SAGE Based Suboptimal Receiver for Downlink MC-CDMA Systems

Nihat Kabaoğlu, Member, IEEE

Abstract—An iterative joint data detection and channel estimation algorithm for downlink multi-carrier code division multiple access (MC-CDMA) systems is proposed. The resulting algorithm is a space alternating generalized expectation-maximization (SAGE) algorithm which updates the data sequences serially and the channel parameters in parallel, leading to a receiver structure that also incorporates interchannel interference cancellation. Its performance is compared with various Minimum Mean Square Error (MMSE) estimators for a multipath frequency selective fading channel using simulations. It is illustrated that the proposed SAGE algorithm performs better than the MMSE estimators considered in this paper.

Index Terms—Suboptimal receiver, joint channel estimation and data detection, downlink, MC-CDMA, SAGE algorithm.

I. INTRODUCTION

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C-CDMA is preferred for the downlink of a system because of its high spectral efficiency and its low receiver complexity [1]. In a multicarrier communication, severe multiple-access interference (MAI) and inter-symbol-interference (ISI) arise due to multiuser multipath propagation when the channel delay exceeds the duration of a symbol. Thus, the capacity of a multicarrier system is limited. MAI can be reduced when orthogonal codes are used. Still, it is not possible to remove MAI entirely due to the effects of time delay and corruption on the orthogonality of users’ spreading codes. Nevertheless, these effects on MAI can be decreased by using channel estimation and data detection of all active users. This turns away the data detection work related to CDMA based system to the multi-user detection. Moshavi’s work presented in [2] has a far reaching influence on this orientation.

Iterative receivers including parallel interference canceller (PIC) detectors are adopted to improve the performance of MC-CDMA systems [3]-[5]. Since their performance is sensitive to how they are initialized, they require high quality data and channel estimation during the initial stage. This is the main motivation behind researchers to develop algorithms having high performance such as expectation-maximization (EM) and SAGE that provide a numerical solution to the maximum likelihood (ML) estimator [4], [5]. This is also the main reason for us to propose this type of iterative algorithm. Although an ML type algorithm gives an opportunity to jointly estimate data and channel coefficients, it is a computationally complex and slower algorithm if the interest is in both parameters. Hence, ML type joint estimation methods for CDMA based systems are usually exploited in the uplink communication [6]. Considering that new developments in capabilities of users’ equipments and their requirement on high data rate with low error probability, the usage of joint channel estimation and data detection algorithm for them is inevitable. For this reason, in this paper, we focus on joint channel estimation and data detection for downlink MC-CDMA systems. Next, we propose a SAGE based joint algorithm.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters; all vectors are column vectors. \((\cdot)^{-1}\), \((\cdot)^T\) and \((\cdot)^H\) denote the matrix inversion, transpose and conjugate transpose, respectively. \(|\cdot|\) denotes the absolute value. \(tr\{\cdot\}\) denotes the trace of a matrix. \(I_N\) denotes the \(N \times N\) identity matrix. \(\Re\{\cdot\}\) denotes the real part of its argument.

II. SIGNAL MODEL FOR DOWNLINK OF AN MC-CDMA SYSTEM

In this work, a synchronous downlink MC-CDMA system is considered since it is computationally efficient to add the spread signals of \(K\) users before the OFDM operation. The channel is a frequency selective multipath fading channel whose impulse response is expressed as \(h_k(t) = \sum_{s=1}^{S} g_{k,s} \delta(t - \tau_{k,s})\) for \(k\)th user. Here, \(S\) is the number of paths; \(g_{k,s}\) and \(\tau_{k,s}\) are complex fading coefficients and delay of the \(s\)th path, respectively. In the \(k\)th mobile unit, the received signal is sampled at an adequate-rate, and it is converted from serial-to-parallel (S/P). In what follows, cyclic prefix is removed, and discrete Fourier transform is applied to the resulting discrete time signal. The obtained signal at the output of the \(k\)th user’s matched filter in the \(m\)th symbol interval is

\[
y(m) = \sum_{n=1}^{K} b_n(m)\rho_{nk}Fh_k + \eta(m) \tag{1}
\]

where \(b_n(m)\) is \(m\)th symbol of \(n\)th user; \(\rho_{nk} \in \mathbb{C}^{1 \times P}\) represents the cross correlation between \(n\)th user’s spreading code and \(k\)th user’s spreading code, the length of which is \(P\); \(F \in \mathbb{C}^{P \times S}\) represents the Fourier transform matrix; \(h_k = [h_k(1), h_k(2), \ldots, h_k(S)]^T \in \mathbb{C}^{S \times 1}\) is \(k\)th user’s discrete channel impulse response vector which is normally distributed according to \(N(0, C_{h_k})\); \(\eta(m)\) is noise which represents the output of the matched filter when the zero-mean, i.i.d. complex Gaussian vector with variance \(\sigma^2/2\) per dimension is applied to the matched filter. (1) can be written in a more compact form as follows:

\[
y = Qh_k + \eta. \tag{2}
\]

where \(y = [y(1), y(2), \ldots, y(M)]^T\); \(Q = \sum_{n=1}^K Q_{nk}\); \(Q_{nk} = b_n \otimes (\rho_{nk}F)\); \(b_n = [b_n(1), b_n(2), \ldots, b_n(M)]^T\); \(\eta = [\eta(1), \eta(2), \ldots, \eta(M)] \in \mathbb{C}^{M \times 1}\) is the complex Gaussian noise vector with zero mean and covariance matrix \(C_\eta = \frac{\sigma^2}{2} I_M\).

III. SAGE BASED DATA DETECTION

When the observation \(y\) is given, joint Maximum Likelihood (ML) estimation of \(k\)th user’s data vector \(b_k\) and channel
vector $h_k$ can be written as 
\[
(b_k, h_k) = \arg \max_{b_k, h_k} \ell(y; b_k, h_k)
\]
where $\ell(y; b_k, h_k)$ is the likelihood function. Direct maximization of (3) has computational complexity for large values of $K$ and $M$, but it can be easily solved iteratively. The goal of this work is to obtain a receiver architecture that iterates between soft-data and channel estimation without complete channel state information, employing the signal model in (1). An appropriate approach for applying the SAGE algorithm for non-pilot data symbols is to decompose the received signal in (1) into the sum 
\[
y(m) = b_k(m)\rho_{kk}Fh_k + \eta(m) + \sum_{n=1, n \neq k}^{K} b_n(m)\rho_{nk}Fh_k.
\]
(4) can be written in a vector form as follows:
\[
y = r + \hat{r},
\]
where $r = Q_{kk}h_k + \eta$ and $\hat{r} = \sum_{n=1, n \neq k}^{K} Q_{nk}h_k$.

Since SAGE algorithm needs complete and incomplete data sets, they should be defined first. A natural choice for the complete and incomplete data sets for the signal model in (5) are $\{r, h_k\}$ and $y$, respectively. The vector to be estimated is $b_k$ in the set $b \triangleq \{b_1, b_2, \ldots, b_K\}$. The SAGE algorithm is required to find the parameter vector $b_k$ that maximizes the expected average of the log-likelihood function with respect to complete data set given the incomplete data set under the current estimate of the parameter $b$ except $b_k$. Hence, the SAGE algorithm seeks to find the MLE of the marginal likelihood by iteratively applying the following two steps.

A. Expectation Step:

The SAGE algorithm is based on the EM algorithm. The first step to execute the EM algorithm is to find expected average of log-likelihood function that can be expressed as 
\[
\mathcal{L}(b_k|b^{(q)}) = E_{h_k|y, b^{(q)}}[\ln p(r|b_k, b^{(q)}, h_k)],
\]
where $b^{(q)}_n$ is the estimation of $b_n$ at the $q$th iteration and $b_n$ denotes the symbol vectors of other users except user $k$. After discarding the terms independent from $b_k$, likelihood function can be approximately defined by
\[
\ln p(r|b_k, b^{(q)}, h_k) \approx \sum_{m=1}^{M} \left[ \ln p(b^*_k(m)|h_k)F^H\rho_{kk}^T r(m) \right] \\
- \frac{1}{2} |b_k(m)|^2 F^H\rho_{kk}^T Fh_k.
\]
(7) Inserting (7) in (6), we have
\[
\mathcal{L}(b_k|b^{(q)}) = \sum_{m=1}^{M} \left[ \ln \left\{ b^*_k(m) \left[ \xi^{(q)}(m) \right] \right\} \\
+ E_{h_k|y, b^{(q)}}[h_kF^H\rho_{kk}^T r(m)] \right] \\
- \frac{1}{2} |b_k(m)|^2 E_{h_k|y, b^{(q)}}[h_kF^H\rho_{kk}^T Fh_k].
\]
(8) where
\[
\xi^{(q)}(m) = b^{(q)}_k(m) \left\{ E_{h_k|y, b^{(q)}}[h_kF^H\rho_{kk}^T \rho_{kk}Fh_k] \\
- \sum_{n=1, n \neq k}^{K} b^{(q)}_n(m) E_{h_k|y, b^{(q)}}[h_kF^H\rho_{kk}^T \rho_{nk}Fh_k] \right\}.
\]
(9) can be considered as joint equalization and ICI cancellation in the time-domain once estimate of the channel is obtained. In order to completely solve (8), we need to compute the three conditional expected values $E_{h_k|y, b^{(q)}}[h_k^H F^H \rho_{kk}^T \rho_{kk} F h_k]$ and $E_{h_k|y, b^{(q)}}[h_k^H F^H \rho_{kk}^T \rho_{nk} F h_k]$ and $E_{h_k|y, b^{(q)}}[h_k^H F^H \rho_{kk}^T \rho_{nk} F h_k]$. The conditional mean of $h_k$ can be obtained from the posterior probability density function, and it is given as follows
\[
p(h_k|y, b^{(q)}) \propto p(y|h_k, b^{(q)})p(h_k).
\]
(10) Since $p(y|h_k, b^{(q)})$ in (10) can be obtained using the signal model in (2) and $p(h_k)$, which is the probability density function of $h_k$, is known as apriori by the receiver, $p(h_k|y, b^{(q)})$ can easily be obtained. After some algebra, it can be shown that
\[
p(h_k|y, b^{(q)}) \sim \mathcal{N}(\mu^{(q)}_{h_k}, \Sigma^{(q)}_{h_k}),
\]
(11) where
\[
\mu^{(q)}_{h_k} = \frac{1}{\sigma^2} \Sigma^{(q)}_{h_k} Q^{H} y,
\]
(12)
\[
\Sigma^{(q)}_{h_k} = \left( C_h^{-1} + \frac{1}{\sigma^2} Q^{H} Q \right)^{-1}.
\]
It is noticed that (12) is a maximum a posteriori estimate of the channel vector $h_k$. Now we can easily compute the first necessary expected value using (12) as
\[
E_{h_k|y, b^{(q)}}[h_k^H] = \mu^{(q)H}_{h_k}.
\]
(14) By defining $\Gamma_{kk} = F^H \rho_{kk}^T \rho_{nk} F$, the second and third conditional expected values can be easily obtained as follows:
\[
E_{h_k|y, b^{(q)}}[h_k^H \Gamma_{kk} h_k] = tr \left( \Gamma_{kk} \Sigma^{(q)}_{h_k} + \mu^{(q)H}_{h_k} \Gamma_{kk} \mu^{(q)}_{h_k} \right)
\]
and
\[
E_{h_k|y, b^{(q)}}[h_k^H \Gamma_{kk} h_k] = tr \left( \Gamma_{kn} \Sigma^{(q)}_{h_k} + \mu^{(q)H}_{h_k} \Gamma_{kn} \mu^{(q)}_{h_k} \right)
\]
When the calculated conditional expected values are inserted in (8), we obtain
\[
\mathcal{L}(b_k|b^{(q)}) = \sum_{m=1}^{M} \left[ \ln \left\{ b^*_k(m) \left[ \xi^{(q)}(m) + \mu^{(q)H}_{h_k} F^H \rho_{kk}^T r(m) \right] \right\} \\
- \frac{1}{2} |b_k(m)|^2 \left[ tr \left( \Gamma_{kk} \Sigma^{(q)}_{h_k} \right) + \mu^{(q)H}_{h_k} \Gamma_{kk} \mu^{(q)}_{h_k} \right] \right] .
\]
(17)
discrete, belonging to a signal constellation point, the obtained summation in the right hand side of (17). However, can be separately obtained by maximizing the corresponding other users’ symbol are kept

B. Maximization Step:

In the second step of the proposed SAGE algorithm, while other users’ symbol are kept fixed, the parameter \(b_k\) at the \((q+1)\)th iteration step is updated over non-pilot data symbols of \(b_k(m)\) for \(m = 1, 2, ..., M\) according to

\[
b_k^{(q+1)} = \arg \max_{b_k} \mathcal{L}(b_k | b_k^{(q)}).
\] (18)

Assuming that there is no coding, each component of \(b_k\) can be separately obtained by maximizing the corresponding summation in the right hand side of (17). However, \(b_k(m)\) is discrete, belonging to a signal constellation point, the obtained \(b_k^{(q+1)}\) value after maximization step must be quantized to the nearest constellation point in each iteration. Therefore, the update rule of the proposed SAGE algorithm takes the following term

\[
b_k^{(q+1)}(m) = \text{Quant} \left\{ \frac{\xi(q)(m) + \mu_{b_k}^{(q)} H F H \rho_{b_k}^H y(m)}{tr \left( \Gamma_{b_k} \Sigma_{b_k}^{(q)} \right) + \mu_{b_k}^{(q)} H \Gamma_{b_k} \mu_{b_k}^{(q)}} \right\},
\]

where \(\text{Quant}(.\)\) is a quantization process that quantizes its argument to its nearest data symbol constellation point.

IV. COMPUTER SIMULATION

We consider a power controlled cellular MC-CDMA system. The simulation parameter is chosen as the number of users \(K = 6\); the length of each of users’ data frame \(M = 128\); the number of paths \(S = 3\). The orthogonal Walsh sequences are selected as a spreading code and the processing gain \(P\) is equal to the number of subcarriers \(N_c\) \((P = N_c = 64)\). Channel characteristic is frequency selective fading channel. We assume that QPSK modulation is used.

In Figure 1, symbol-error rate (SER) performance as a function of \(SNR\) for MMSE separate data detection and channel estimation (MMSE-SDDCE) method, MMSE successive detection (MMSE-SucDDCE) method, MMSE with PIC (MMSE-PIC) method and the proposed SAGE based joint channel estimation and data detection algorithm (J-SAGE) for the considered downlink MC-CDMA system. From this figure, it can be seen that the proposed SAGE algorithm outperforms these MMSE methods.

To investigate the convergence rate of the proposed SAGE algorithm, the SER performance are investigated as a function of the number of iterations for the different values of \(SNR\) in Figure 2. It is concluded from these curves that the SER performance of our proposed SAGE algorithm converges within 2 – 3 iterations, depending on initial values and \(SNR\).

V. CONCLUSIONS

In this paper, an efficient SAGE based suboptimal receiver is derived in closed form for the downlink MC-CDMA systems operating in the presence of frequency selective multipath fading channels. It is capable of data detection which incorporate channel estimation as well as partial interference cancellation. Simulation results show that the proposed algorithm has an excellent SER performance and significantly outperforms all MMSE methods in terms of SER.

REFERENCES