] (26)

Hence, \( \bar{U} \) is the average sum rate that the EH transmitter can transmit to the \( M \) receivers. Our aim is to maximize \( \bar{U} \). To this end, using Theorem 3, we transform the EH optimization problem in (19) into the non-EH optimization problem in (20) by setting \( \bar{P}_{\text{lim, non-EH}} = \bar{P}_{\text{in}} \). Then, the solution to the non-EH optimization problem is given by [24] \(^7\)

\[
P_{d,k}(i) = \begin{cases} 
1/\lambda - 1/\gamma_k(i), & \text{if } \gamma_k(i) = \max\{\gamma_1(i), \gamma_2(i), ..., \gamma_M(i)\} \text{ AND } \gamma_k(i) \geq \lambda \\
0, & \text{otherwise},
\end{cases}
\] (27)

where \( \lambda \) is found as the solution to \( \sum_{k=1}^{M} E\{P_{d,k}(i)\} = \bar{P}_{\text{in}} \). As a result of Theorem 3, (27) is also the solution to the EH optimization problem in (19). Hence, the \( i \)-th codeword from the EH transmitter to the \( k \)-th receiver is transmitted with power \( P_{\text{out},k}(i) = \min\{B(i-1), P_{d,k}(i)\} \) where \( P_{d,k}(i) \) is given in (27). Next, we provide an example for power allocation in a multiple-access EH network.

\(^7\)The result in [24] is derived for the multiple-access channel, however, due to the duality of the multiple-access and the broadcast channels, a similar result holds for the broadcast channel.
Example 4: In the multiple-access network, the square of the fading gain of the channel from the \( k \)-th EH transmitter, \( k = 1, \ldots, M \), to the receiver in the \( i \)-th time slot is denoted by \( \gamma_k(i) \). We assume that the average power harvested by the \( k \)-th EH transmitter is \( \bar{P}_{\text{in},k} \) and that \( \bar{P}_{\text{lim},k} \geq \bar{P}_{\text{in},k} \), \( \forall k \). Furthermore, we assume that \( P_{d,k}(i) \), can only take one arbitrary constant value which is fixed \( \forall i \).

We assume that the utility function \( U(i) \) is the bit error probability at the receiver in the \( i \)-th time slot assuming all \( M \) EH transmitters transmit the same symbol using binary phase shift keying (BPSK) and beamforming\(^8\). As a result, the utility function, \( U(i) \), is given by

\[
U(i) = Q \left( \sqrt{2 \sum_{k=1}^{M} \gamma_k(i) P_{\text{out},k}(i)} \right),
\]

where \( Q(\cdot) \) is the Gaussian Q-function. Hence, \( \bar{U} \) is the average bit error rate at the receiver. Since, \( U(i) \) is a decreasing function of each \( P_{\text{out},k}(i) \), \( \forall k \), according to Remark 2, the average utility function, \( \bar{U} \), has to be minimized. Hence, we replace the maximization in (24) and (25) with a minimization. Now, in order to solve the minimization problem in (24), we use Theorem 4 and transform (24) into the non-EH minimization problem in (25) by setting \( \bar{P}_{\text{lim,non-EH},k} = \bar{P}_{\text{in},k} \), \( \forall k \), and force \( P_{d,k}(i) \) to be constant \( \forall i \). Then, the solution to the non-EH optimization problem is straightforward and given by \( P_{d,k}(i) = \bar{P}_{\text{in},k} \), \( \forall k, i \). Moreover, this is also the solution to the EH optimization problem in (24). Hence, the transmit power of the \( k \)-th EH transmitter for the \( i \)-th symbol, is given by \( P_{\text{out},k}(i) = \min\{B_k(i-1), \bar{P}_{\text{in},k}\} \), \( \forall k \).

Numerical results for both examples are provided in Section V.

IV. THE GENERAL EH NETWORK

In this section, we extend the framework developed in the previous sections further and derive the asymptotically optimal power allocation for a general EH network.

A. Asymptotically Optimal Power Allocation

We consider a network comprised of \( M \) EH transmitters. For the \( k \)-th EH transmitter, \( k = 1, \ldots, M \), in the \( i \)-th time slot, we denote the harvested power, the actual transmit power, the

\(^8\)This case occurs in the second hop of a decode-and-forward cooperative network with \( M \) EH relays if all EH relays receive error-free from a source and then use beamforming to transmit the information to the destination.
desired transmit power, and the amount of stored power in the battery by $P_{in,k}(i)$, $P_{out,k}(i)$, $P_{d,k}(i)$, and $B_k(i)$, respectively. Each EH transmitter uses the harvested power to transmit to its designated receiving nodes. We collect the indices of the receiving nodes of the $k$-th EH transmitter in set $R_k$. Then, in the $i$-th time slot, we decompose the transmit power of the $k$-th EH node, $P_{out,k}(i)$, into $|R_k|$ transmit powers as

$$P_{out,k}(i) = \sum_{j \in R_k} P_{out,k \rightarrow j}(i), \quad (29)$$

where $P_{out,k \rightarrow j}(i)$ is the transmit power of the codeword sent from the $k$-th EH transmitter to its $j$-th receiver in the $i$-th time slot. Similarly, in the $i$-th time slot, we decompose the desired transmit power of the $k$-th EH transmitter, $P_{d,k}(i)$, into $|R_k|$ desired transmit powers as

$$P_{d,k}(i) = \sum_{j \in R_k} P_{d,k \rightarrow j}(i), \quad (30)$$

where $P_{d,k \rightarrow j}(i)$ is the desired transmit power for the codeword sent from the $k$-th EH transmitter to its $j$-th receiver in the $i$-th time slot. Using (29) and (30), we write the average transmit power and the average desired transmit power of the $k$-th EH transmitter as

$$\bar{P}_{out,k} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} P_{out,k}(i) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \sum_{j \in R_k} P_{out,k \rightarrow j}(i) \quad (31)$$

and

$$\bar{P}_{d,k} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} P_{d,k}(i) = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \sum_{j \in R_k} P_{d,k \rightarrow j}(i), \quad (32)$$

respectively. The average harvested power of the $k$-th EH transmitter is given by

$$\bar{P}_{in,k} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} P_{in,k}(i). \quad (33)$$

Let $\bar{P}_{lim,k}$ denote the upper limit on the average transmit power of the $k$-th EH transmitter. Then, $\bar{P}_{out,k} \leq \bar{P}_{lim,k}$ has to be satisfied.

Similar to our model of the broadcast EH network in Section III, and without loss of generality, we assume that in each time slot the $k$-th EH transmitter extracts power from its battery in the following predefined sequential manner. For each time slot, the $k$-th EH transmitter first extracts the power for transmission of the codeword intended for the first receiving node in $R_k$, then, from the leftover power in the battery, the $k$-th EH transmitter extracts the power for transmission of the codeword intended for the second receiving node in $R_k$, and so on until, from the leftover
power in its battery, it extracts power for transmission of the codeword intended for the $|R_k|$-th receiving node in $R_k$. Then, $P_{out,k\rightarrow j}(i)$ is given by

$$P_{out,k\rightarrow j}(i) = \begin{cases} P_{d,k\rightarrow j}(i), & \text{if } B_k(i - 1) \geq \sum_{l \in R_k, l \leq j} P_{d,k\rightarrow l} \\ B_k(i - 1) - \sum_{l \in R_k, l < j} P_{out,k\rightarrow l}, & \text{otherwise}, \end{cases}$$

(34)

where $B_k(i)$ is the amount of power stored in the battery of the $k$-th EH transmitter at the $i$-th time slot and is given by

$$B_k(i) = B_k(i - 1) + P_{in,k}(i) - \sum_{j \in R_k} P_{out,k\rightarrow j}(i).$$

(35)

In a communication network comprised of multiple nodes, codewords may arrive at the intended receiver with a certain delay from the moment of transmission if multiple hops are involved. For example, a codeword originating from transmitter $k$ in time slot $l$ may pass through several hops before arriving at the intended receiver in time slot $i$, where $i \geq l$. In order to model this delay, in the following, for the $i$-th time slot, we develop a generalized utility function, $U(i)$, which may be a function of the transmit powers in time slots prior to $i$.

Let $U(i)$ denote the utility function in the $i$-th time slot. In order to generalize the utility function, $U(i)$, we introduce a delay $\Delta_{k\rightarrow j}$ assigned to the pair of the $k$-th EH transmitting node and the $j$-th receiving node. The delay $\Delta_{k\rightarrow j}$ is a constant integer number satisfying $0 \leq \Delta_k \leq N$, $\forall i$, i.e., the delay between nodes $k$ and $j$ does not change with time$^9$. Now, we define the utility function for the general network, $U(i)$, to be a function of $P_{out,k\rightarrow j}(i - \Delta_{k\rightarrow j})$, $\forall k, j$. Using $R_k$, defined previously, we construct the following vector of powers

$$P_{out,k}(i) = [P_{out,k\rightarrow 1}(i - \Delta_{k\rightarrow 1}), P_{out,k\rightarrow 2}(i - \Delta_{k\rightarrow 2}), \ldots, P_{out,k\rightarrow |R_k|}(i - \Delta_{k\rightarrow |R_k|})].$$

(36)

Using vectors $P_{out,k}(i)$, for $k = 1, \ldots, M$, we express the dependence of $U(i)$ on $P_{out,k\rightarrow j}(i - \Delta_{k\rightarrow j})$, $\forall k, j$, as$^{10}$

$$U(i) = U(P_{out,1}(i), P_{out,2}(i), \ldots, P_{out,M}(i)).$$

(37)

$^9$We note that a similar mathematical framework could be developed for time-varying delays $\Delta_{k\rightarrow j}$. However, this would make the presentation much more involved. Hence, for simplicity, we assume $\Delta_{k\rightarrow j}$ to be constant $\forall i$.

$^{10}$For simplicity of presentation, we have assumed that $U(i)$ depends only on the transmit power of node $k$ to node $j$ in one time instant, $\forall k, j$. However, the proposed framework can also be extend to the case when $U(i)$ depends on the transmit powers of node $k$ to node $j$ in multiple time slots, as would be the case if, for example, an automatic repeat request (ARQ) protocol was employed. To this end, defining a corresponding equivalent non-EH network is required.
We note that \( U(i) \) as a function of any individual \( P_{\text{out},k\rightarrow j}(i-\Delta_{k\rightarrow j}) \) has the properties outlined for the point-to-point case in Definition 1. Adopting \( U(i) \) in (37), the average utility function \( \bar{U} \) is given by (5).

For the general EH network, given a limit on the average transmit power of each EH transmitter, \( \bar{P}_{\text{lim},k} \), our objective is to determine the optimal desired powers \( P_{d,k\rightarrow j}(i-\Delta_{k\rightarrow j}) \), \( \forall k, j, i \), which produce the corresponding transmit powers \( P_{\text{out},k\rightarrow j}(i-\Delta_{k\rightarrow j}) \), such that \( \bar{P}_{\text{out},k} \leq \bar{P}_{\text{lim},k} \) hold \( \forall k \) and the average utility function \( \bar{U} \) is maximized. We define this rigorously in the following maximization problem for \( N \rightarrow \infty \)

\[
\begin{align*}
\text{Maximize :} & \quad \frac{1}{N} \sum_{i=1}^{N} U(i) \\
\text{Subject to :} & \quad C1 : P_{\text{out},k\rightarrow j}(i) = \begin{cases} 
P_{d,k\rightarrow j}(i), & \text{if } B_k(i-1) \geq \sum_{l \in \mathcal{R}_k, l \leq j} P_{d,k\rightarrow l} \\
B_k(i-1) - \sum_{l \in \mathcal{R}_k, l < j} P_{\text{out},k\rightarrow l}, & \text{otherwise},
\end{cases} \\
C2 : & \quad \bar{P}_{\text{out},k} \leq \bar{P}_{\text{lim},k} \\
C3 : & \quad \text{Optional constraints on } P_{d,k\rightarrow j}(i) \\
C4 : & \quad B_k(i) = B_k(i-1) + P_{\text{in},k}(i) - \sum_{j \in \mathcal{R}_k} P_{\text{out},k\rightarrow j}(i),
\end{align*}
\]

(38)

where \( P_{\text{in},k}(i) \) is known causally at the \( k \)-th EH transmitter only. On the other hand, since for the equivalent non-EH system \( P_{\text{out},k\rightarrow j}(i) = P_{d,k\rightarrow j}(i) \), \( \forall i, k, j \), the optimization problem for \( N \rightarrow \infty \) is given by

\[
\begin{align*}
\text{Maximize :} & \quad \frac{1}{N} \sum_{i=1}^{N} U(i) \\
\text{Subject to :} & \quad C1 : P_{\text{out},k\rightarrow j}(i) = P_{d,k\rightarrow j}(i) \\
C2 : & \quad P_{\text{out},k} \leq \bar{P}_{\text{lim,non-EH},k} \\
C3 : & \quad \text{Optional constraints on } P_{d,k\rightarrow j}(i),
\end{align*}
\]

(39)

where \( U(i), \forall i \), and C3 are the same as in (38).

Depending on the values of \( \bar{P}_{\text{in},k} \) and \( \bar{P}_{\text{lim},k} \), as explained in Lemma 1, the battery of the \( k \)-th EH transmitter can be absorbing or non-absorbing. Let us assume a general case in which a fraction of the \( M \) EH transmitters have an absorbing battery and the rest have a non-absorbing battery. Then, let us put the indices of the nodes with an absorbing battery into set \( \mathcal{A} \) and the indices of the nodes with a non-absorbing battery into set \( \mathcal{A} \). Using sets \( \mathcal{A} \) and \( \mathcal{A} \), we are now ready to provide the framework for solving (38).
Theorem 5: Maximization problem (38) can be transformed to maximization problem (39) by choosing $P_{\text{lim,non-EH,k}}$ in (39) as $P_{\text{lim,non-EH,k}} = P_{\text{lim,k}}$, $\forall k \in \mathcal{A}$ and $P_{\text{lim,non-EH,k}} = P_{\text{in,k}}$, $\forall k \in \bar{\mathcal{A}}$.

Proof: Please see Appendix F. 

B. Example for General EH Network

In the following, we present an example for online power allocation in a general EH network.

Example 4: In this example, we consider a multi-hop amplify-and-forward (AF) relay network comprised of one EH transmitter source, $M-2$ AF half-duplex EH relays, and a receiver. For simplicity, we assume that $M$ is an odd number. We numerate the nodes with numbers such that the source node is node 1, the destination node is node $M$, and the $M-2$ relays are numerated from 2 to $M-1$. The network operates in slow time-continuous fading and the AWGN at each receiver has unit variance. In the $i$-th time slot, the square of the fading gain of the channel from the $k$-th EH node to the $(k+1)$-th EH node is denoted by $\gamma_{k \rightarrow k+1}(i)$ where $k = 1, \ldots, M-1$. We assume that all nodes have full CSI. The transmission from the source via the relays to the destination is carried out in the following manner. In odd (even) time slots, the $k$-th node, where $k$ is an odd (even) number, transmits to the $(k+1)$-th node. When a relay transmits in time slot $i$, it transmits a scaled version of the codeword received in time slot $i-1$. Let $U(i)$ represent the information rate received at the destination at time slot $i$. Then, $U(i)$ is given by

\[
U(i) = \begin{cases} 
\log_2(1 + SNR_{\text{eq}}(i)) & \text{if } i \geq M-1 \text{ and } i \text{ is even} \\
0 & \text{otherwise},
\end{cases}
\]  

(40)

where the equivalent SNR at the destination, $SNR_{\text{eq}}(i)$, is given by [25, Eq. (17)]

\[
SNR_{\text{eq}}(i) = \left( \prod_{m=1}^{M-1} \left( 1 + \frac{1}{P_{\text{out},m \rightarrow m+1}(i-M+1+m)\gamma_{m \rightarrow m+1}(i-M+1+m)} \right) - 1 \right)^{-1}.
\]  

(41)

Hence, $\bar{U}$ is the average information rate. Let $\bar{P}_{\text{in,k}}$ be the average harvested power of the $k$-th EH node, and let $\bar{P}_{\text{lim,k}} \geq P_{\text{in,k}}$, for $k = 1, \ldots, (M-1)/2$ and $\bar{P}_{\text{lim,k}} < P_{\text{in,k}}$, for $k = (M-1)/2+1, \ldots, M-1$. Hence, the first half of EH transmitters have non-absorbing batteries and the second half have absorbing batteries, cf. Lemma 1. Furthermore, we assume that $P_{d,k \rightarrow k+1}(i)$ is either zero or assumes a constant value identical $\forall i$ for which it is not zero. Then, we use Theorem 5 to solve (38). Thereby, we transform the EH optimization problem (38) to the non-EH optimization problem (39) by setting $\bar{P}_{\text{lim,non-EH,k}} = \bar{P}_{\text{in,k}}$, for $k = 1, \ldots, (M-1)/2$ and
\[ P_{\text{lim, non-EH}, k} = P_{\text{lim}, k}, \text{ for } (M - 1)/2 + 1, \ldots, M - 1 \] and insert the constraint on \( P_{d,k\rightarrow k+1}(i) \).

The solution to this non-EH optimization problem is then straightforward and given by

\[ P_{d,k\rightarrow k+1}(i) = \begin{cases} 
2\bar{P}_{\text{in},k} & \text{for odd } i \text{ and odd } k \\
2\bar{P}_{\text{in},k} & \text{for even } i \text{ and even } k \\
0 & \text{for even } i \text{ and odd } k \\
0 & \text{for odd } i \text{ and even } k 
\end{cases}, \quad k = 1, \ldots, (M - 1)/2 
\] (42)

and

\[ P_{d,k\rightarrow k+1}(i) = \begin{cases} 
2\bar{P}_{\text{lim},k} & \text{for odd } i \text{ and odd } k \\
2\bar{P}_{\text{lim},k} & \text{for even } i \text{ and even } k \\
0 & \text{for even } i \text{ and odd } k \\
0 & \text{for odd } i \text{ and even } k 
\end{cases}, \quad k = (M - 1)/2 + 1, \ldots, M - 1. 
\] (43)

According to Theorem 5, the desired transmit powers in (42) and (43) are also the solution to the EH optimization problem (38). Hence, the transmit powers of the \( k \)-th EH node in the \( i \)-th time slot are given by \( P_{\text{out},k\rightarrow k+1}(i) = \min\{B_k(i - 1), P_{d,k\rightarrow k+1}(i)\} \), where \( P_{d,k\rightarrow k+1}(i) \) for \( k = 1, \ldots, (M - 1)/2 \) and \( k = (M - 1)/2 + 1, \ldots, M - 1 \) are given by (42) and (43), respectively.

Numerical results for this example are provided in the following section.

V. NUMERICAL EXAMPLES

In the following, we discuss the applicability of the developed framework, and, for the examples introduced in Sections II - IV, we provide numerical results for different numbers of time slots \( N \) and/or battery capacities \( B_{\text{max}} \). For all of the examples, we assume that the channels are Rayleigh fading with unit variance. Furthermore, we assume that the power harvested by the EH transmitters is an exponentially distributed random variable with mean \( \bar{P}_{\text{in}} \). All figures are obtained via Monte-Carlo simulation.

A. Applicability of the Developed Framework

In practical EH systems each EH node transmits in a finite number of time slots \( N \) and has a finite battery capacity \( B_{\text{max}} \). In this case, for the proposed solution to be applicable, \( N \) has to be sufficiently large. The numerical examples in the following subsections will illustrate what ”sufficiently large” means in this context. In terms of \( B_{\text{max}} \), the proposed solution is applicable in two cases. The first case is when the maximum capacity of the battery, \( B_{\text{max}} \), is much larger
than the average harvested power $\bar{P}_{\text{in}}$, i.e., when $B_{\text{max}} \gg \bar{P}_{\text{in}}$ holds. This is intuitive, since if $B_{\text{max}} \gg \bar{P}_{\text{in}}$ then the battery is practically never fully filled. Hence, the finiteness of $B_{\text{max}}$ has no effect. The second case is when the maximum capacity of the battery is much larger than the upper limit on the average transmit power, i.e., when $B_{\text{max}} \gg \bar{P}_{\text{lim}}$ holds. In this case, either $\bar{P}_{\text{in}} \leq \bar{P}_{\text{lim}}$ or $\bar{P}_{\text{in}} > \bar{P}_{\text{lim}}$ holds. If $\bar{P}_{\text{in}} > \bar{P}_{\text{lim}}$ then the battery is absorbing and the framework holds, whereas, if $\bar{P}_{\text{in}} \leq \bar{P}_{\text{lim}}$ then $\bar{P}_{\text{in}} \leq \bar{P}_{\text{lim}} \ll B_{\text{max}}$ and the battery is practically never fully filled, hence, the finiteness of $B_{\text{max}}$ has no effect. For example, in today’s mobile phones the maximum capacity of the battery is much larger than the average transmit power of a codeword. This means that $B_{\text{max}}$ is much larger than the upper limit on the average transmit power $\bar{P}_{\text{lim}}$. Hence, this corresponds to the second case and independent of how large the average harvested power is, the proposed solution is applicable in terms of $B_{\text{max}}$.

B. Numerical Results for Example 1

In Fig. 1, we plot the outage probability, $U$, of the point-to-point EH system, discussed in Example 1 in Section II, for $B_{\text{max}} = 200 \bar{P}_{\text{in}}$ and different $N$, and compare it to the outage probability of the non-EH system with an average transmit power $\bar{P}_{\text{in}}$ and infinite $N$. From Fig. 3, we observe that even for $N = 10^2$, the loss in outage probability performance of the proposed online solution is relatively small (less than 3 dB in the entire considered range of $\bar{P}_{\text{in}}$) compared to the performance of the non-EH system. This performance loss becomes almost negligible for $N = 10^4$. Hence, for the point-to-point EH system the proposed online solution, although sub-optimal for finite $N$, yields a high performance at low complexity even for small $N$.

C. Numerical Results for Example 2

In Fig. 2, we show the average information rate, $\bar{U}$, of the point-to-point EH system discussed in Example 2 in Section II, for different $N$ and $B_{\text{max}} = 200 \bar{P}_{\text{in}}$, and compare it to the average information rate of the equivalent non-EH system with average transmit power $\bar{P}_{\text{in}}$ and infinite $N$. An ideal transmitter with $\varepsilon = 1$ and $P_C = 0$ is assumed. Fig. 2 shows that even for finite $N$ and finite $B_{\text{max}}$, the loss in rate of the point-to-point EH system is small compared to the

Note that also the non-EH system achieves a lower average information rate for finite $N$ than for $N \to \infty$, i.e., even for the non-EH system $N \to \infty$ has to be assumed for the power allocation in (12) to be optimal.
non-EH system. For $N = 10^2$, as a performance upper bound, we also show the average rate obtained with the offline solution from [4]. We note that the offline solution, however, needs noncausal knowledge of the harvested powers and the fading gains in all $N = 10^2$ time slots. In contrast, our proposed solution, although non-optimal for finite $N$, is an online solution and requires only causal knowledge of the fading gain in the current time slot, the average harvested power, and the average fading gain. The optimal online solution for $N = 10^2$ would result in a curve which is between the curve obtained with the optimal offline solution from [4] and the curve obtained with our proposed solution. However, the optimal online solution for finite $N$ is based on dynamic programing and its exponentially increasing computational complexity becomes prohibitive for $N = 10^2$.

In Fig. 3, we plot the average information rate, $\bar{U}$, of a point-to-point EH system for $\varepsilon = 5$ and $P_C = -25$ dB. We plot $\bar{U}$ for fixed $N = 10^4$ and different $B_{\text{max}}$, and compare it to the average information rate of the non-EH system with an average transmit power $\bar{P}_{\text{in}}$ and infinite $N$. The figure shows that, for the adopted $N$, even with $B_{\text{max}} = 20\bar{P}_{\text{in}}$, the loss in rate of the point-to-point EH system is small compared to the non-EH system. We note that in the figure, all rates are zero for $P_{\text{in}} \leq -25$ dB as $P_C = -25$ dB.

**D. Numerical Results for Example 3**

In Fig. 4, we plot the sum information rate, $\bar{U}$, of the broadcast EH network with $M$ receivers, see Example 3 in Section III, for $B_{\text{max}} = 200\bar{P}_{\text{in}}$ and different $N$, and compare it to the sum rate of the equivalent broadcast non-EH network with an average transmit power $\bar{P}_{\text{in}}$ and infinite $N$. It can be seen that, for the considered broadcast scenario, the performance loss with the proposed power allocation solution compared to the broadcast non-EH network, is independent of the number of receivers $M$. In other words, for fixed $N$, the same loss in performance compared to the non-EH network is obtained for $M = 2$ and $M = 25$. This is expected since, for the broadcast EH network, there is only one battery from which we drain power. How this power is divided among the receivers is irrelevant given that enough power is drained from the battery for all receivers. Furthermore, even for $N = 10^2$, the performance of the proposed power allocation solution is close to the optimal non-EH performance. Moreover, in Fig. 4, as a performance upper bound we also show the average rate obtained with the offline solution from [11] for $M = 2$ and $N = 10^2$, which is, to the best of our knowledge, the only available offline solution for the
broadcast network with fading. We emphasize again that the offline solution needs noncausal knowledge of the harvested powers and the fading gains of both channels in all $N = 10^2$ time slots.

**E. Numerical Results for Example 4**

In Fig. 5, we plot the average BER, $\bar{U}$, of the multiple-access EH network with $M$ receivers, cf. Example 4 in Section III, for $B_{\text{max}} = 200\bar{P}_{\text{in}}$ and different $N$, and compare it to the average BER of the equivalent multiple-access non-EH network with average transmit power $\bar{P}_{\text{in}}$ and infinite $N$. The figure shows that the difference between the BER of the multiple-access EH network with $N = 10^2$ and the non-EH network is relatively small (less than 3 dB in the entire considered range of $\bar{P}_{\text{in}}$) and becomes almost negligible for $N = 10^4$. Fig. 5 also reveals that for fixed $\bar{P}_{\text{in}}$, $\bar{P}_{\text{in}} = 5$ dB for example, the loss in performance of the EH network compared to the non-EH network increases as the number of transmitting nodes, $M$, increases. However, for $N = 10^4$ this loss becomes negligible. This is expected from the proof of Theorem 4, cf. Appendix E, where it is shown that for finite $N$ the number of slots in which $P_{d,k}(i) \neq P_{\text{out},k}(i)$ for any $k \in \{1, \ldots, M\}$, would increases with $M$. However, although the number of slots in which $P_{d,k}(i) \neq P_{\text{out},k}(i)$ for any $k \in \{1, \ldots, M\}$, increases with $M$, it becomes negligible for $N \rightarrow \infty$.

**F. Numerical Results for Example 5**

In Fig. 6, we plot the average information rate, $\bar{U}$, of the multihop relay EH network, considered in Example 5 in Section IV, for $B_{\text{max}} = 200\bar{P}_{\text{in}}$, different $N$, and different numbers of relays, and compare it to the average information rate of the equivalent relay non-EH network with infinite $N$. For the absorbing nodes, we assume that $P_{\text{lim},k} = \bar{P}_{\text{in}}/2$, $\forall k = (M - 1)/2 + 1, \ldots, M - 1$. Furthermore, we assume that the distance between source and destination is fixed, and that the relays are equidistantly spaced on the line between the source and destination. Hence, if we assume a path loss model with a path loss exponent equal to two, and that the fading between source and destination has unit average power, then, for $M - 1$ relays between source and destination, the fading between neighboring nodes has average power $M^2$. Note that each relay has an average harvested power of $\bar{P}_{\text{in}}$. The figure shows that, for fixed $N$, as the number of relays increases, the performance loss of the proposed power allocation solution also
increases compared to the performance of the non-EH network. However, as \( N \) increases this performance loss becomes negligible as highlighted in Fig. 6 for \( N = 10^4 \) and 15 relays. Hence, even if the utility function \( U(i) \) has a relatively complicated form, as is the case in this example, the performance of the proposed online solution for the general EH network is almost identical to the performance of the equivalent non-EH network even for moderate \( N \).

VI. Conclusion

We have shown that the maximum average performance of an EH network, utilizing optimal online power allocation, converges to the maximum average performance of an equivalent non-EH network with appropriately chosen average transmit power when the number of transmit time slots, \( N \), and the battery capacities at each EH node, \( B_{\text{max}} \), satisfy \( N \to \infty \) and \( B_{\text{max}} \to \infty \). We have derived the asymptotically optimal online power allocation which for a general EH network optimizes a general utility function for \( N \to \infty \) and \( B_{\text{max}} \to \infty \). The considered family of utility functions is general enough to include the most important performance measures in communication theory such as the ergodic information rate, outage probability, average bit error probability, and average signal-to-noise ratio. The optimal online power allocation solution is obtained by solving the power allocation problem of an equivalent non-EH network with nodes having infinite energy available for the transmission of their codewords. Interestingly, the optimal solution only requires knowledge of the average harvested energy but not of the amount of harvested energy in past, present, or future time slots. Although asymptotic in nature, the proposed solution is applicable to EH systems transmitting in a large but finite number of time slots and having nodes with battery capacities much larger than the average harvested power and/or the maximum average transmit power.

APPENDIX

A. Proof of Lemma 1

Taking the expectation of both sides of (3), we obtain

\[
E\{B(i)\} = E\{B(i - 1)\} + \bar{P}_{\text{in}} - \bar{P}_{\text{out}}. \tag{44}
\]

From (44) we observe that \( E\{B(i)\} > E\{B(i - 1)\} \), i.e., the battery is absorbing, if and only if \( \bar{P}_{\text{in}} > \bar{P}_{\text{out}} \). Since \( \bar{P}_{\text{lim}} \geq \bar{P}_{\text{out}} \) holds always, it follows that if \( \bar{P}_{\text{lim}} < \bar{P}_{\text{in}} \), then \( \bar{P}_{\text{out}} < \bar{P}_{\text{in}} \) is satisfied and therefore the battery is absorbing. The battery is non-absorbing if \( \bar{P}_{\text{in}} = \bar{P}_{\text{out}} \). The
non-absorbing condition, $\bar{P}_{\text{in}} = \bar{P}_{\text{out}}$, is satisfied if and only if $\bar{P}_{\text{lim}} \geq \bar{P}_{\text{in}}$. To prove this, note
that if $P_d \geq P_{\text{in}}$, then $\bar{P}_{\text{in}} = \bar{P}_{\text{out}}$ holds. It is easy to see that this claim is true from the law of
conservation of flow for $\bar{P}_d > \bar{P}_{\text{in}}$, i.e., although on average we desire to extract more power
than what is being harvested, the battery, due to the law of conservation of flow, can only supply
an average power of $\bar{P}_{\text{in}}$. On the other hand, we prove in Appendix C that for $\bar{P}_d = \bar{P}_{\text{in}}$, we
obtain $\bar{P}_{\text{out}} = \bar{P}_{\text{in}}$, i.e., the battery is non-absorbing. Now, given our assumptions, if $\bar{P}_{\text{lim}} \geq \bar{P}_{\text{in}}$ it follows that $\bar{P}_d \geq \bar{P}_{\text{in}}$ is achievable, leading to $\bar{P}_{\text{in}} = \bar{P}_{\text{out}}$, i.e., to a non-absorbing battery.

B. Proof of Theorem 1

Since the battery is absorbing, after some negligible transient period, the battery is able to
satisfy the desired powers $P_d(i)$. In the following, we prove this rigorously.

Taking the expectation of both sides in (3), we obtain (44). Since in (44) $\bar{P}_{\text{in}} > \bar{P}_{\text{out}}$ (due to
the absorbing battery), the following must hold

$$E\{B(i)\} > E\{B(i-1)\}$$

$$\lim_{i \to \infty} E\{B(i)\} = \infty.$$  \hspace{1cm} (45)

(46)

Since (46) holds, and since $P_d(i), \forall i$, is finite, there must be some time slot, denoted by $j$, after
which for $i > j$ the probability of the event $P_{\text{out}}(i) = \min\{B(i-1), P_d(i)\} = B(i-1)$ is zero,
and for $i > j$, $P_{\text{out}}(i) = \min\{B(i-1), P_d(i)\} = P_d(i)$ holds with probability one. Furthermore,
$j$ must satisfy

$$\lim_{N \to \infty} \frac{j}{N} = 0.$$  \hspace{1cm} (47)

We prove this by contradiction. Assume $P_{\text{out}}(j) = \min\{B(j-1), P_d(j)\} = B(j-1)$ holds
with probability larger than zero and (47) does not hold, i.e., $j \to \infty$ holds instead. Then, since
$B(j-1) < P_d(j)$ and since $j \to \infty$, we obtain

$$\lim_{j \to \infty} E\{B(j-1)\} < E\{P_d(j)\} < \infty$$

and thereby violate (46). Hence, (47) must hold.

Now, if (47) holds, then $\bar{P}_{\text{out}}$ can be represent as

$$\bar{P}_{\text{out}} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{j} \min\{B(i-1), P_d(i)\} + \lim_{N \to \infty} \frac{1}{N} \sum_{i=j+1}^{N} P_d(i).$$  \hspace{1cm} (48)
Now, let $C$ be defined as

$$C = \max_{1 \leq i \leq j} \{ \min\{B(i - 1), P_d(i)\} \}.$$  

Since $C$ is finite and since (47) holds, the first sum in (48) is upper bounded by

$$\lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{j} \min\{B(i - 1), P_d(i)\} \leq \lim_{N \to \infty} \frac{j}{N} \times C = 0. \quad (49)$$

Therefore, $\bar{P}_{\text{out}}$ in (48) can be written as

$$\bar{P}_{\text{out}} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} P_d(i), \quad (50)$$

i.e., only the powers after the $j$-th slot contribute to the average transmit power, the other powers are negligible. Furthermore, since $P_d(i), \forall i,$ is finite, we can add the following zero sum to (50),

$$\lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{j} P_d(i) = 0,$$

and obtain

$$\bar{P}_{\text{out}} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=j+1}^{N} P_d(i). \quad (51)$$

This means that practically all codewords are transmitted with power $P_d(i)$.

Similarly, since $U(i), \forall i,$ is finite, following the same approach as above, we obtain

$$\bar{U} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{j} U(i) + \lim_{N \to \infty} \frac{1}{N} \sum_{i=j+1}^{N} U(i)$$

$$= \lim_{N \to \infty} \frac{1}{N} \sum_{i=j+1}^{N} U(i), \quad (52)$$

i.e., only the codewords after the $j$-th slot contribute to the average utility function, the contribution of the other codewords is negligible. Therefore, we can write (6) as (7) with $\bar{P}_{\text{lim,non-EH}} = \bar{P}_{\text{lim}}$. This completes the proof.

**C. Proof of Theorem 2**

We first prove that if (8) holds, then

$$\bar{P}_{\text{out}} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \min\{B(i - 1), P_d(i)\}$$

$$= \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} P_d(i) = \bar{P}_{\text{in}}. \quad (53)$$

We prove this by contradiction. Assume that (8) holds, whereas (53) does not hold, and instead
\[
\bar{P}_{\text{out}} = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} \min\{B(i-1), P_d(i)\}
\]

\[
< \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} P_d(i) = \bar{P}_{\text{in}}
\]  \hspace{1cm} (54)

holds. Then, since \( P_{\text{out}} < P_{\text{in}} \) holds (i.e., the battery is absorbing), according to the proof of Theorem 1, \( \bar{P}_{\text{out}} \) is given by (51). However, since \( \bar{P}_{\text{out}} \) is given by (51), (54) cannot hold and (53) must hold instead. This concludes the proof of (53).

Now, the only way that (53) can hold is if the number of slots for which \( P_{\text{out}}(i) = B(i-1) \) holds, denoted by \( \Delta \), are negligible compared to the number of slots for which \( P_{\text{out}}(i) = P_d(i) \) holds, i.e., \( \Delta \) is such that \( \lim_{N \to \infty} \Delta/N = 0 \) holds. As a result, the codewords for which \( P_{\text{out}}(i) = B(i-1) \) occurs have negligible effect on the average utility function \( \bar{U} \) compared to the codewords for which \( P_{\text{out}}(i) = P_d(i) \) occurs. This can be proved as follows. Put all time slots \( i \) for which \( P_{\text{out}}(i) = B(i-1) \) occurs into set \( I \). Then, let \( C = \max_{i \in I} U(i) \). Since \( C \) is finite it follows that the contribution of the codewords with powers \( P_{\text{out}}(i) = B(i-1) \) to the average utility function is

\[
\lim_{N \to \infty} \frac{1}{N} \sum_{i \in I} U(i) \leq \lim_{N \to \infty} \frac{\Delta}{N} \times C = 0.
\]  \hspace{1cm} (55)

Consequently, we can write (6) as (7) with \( \bar{P}_{\text{lim,non-EH}} = \bar{P}_{\text{in}} \). This completes the proof.

D. Proof of Theorem 3

Following the same procedure as for the proofs in Appendices B and C, we can prove that for an absorbing and a non-absorbing battery with \( P_d \) adjusted to \( \bar{P}_d = \bar{P}_{\text{lim}} \) and to \( \bar{P}_d = \bar{P}_{\text{in}} \), respectively, the total transmit power over all \( M \) channels, \( P_{\text{out}}(i) \), satisfies \( P_{\text{out}}(i) = P_d(i) \) for practically all time slots when \( N \to \infty \), i.e., the fraction of time slots, denoted by \( \Delta \), for which \( P_{\text{out}}(i) = P_d(i) \) does not hold satisfies \( \lim_{N \to \infty} \Delta/N = 0 \). Now, for each time slot \( i \) for which \( P_{\text{out}}(i) = P_d(i) \) holds, \( P_{\text{out},k}(i) = P_{d,k}(i) \), \( \forall k \), also holds. Therefore, it follows that \( P_{\text{out},k}(i) = P_{d,k}(i) \), \( \forall k \), holds for practically all time slots when \( N \to \infty \). Consequently, we can write (19) as (20) with appropriately adjusted \( P_{\text{lim,non-EH}} \) as explained in Theorem 3.

E. Proof of Theorem 4

In this proof, set \( \mathcal{A} \) comprises the indices of the nodes with absorbing batteries and set \( \mathcal{A} \) comprises the indices of the nodes with non-absorbing batteries. We first prove that for the nodes
the number of time slots for which \( P_{\text{out},k}(i) = B_k(i-1) \) holds is negligible compared to the number of time slots with \( P_{\text{out},k}(i) = P_{d,k}(i) \). To this end, we use the proof in Appendix B, where it is shown that an individual absorbing node \( k \in A \), after some finite number of time slots \( j_k \), transmits with power \( P_{\text{out},k}(i) = P_{d,k}(i) \), \( \forall i > j_k \), where \( \lim_{N \to \infty} j_k/N = 0 \). Now, let \( j = \max_k\{j_k\} \). Then, \( \lim_{N \to \infty} j/N = 0 \) holds. Furthermore, after this \( j \)-th time slot, all of the nodes with absorbing batteries \( k \in A \) transmit with power \( P_{\text{out},k}(i) = P_{d,k}(i) \), \( \forall i > j \), i.e., these nodes transmit with power \( P_{\text{out},k}(i) = P_{d,k}(i) \) for practically all time slots. This completes the proof for the nodes with absorbing batteries. Now, we are only left to prove that for the nodes \( k \in \bar{A} \), the number of time slots in which \( P_{\text{out},k}(i) = B_k(i-1) \) holds is negligible compared to the number of time slots with \( P_{\text{out},k}(i) = P_{d,k}(i) \). To this end, for each node \( k \in \bar{A} \), let us create a set \( \mathcal{I}_k \) in which we put the time slots \( i \) for which \( P_{\text{out},k}(i) = B_k(i-1) \) holds. Furthermore, let us create the set \( \mathcal{I} \) in which we put the time slots \( i \) for which \( P_{\text{out},k}(i) = B_k(i-1) \) holds for at least one of the nodes \( k \in \bar{A} \), i.e., \( \mathcal{I} \) is the union of all \( \mathcal{I}_k \) for \( k \in \bar{A} \). Thereby, the cardinality of \( \mathcal{I} \) is upper bounded by the sum of the cardinalities of \( \mathcal{I}_k \), \( \forall k \in \bar{A} \), i.e., the cardinality of \( \mathcal{I} \), denoted by \( \Delta \), is upper bounded as

\[
\Delta = |\mathcal{I}| \leq \sum_{k \in \bar{A}} |\mathcal{I}_k|.
\]

According to Theorem 2, if \( \bar{P}_{d,k} = \bar{P}_{\text{in},k} \) holds for a non-absorbing node \( k \), then the number of time slots in which \( P_{\text{out},k}(i) = B_k(i-1) \) holds is negligible compared to the number of time slots with \( P_{\text{out},k}(i) = P_{d,k}(i) \). Hence, let us adopt \( \bar{P}_{d,k} = \bar{P}_{\text{in},k} \), \( \forall k \in A \). Now, since \( |\mathcal{I}_k| \) is the number of time slots in which \( P_{\text{out},k}(i) = B_k(i-1) \) holds for node \( k \in A \), according to Theorem 2, \( |\mathcal{I}_k| \) satisfies

\[
\lim_{N \to \infty} |\mathcal{I}_k|/N = 0 \quad \forall k \in \bar{A}.
\]

Now, combining (56) and (57) we find that the cardinality of \( \mathcal{I} \) satisfies

\[
\lim_{N \to \infty} \Delta/N \leq \sum_{k \in A} \lim_{N \to \infty} |\mathcal{I}_k|/N = 0.
\]

Therefore, the cumulative effect of \( P_{\text{out},k}(i) = B_k(i-1) \) on \( \bar{U} \) from the absorbing nodes and the non-absorbing nodes is negligible since \( \lim_{N \to \infty} (j + \Delta)/N = 0 \) holds. Therefore, we can write (24) as (25) with appropriately adjusted \( P_{\text{lim,non-EH}} \) as explained in Theorem 4. This completes the proof.
F. Proof of Theorem 5

The proof for the general EH network is identical to that of the multiple-access EH network given in Appendix E, however, the powers $P_{\text{out},k}(i)$ and $P_{\text{d},k}(i)$ now have different meanings and are given by (29) and (30), respectively. In particular, $P_{\text{out},k}(i)$ and $P_{\text{d},k}(i)$ now are the total transmit and the total desired powers in time slot $i$ of the $k$-th EH transmit node to all receiving nodes. Hence, by following the same procedure as for the proof in Appendix E, we can prove that, for the absorbing nodes $k \in A$ with $P_{\text{d},k}$ tuned to $P_{\text{d},k} = \bar{P}_{\text{lim},k}$ and the non-absorbing nodes $k \in \bar{A}$ with $P_{\text{d},k}$ tuned to $P_{\text{d},k} = \bar{P}_{\text{in},k}$, the number of slots in which $P_{\text{out},k}(i) = B_k(i-1)$ holds is negligible compared to the number of slots in which $P_{\text{out},k}(i) = P_{\text{d},k}(i)$ holds. As a result, the number of slots in which $P_{\text{out},k\rightarrow_j}(i) \neq P_{\text{d},k\rightarrow_j}(i)$ holds $\forall i, k, j$ is negligible compared to the number of time slots in which $P_{\text{out},k\rightarrow_j}(i) = P_{\text{d},k\rightarrow_j}(i)$ holds $\forall i, k, j$. Hence, Theorem 5 follows.

References


Fig. 1. Outage probability for the point-to-point EH and the equivalent non-EH systems for different $N$ and $B_{\text{max}} = 200P_{\text{in}}$. 
Fig. 2. Average rates for the point-to-point EH and the equivalent non-EH systems for different $N$ and $B_{\text{max}} = 200 \bar{P}_{\text{in}}$. 
Fig. 3. Average rates for the point-to-point EH system and the equivalent non-EH systems for different battery capacity $B_{\text{max}}$, and $N = 10^4$. A realistic transmitter model is adopted with power efficiency $\varepsilon = 20\%$ and constant circuit power consumption $P_C = -25$ dB.
Fig. 4. Average rate for the broadcast EH network and the equivalent broadcast non-EH network for different $N$ and $B_{\text{max}} = 200P_{\text{in}}$. 
Fig. 5. Average BER for the multiple-access EH network and the equivalent multiple-access non-EH network for different $N$ and $B_{\text{max}} = 200\bar{P}_{\text{in}}$. 
Fig. 6. Average rate for multihop relay EH network and equivalent multihop non-EH network for different $N$, $B_{\text{max}} = 200\bar{P}_{\text{in}}$, and different numbers of relays.