Modelling Angle Spread Autocorrelations and the Impact on Multi-user Diversity Gains

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Abstract—One way of modelling the wireless channel is in a statistical manner, based on a few parameters describing the characteristics of the environment. In most current wireless channel models, these key parameters are assumed independent between separate links, i.e. on the channels modelling the propagation between one base station and several mobile stations, or one mobile station and several base stations. In practice, dependencies between these wireless channels is expected and as a consequence, system performance evaluations based on models with independent links may be inaccurate. Herein, we consider simulations of a system that depend on the spatial nature of the channel. In particular, we study a system with multi-user scheduling using a single carrier. We investigate the impact of angle spread correlations on multi-user diversity gains using opportunistic scheduling. To facilitate this, a novel method of modelling the angle spread correlations for multi-user system simulations is developed. It is shown that in systems with multiple user scheduling, modelling the angle spread autocorrelation is necessary to obtain reliable system performance results, especially as the number of simultaneous scheduled users increases.

I. INTRODUCTION

Future wireless communication systems will utilize the spatial properties of the wireless channel to improve the spectral efficiency and thus increase system capacity. This is realized by deploying multiple antennas at both the transmitter and receiver. Utilizing more, and different, dimensions of the channel places new demands on the underlying channel models used for simulation and validation. The nature of the wireless channel is highly unpredictable, thus it is useful to model the channel in a statistical fashion. Channel models like the SCM model within 3GPP, [1], or the WINNER-I model in the WINNER project, [2], are statistical models that are partly based on some bulk parameters, called large scale parameters (LS), describing the characteristics of the channel. Such parameters may include shadow fading (SF), angle spread (AS) and delay spread. Much research has been devoted to find appropriate distributions and statistics for modelling the LS parameters for simulations, through analyzing measurement data. The LS parameters are assumed dependent on the local environment around the base station (BS) and mobile station (MS) as well as the intermediate environment between them. Therefore, a small change in the environment should give a small change in the LS parameters used to describe the channel. One of the first papers studying the variation of one of the LS parameters for a moving user is [3], where the spatial autocorrelation of shadow fading was found to be well modelled by an exponential decay as a function of the distance between two locations. A similar study was reported in [4], where the exponential decay was shown to model the autocorrelation of the angle spread as well. These results have also been observed in [5]. Based on the exponential autocorrelation observation [6], among some, suggests that the LS parameters for a moving user could be modelled as an autoregressive (AR) process. However, this leads to inconsistencies such as a moving mobile not experiencing the same shadow fading or angle spread at the same location at two different time instants, which contradicts the assumptions made for the LS parameters [1], [2]. Further, using an AR approach to model the variation of the LS parameters in a multi-user scenario results in uncorrelated large scale processes for each user even if they are closely located. In [7] the effect of spatially correlated shadowing was studied by investigating the call dropping ratio as a function of the decorrelation distance. It was shown that the percentage of dropped calls was highly dependent on the decorrelation distance of the shadow fading. In this paper a similar study is made where we consider the effect of correlated angle spread and the impact on transmit power requirements in a multi-user scheduling scenario. To facilitate this, a novel method of modelling the angle spread correlations for multi-user simulations, which does not suffer from the inconsistencies mentioned earlier, is proposed. The technique is well suited for evaluating system performance of multi-user wireless systems. The exponential shape of the autocorrelation function presented in [3], [4] and [5] is assumed valid, and based on this synthetic two dimensional (2D) maps of angle spread patterns are generated. In the simulations the users draw their angle spread values from the precomputed map, in similarity to reality where the users experience angle spreads determined by their physical location. By using previously reported opportunistic multi-user selection schemes [8], it is shown that the increase in multi-user diversity gain with increasing number of users, reported in for example [9], through various opportunistic schemes are bounded by physical diversity properties of the underlying environment. This is done by studying the transmit power necessary to provide a reasonable signal power level at the receiver. It is further shown that the need for modelling the
angle spread correlation increases with increasing number of users.

This paper is structured as follows. In Section II the mathematical description used for the angle spread and the autocorrelation is given, and Section III explains the model used for generating the syntectic data. The simulations are explained in Section IV, and results are given in Section V, followed by the conclusions in Section VI.

II. MATHEMATICAL DESCRIPTION

This section describes the definitions of the angle spread and its autocorrelation function used within this paper. These definitions are the same as those used in [8], [10].

A. Angle Spread

The angle spread (AS) is a measure of how the multipath components (MCs) arrive (depart) relative to the mean angle of arrival (AoA) or angle of departure (AoD) respectively. The spread is commonly measured in degrees (or radians) and describes the width of the sector from which most of the received signal power arrives. Assuming N multipath components, the angular spread is defined as [10]

$$\sigma_{AS} = \sqrt{\frac{\sum_{i=1}^{N} P_A(\phi_i)(\phi_i - \bar{\phi})^2}{\sum_{k=1}^{N} P_A(\phi_i)}}$$

(1)

where $\phi_i$ is the i:th MC's angle of arrival (departure), $P_A(\phi_i)$ its power, and $\bar{\phi}$ is defined as

$$\bar{\phi} = \frac{\sum_{i=1}^{N} P_A(\phi_i)\phi_i}{\sum_{k=1}^{N} P_A(\phi_i)}.$$  

(2)

From measurements, like [4] and [11] among some, it is found that the log-angle spread is well modelled by a normal distribution, with mean $\mu_{AS} = \mathbb{E}[-\log_{10}(\sigma_{AS})]$ and standard deviation $\varepsilon_{AS} = \text{std}(\log_{10}(\sigma_{AS}))$. This implies that a random variable $Z$, describing the local angle spread may be expressed as

$$Z = 10^{\varepsilon_{AS}X + \mu_{AS}},$$

(3)

where $X \in N(0,1)$.

B. Autocorrelation

The angle spread is assumed constant for a given mobile base station pair at a specific location. As the mobile moves, the surrounding and intermediate environment changes slowly, and hence the LS parameters describing the channel will change slowly. To evaluate at which rate the angle spread varies it is common to study the autocorrelation function

$$R(\Delta d) = \frac{\mathbb{E}[(w(d) - \mu)(w(d + \Delta d) - \mu)]}{\sigma^2},$$

(4)

where $w(d)$ is the angle spread at a given location or time $d$. The parameters $\mu$ and $\sigma^2$ are the mean and the variance of the angle spread. Several works have studied the variation of angle spread from measurements, for example [2], [4], [5], [11]. Therein the autocorrelation function is shown to be well modelled by an exponential decay as

$$R(\Delta d) = e^{-\frac{|\Delta d|}{d_{decorr}}},$$

(5)

where $\Delta d$ is the distance separation and $d_{decorr}$ is the decorrelation distance, that depends on the propagation environment. The decorrelation distance is commonly defined as the distance to which the correlation coefficient has dropped to $e^{-1}$.

III. MATHEMATICAL MODELLING

The generation of angle spread data with a given autocorrelation function is in the proposed scheme realized in two steps. Assume a 2D lattice representation of the area of interest. For the simulations herein, square areas with cartesian coordinates are used, but it may be generalized to arbitrary coordinate systems. At first a 2D "map" $\mathbb{Z} = \{z(x,y), \forall x,y = 1...N\}$ is randomized where $z(x,y) = N(0, \varepsilon_{AS})$. The given set of data are independent samples with white slow fading, characterized by unit autocorrelation for zero spatial displacement and null otherwise. To introduce the spatial autocorrelation described in (5) a two-dimensional filter $h(x,y)$ is applied. Assuming that $w(x,y)$ and $S_w(f_x,f_y)$ is the desired spatially correlated data and the corresponding power spectral density, the filter $h(x,y)$ may be defined as

$$h(x,y) = \hat{s}^{-1}\{\sqrt{\frac{S_w(f_x,f_y)}{\varepsilon_{AS}}}\},$$

(6)

where $\hat{s}\{\cdot\}$ is the fourier transform of $\{\cdot\}$. To show (6), let $w(x,y)$ be a signal $z(x,y)$ filtered through $h(x,y)$, and recall that the spectrum $W(f_x,f_y)$ of $w(x,y)$ can be expressed as

$$W(f_x,f_y) = H(f_x,f_y)Z(f_x,f_y),$$

(7)

where $Z(f_x,f_y)$ is the spectrum of $z(x,y)$ and $H(f_x,f_y)$ is the frequency response of $h(x,y)$.

Let $z(x,y)$, be the spatially uncorrelated data as given above, and $w(x,y)$ be the desired spatially correlated data. Assuming that the frequency response of the filter $h(x,y)$ is
known the power spectral density $S_w(f_x, f_y)$, of $w(x, y)$, can be expressed as

$$S_w(f_x, f_y) = S_z(f_x, f_y) |H(f_x, f_y)|^2.$$  

Further, since the signal $z(x, y)$ is white with unit autocorrelation at zero displacement and zero otherwise, its power spectral density is constant,

$$S_z(f_x, f_y) = \varepsilon_{AS}^2.$$  

Hence, (8) can be written as

$$S_w(f_x, f_y) = \varepsilon_{AS}^2 |H(f_x, f_y)|^2.$$  

Here $\varepsilon_{AS}^2$ is a controllable system design parameter describing the standard deviation of the angle spread. Moreover, since $w(x, y)$ is the desired signal, we actually know its power spectral density as the Fourier transform of its autocorrelation function defined by (5), which is also a design parameter. Thus, from (10) we see that the filter $h(x, y)$, may be expressed as given by (6).

Using this we may generate our two-dimensional map of correlated angle spread data as

$$\Xi(x, y) = 10^{w(x, y) + \mu_{AS}},$$

where $w(x, y) = h(x, y) * z(x, y)$ is the convolution of $h(x, y)$ and $z(x, y)$, and $\mu_{AS}$ is the mean value of the angle spread.

A. Validation of Channel Simulator

To validate the simulation model, four separate sets of angle spread data is generated using fixed decorrelation distances of $\{0, 50, 100, 200\}$ meters respectively. The autocorrelation functions for each of these sets of data are calculated and shown in Fig. 2. As can be seen the results correspond well to the input parameters used.

IV. SYSTEM SIMULATION

In this section we evaluate how the angle spread autocorrelation may affect system performance. To do this we study the transmit power minimization in conjunction with beamforming that guarantees a certain signal to noise interference ratio (SINR) at the receiver. For simplicity, we consider a single carrier downlink system consisting of one transmitter with $N$ antennas, scheduling simultaneous transmission for $K$ single antenna receivers.

A. Channel Model

The wireless channel is modelled as quasi-static, and the baseband equivalent received signal at user $i$ can be expressed as:

$$y_i = h_i^* \sum_{j=1}^{K} w_j x_j + n_i,$$  

where $h_i$ is the $(N \times 1)$ baseband equivalent channel from the base station, with $N$ antennas, to the single antenna user $i$, and $\{\cdot\}^*$ is the complex conjugate transpose. The transmitted signal $x_j$ is modelled as a scalar with unit energy and the beamforming vector $w_j \in \mathbb{C}^N$. The noise, $n_i$, is modelled as additive white Gaussian (AWGN), with variance $\sigma_n^2$.

From (12) it follows that the SINR at user $i$ can be expressed as:

$$SINR_i = \frac{|h_i^*|^2}{\sum_{j \neq i} |w_j^* h_j|^2 + \sigma_n^2}.$$  

From the operators point of view, it is of interest to minimize the radiation power, to minimize interference and save cost, while maintaining high SINR, to provide a reasonable quality of service. Thus, the following problem is considered

minimize $\|w_i\|^2$
subject to $SINR_i \geq p, \quad \forall i \in S$,

where $S$ is the set of all scheduled users, and $p$ is the target signal to interference noise ratio. The channels to each user are modelled as independent with a fixed covariance matrix, determined by the angle of departure (AoD) and its spread as

$$h_i = C_i(\phi_i, \sigma_{AS}, \mathbf{g}_i),$$

where $C_i(\phi_i, \sigma_{AS}, \mathbf{g}_i)$ is the Cholesky factor of the covariance matrix, for user $i$, $R_i(\phi_i, \sigma_{AS}, \mathbf{g}_i) = C_i C_i^H$, and $\mathbf{g}_i$ is a $(N \times 1)$ complex vector with i.i.d. elements distributed as $\mathcal{CN}(0, 1)$. The variables $\phi_i$ and $\sigma_{AS}$, are the direction to user $i$ and the corresponding angle spread for that link. The covariance matrix is calculated using the Gaussian Angle of Arrival, one cluster (GAAO), assumption [12]. This channel model is reasonable in a non-line-of-sight (NLOS) macro cellular scenario where the base station is elevated and the mobile, at ground level, is surrounded by several scatterers. Such a scenario for a single MS single BS is depicted in Fig. 3. As shown in [12], the element $[\cdot]_{e,k}$ of the covariance matrix $R(\phi, \sigma_{AS})$, under the Gaussian assumption, can be expressed as:

$$[R(\phi, \sigma_{AS})]_{e,k} = e^{-\frac{\pi^2 \varepsilon_e^2}{2} (e-k)^2} e^{i(e-k) \frac{\pi \Delta \sin(\phi)}{2}},$$

where $\varepsilon_e$ is the desired signal, we actually know its power

Fig. 2. Autocorrelation functions for the data generated using fixed decorrelation distances of $\{0, 50, 100, 200\}$ meters respectively.
where
\[ \tilde{\sigma} = \frac{\pi^2 \Delta}{90^\circ \lambda} \cos(\phi) \sigma_{AS}. \] (17)

The parameter \( \lambda \) is the wavelength, and \( \Delta \) is the distance between adjacent antenna elements. In the current simulation an antenna spacing of \( \Delta = 0.5\lambda \) is used at a frequency of \( f = 1760 \text{MHz} \).

### B. User Selection Methods

Three different user selection (US) schemes are studied. These are semi orthogonal, norm based and random user selection. All these methods are suboptimal as explained in [8], since optimal scheduling requires exhaustive search which is considered prohibitive. The semi orthogonal user selection method tries to select users which are as orthogonal as possible while maintaining as high channel gain as possible. This is done by selecting one user at a time, and each time try to maximize the channel projection onto the orthogonal subspace spanned by the channels of the users who are already selected. Ignoring the orthogonal criterion and only maximizing the channel gains leads to norm based user selection scheme. Finally the random user selection method picks users at random disregarding the channel conditions. For more information of these scheduling schemes, and user selection algorithms see [8].

### C. Simulation Description

To evaluate the effect of angle spread autocorrelations on the transmit power \( N_{m} \) LS parameter maps were studied. For each generated map \( N_{d} \) drops, each consisting of \( N_{c} \) channel realizations, were evaluated. Within each drop the users are uniformly distributed within a square area as shown in Fig. 4. This means that all the users has a fixed position, thus a fixed angle spread and angle from the base, and therefore a fixed covariance matrix during a drop. Thus, each channel realization is independent, but with a fixed covariance matrix, generated as given in (15). In other words, the channels to each user are independent, but the channel statistics are correlated.

For the simulations below, unless otherwise noted, the base station is equipped with \( N = 10 \) antenna elements, and located \( d = 320 \text{m} \) from the center of the square area of size \( 320 \times 320 \text{m} \), as depicted in Fig. 4. A mean angle spread of \( \mu_{AS} = 1.08 \) with standard deviation \( \varepsilon_{AS} = 0.25 \) is used.

This corresponds to an angle spread of about \( 12^\circ \), which is a reasonable value in outdoor macro cellular environment [11]. The simulation parameter used are tabulated in Table I. Further, we assume that full channel state information (CSI) is available at the base station, i.e. the BS knows the channel to each user perfectly in each realization.

### V. Simulation Results

To study the effect of angle spread autocorrelation on the different user selection schemes, consider the case when the base selects \( K_s = 4 \) out of \( K = 8 \) users for transmission. Fig. 5 shows the average beamforming (BF) power needed to achieve an SINR of 10dB at the receiver as a function of the decorrelation distance for the three different scheduling modes. As noted in [8], the semi orthogonal user selection method is the most efficient method of the three. However, all three methods require higher transmit power as the angle spread decorrelation distance increases, i.e. as the channels statistics to the separate users become more correlated. The norm based user selection scheme is the most affected of the three. Table II shows the increase in transmit power needed when the angle spread autocorrelation increases from 0 to 100m. This increase in transmit power depends of course on the average angle to, and between the users, i.e the distance and angle to the area where the users are located. Further, the increase in transmit power as a function of the decorrelation distance is affected by the number of users selected. Fig. 6 shows the average transmit power needed to guarantee the 10dB SINR criterion, for the cases when selecting \( K_s = \{1,2,4,8\} \) out of \( K = 8 \) users, using the semi orthogonal user selection method. To highlight the increase in power required as a function of decorrelation distance the results in Fig. 6 are normalized with respect to the transmit power needed for an angle spread decorrelation distance of 0m. These transmit power normalization coefficients are tabulated in Table III.
### Table III

**Average Transmit Power (in dB) needed to Guarantee 10dB SINR at the Receivers for When Selecting n/8 Users and for 0m Decorrelation Distance.**

<table>
<thead>
<tr>
<th>1/8 users</th>
<th>2/8 users</th>
<th>4/8 users</th>
<th>8/8 users</th>
</tr>
</thead>
<tbody>
<tr>
<td>−15.7dB</td>
<td>−11.3dB</td>
<td>3.0dB</td>
<td>33.3dB</td>
</tr>
</tbody>
</table>

Table. IV shows the increase in transmit power needed when the angle spread autocorrelation increases from 0 to 100m for the four cases when selecting $K_s = \{1, 2, 4, 8\}$ out of $K = 8$ users with the semi-orthogonal user selection method.

As observed in Fig. 6 and Table. IV, the increase in power as a function of the decorrelation distance when selecting $K_s = \{1, 2\}$ out of $K = 8$ users is negligible. However as the number of scheduled users increases the effect of the angle spread decorrelation distance on the average transmit power increases rapidly.

This indicates that the increase in multi-user diversity gain with increasing number of users, reported in for example [9], through various opportunistic schemes are bounded by physical diversity properties of the underlying environment. Further it shows the need for taking the spatial correlation of system parameters into consideration for realistic and reliable system evaluation.

It should be noted that the assumption of independent channels in (15) may be questionable in some scenarios. For example, in transmission to two closely-spaced users with distant scatterers affecting the channels, it may not be just the angular spread that is correlated, but the angles of arrival of the multipath components themselves. This will cause more significant effects in the required transmit power than are modelled herein. However, to analyze such effects, further assumptions, and a more detailed description of the wireless channel is needed, as in ray-tracing channel models.

### VI. Conclusions

Herein, a novel method of modelling the correlation among large scale channel parameters is presented. The technique is well suited for evaluating system performance of multi-user wireless systems. Using the proposed scheme, we investigate the impact of angle spread correlations on multi-user diversity gains in opportunistic scheduling. The results indicate that systems utilizing the spatial property of the channel to increase system efficiency by simultaneously scheduling multiple users, must be evaluated taking its channel parameter dependencies into account. As the number of users jointly scheduled increases, ignoring the influence of large scale parameter correlation can lead to over estimating system performance gains.
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