Optimal Random Multiaccess in Energy Harvesting Wireless Sensor Networks

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Abstract—We consider a Wireless Sensor Network where each sensor device is able to harvest energy from the environment, and randomly accesses the channel to transmit packets of random importance to a fusion center. If a collision occurs, the transmission fails and all packets involved are discarded. We design distributed transmission schemes where each sensor node, based on its own energy level and the importance of its own data packet, decides whether to transmit the packet or remain idle, so as to maximize the network utility, defined as the average long-term aggregate network importance of the data packets successfully reported to the fusion center. Due to the generally non-convex structure of the optimization problem, we resort to approximate solutions.

In particular, we use a mathematical artifice based on a game theoretic formulation of the multiaccess problem, where each sensor node is a player that attempts to selfishly maximize the network utility. We characterize the Symmetric Nash Equilibrium (SNE) of this game, where all the sensor nodes employ the same policy. We prove the existence and uniqueness of the SNE, and we show that it is a local maximum of the original optimization problem. Moreover, we derive an algorithm to compute it.

I. INTRODUCTION

In recent years, energy harvesting (EH) has become a vast research topic, spanning different disciplines related to electronics, energy storage, measurement and modeling of ambient energy sources, and energy management [1], [2]. Due to its demonstrated advantages of long-term, autonomous operation, EH technology is increasingly being considered in the design of Wireless Sensor Networks (WSNs), where battery replacement is difficult or cost-prohibitive, e.g., see [3]. However, the random nature of the energy harvested from the environment calls for proper energy management policies, which best manage the energy available in the rechargeable battery so as to maximize a given network utility.

In this paper, we consider an EH-WSN where multiple EH Sensors (EHSs) randomly access a wireless channel to transmit data packets of random importance to a common fusion center. Assuming that data transmission incurs an energy cost and simultaneous transmission from multiple EHSs causes collision and packet loss, we study the problem of designing optimal random access policies, which manage the energy available in the battery so as to maximize the network utility, defined as the average long-term aggregate network importance of the data packets successfully reported to the fusion center. We resort to a mathematical artifice based on a game theoretic formulation of the multiaccess problem, where each EHS selfishly maximizes the network utility. We characterize the Symmetric Nash Equilibrium, and prove its existence and uniqueness. Moreover, we show that it is a local optimum of the original optimization problem, and we derive an algorithm to compute it. As a first step, we consider the case in which both the EH and the data importance processes are i.i.d. over space and time. An example of such setting is a WSN deployed in an industrial environment, where the sensor nodes monitor the operation of different machines and are equipped with piezo-electric harvesters to sustain their operation. We remark that other settings may exhibit different correlation structures, e.g., solar powered sensors measuring the same phenomenon, where both EH and data importance processes may exhibit high temporal and spatial correlation. The extension of the proposed framework to such settings is left for future work.

The model considered in the present paper is a generalization of [4], which addresses the design of optimal energy management policies for a single EHS. Herein, we extend [4] and model the interaction among multiple EHSs, which randomly access the channel. The problem of maximizing the average long-term importance of the reported data in a replenishable sensor is formulated in [5], for a continuous-time model with Poisson EH and data processes. [6] investigates the relaying of packets of different priorities in a network of energy-limited sensors, but does not account for EH capability.

Despite the intense research in the design of optimal energy management policies for a single EH device, e.g., see [7]–[10], the problem of analyzing and modeling the interaction among multiple EH nodes in a network has not received much attention so far. The design of Medium Access Control (MAC) protocols for EH-WSNs is addressed in [11]–[13], focusing on TDMA, (dynamic) framed Aloha, CSMA and polling techniques. Efficient energy management policies that stabilize the data queues, as well as efficient MAC policies, are developed in [14]. A MAC protocol using a probabilistic polling mechanism that adapts to changing energy harvesting rates or node densities is designed in [15]. An on demand MAC protocol which is able to support individual duty cycles for nodes with different energy profiles so as to achieve energy neutral operation is presented in [16]. The problem of dynamic sensor node activation so as to optimize the sensing and event detection performance of the network is addressed in [17].

The rest of the paper is organized as follows. In Section II, we present the system model. Section III defines the control policies and states the optimization problem, which is further developed in Section IV. In Section V, we present some numerical results. Finally, Section VI concludes the paper.

This work has been supported in part by the European Commission through the FP7 EU projects IoT-A (G.A. no. 257521) and SWAP (G.A. no. 251557).
network management issues, e.g. are loss tolerant, since sensing data exhibit redundancy and delay requirement. Note that typical WSN applications by FC to the EHSs. This assumption implicitly models a stringency in the energy harvested in slot $k$, and is modeled as a Bernoulli i.i.d. process taking values in $\{0, 1\}$, i.e., in each time-slot, either one energy quantum is harvested, or no energy is harvested at all. The probability of harvesting one energy quantum is denoted by $\bar{b} = \Pr(B_{u,k} = 1)$. We assume that the components of $B_k = (B_{1,k}, B_{2,k}, \ldots, B_{U,k})$ are i.i.d. across the EHSs. Moreover the energy harvested in time-slot $k$ can be used only in a later time-slot, so that, if the battery is depleted, i.e., $E_{u,k} = 0$, then $Q_{u,k} = 0$.

The state of the system at time $k$ is given by $(E_u, V_k)$, where $E_k = (E_{1,k}, E_{2,k}, \ldots, E_{U,k}) \in \mathcal{E}^U$ is the joint state of the energy levels in all EHSs.

II. SYSTEM MODEL

We consider a network of $U$ wireless EH Sensors (EHSs), which communicate concurrently via a shared wireless link with a Fusion Center (FC), as depicted in Fig. 1. Each EHS collects ambient energy, which is stored in a rechargeable battery and powers the sensing apparatus and the RF circuitry. A processing unit, e.g., a micro-controller, manages the energy consumption of the EHS. Time is slotted, where slot $k$ is the time interval $[kT, kT+T)$, $k \in \mathbb{Z}^+$ and $T$ is the time-slot duration. At each time instant $k$, each EHS (say, $u$) has a new data packet to send to FC. Each data packet has importance $V_{u,k} > 0$. We model $V_{u,k}$ as a continuous random variable with probability density function (pdf) $f_V(v)$, $v \geq 0$ with support $[0, \infty)$, and assume that the components of $\{V_k\}$, where $V_k = (V_{1,k}, V_{2,k}, \ldots, V_{U,k}) \in \mathbb{R}^U$ is the importance vector, are i.i.d. over time and across the EHSs.

We assume a collision model, i.e., if EHS $u$ transmits in time-slot $k$, the packet is successfully delivered to FC if and only if all the other EHSs remain idle. The data packet is discarded if a collision occurs or the EHS decides to remain idle, and no feedback on the transmission outcome is provided by FC to the EHSs. This assumption implicitly models a stringent delay requirement. Note that typical WSN applications are loss tolerant, since sensing data exhibit redundancy and correlation in time. Moreover, we do not deal explicitly with network management issues, e.g., node synchronization and self organization, nodes joining and leaving the network, fault tolerance, which can be solved using standard techniques.

The battery of each EHS is modeled by a buffer. As in previous works [4], [7], [10], we assume that each position in the buffer can hold one energy quantum and that the transmission of one data packet requires the expenditure of one energy quantum. The maximum number of quanta that can be stored in each EHS, i.e., the battery capacity, is $\epsilon_{\text{max}}$ and the set of possible energy levels is denoted by $\mathcal{E} = \{0, 1, \ldots, \epsilon_{\text{max}}\}$. The amount of energy stored in the battery of EHS $u$ at time $k$ is denoted by $E_{u,k}$. The evolution of $E_{u,k}$ over time $k \in \mathbb{Z}^+$ is governed by

$$E_{u,k+1} = \min \{E_{u,k} - Q_{u,k} + B_{u,k}, \epsilon_{\text{max}}\},$$

where $\{B_{u,k}\}$ is the energy arrival process and $\{Q_{u,k}\}$ is the action process at EHS $u$. $Q_{u,k} = 1$ if the current data packet is transmitted by EHS $u$, which results in the expenditure of one energy quantum, and $Q_{u,k} = 0$ otherwise. $B_{u,k}$ models the randomness in the energy harvested in slot $k$, and is modeled as a Bernoulli i.i.d. process taking values in $\{0, 1\}$, i.e., in each time-slot, either one energy quantum is harvested, or no energy is harvested at all. The probability of harvesting one energy quantum is denoted by $\bar{b} = \Pr(B_{u,k} = 1)$. We assume that the components of $B_k = (B_{1,k}, B_{2,k}, \ldots, B_{U,k})$ are i.i.d. across the EHSs. Moreover the energy harvested in time-slot $k$ can be used only in a later time-slot, so that, if the battery is depleted, i.e., $E_{u,k} = 0$, then $Q_{u,k} = 0$.

The state of the system at time $k$ is given by $(E_u, V_k)$, where $E_k = (E_{1,k}, E_{2,k}, \ldots, E_{U,k}) \in \mathcal{E}^U$ is the joint state of the energy levels in all EHSs.

III. POLICY DEFINITION AND OPTIMIZATION PROBLEM

Each EHS is assumed to have only local knowledge about the state of the system. Namely, EHS $u$, at time $k$, only knows its own energy level and data importance $(E_{u,k}, V_{u,k})$, but does not know the energy level and data importance of the other EHSs in the network. Therefore, the decision of EHS $u$ on whether to transmit or remain idle is based solely on $(E_{u,k}, V_{u,k})$, and is independent of $(E_{i,k}, V_{i,k})$, $i \neq u$, and of the collision history (due to the no feedback assumption). In particular, it can be shown (see, e.g., [4], [18]), that, for the problem considered in this work, the following threshold policy is optimal:

$$Q_{u,k} = \chi(V_{u,k} \geq v_{\text{th},u}(E_{u,k})),$$

where $\chi(\cdot)$ is the indicator function, and $v_{\text{th},u}(E_{u,k})$ is some importance threshold, which is a function of the energy level $E_{u,k}$. We denote the corresponding transmission probability of EHS $u$ in energy level $e$, induced by the random importance $V_{u,k}$, by $\eta_u(e)$. This is given by

$$\eta_u(e) = \mathbb{E}_{V_{u,k}}[Q_{u,k}|E_{u,k} = e] = \Pr(V_{u,k} \geq v_{\text{th},u}(e)).$$

Moreover, we denote the expected data importance reported by EHS $u$ to FC in state $e$, assuming that all the other EHSs remain idle (no collisions occur), as $g(\eta_u(e))$. This is given by

$$g(\eta_u(e)) = \mathbb{E}_{V_{u,k}}[Q_{u,k}|E_{u,k} = e] = \mathbb{E}_{V_{u,k}}[\chi(V_{u,k} \geq v_{\text{th},u}(e))] V_{u,k} = \int_{v_{\text{th},u}(e)}^{\infty} \nu f_V(\nu) d\nu.$$

In words, $g(\eta_u(e))$ is the expected reward accrued when only the data with importance larger than $v_{\text{th},u}(e)$ is reported and no collision occurs, where $\eta_u(e)$ is the corresponding transmission probability. The function $g(x)$ has the following properties, which are stated here without proof.

**Lemma 1** The function $g(x)$ is a strictly increasing, strictly concave function of $x$. 
In the following, we refer to $\eta_u$ as the policy of EHS $u$. Moreover, we denote the aggregate policy used by all the EHSs in the network as $\eta = (\eta_1, \eta_2, \ldots, \eta_U)$.

Given an initial state of the energy levels $e_0 = e_0 \in E^U$, we denote the average long-term importance of the data reported by EHS $u$ to FC, under the aggregate policy $\eta$, as

$$R_{\eta}(e_0) = \lim_{K \to \infty} \frac{1}{K} \E \left[ \sum_{k=0}^{K-1} Q_{u,k} V_{u,k} \prod_{i \neq u} \left( 1 - Q_{i,k} \right) \right]$$

$$= \lim_{K \to \infty} \frac{1}{K} \E \left[ \sum_{k=0}^{K-1} g(\eta_u(E_u,k)) \prod_{i \neq u} \left( 1 - \eta_i(E_i,k) \right) \right].$$

(5)

The expectations above are taken with respect to $\{B_k, Q_k, V_k\}$, where $Q_k = (Q_{1,k}, Q_{2,k}, \ldots, Q_{U,k})$ and, at each instant $k$, $Q_{i,k}$ is given by (2) for appropriate threshold $v_{ih,i}(E_{i,k})$, and $E_{i,k}$ evolves according to (1).

In the last step, we have used the fact that $Q_{i,k}$ only depends on $(E_{i,k}, V_{i,k})$ and $V_{i,k}$ is i.i.d. across the EHSs. The term $Q_{u,k} \prod_{i \neq u} (1 - Q_{i,k}) = 1$ if and only if EHS $u$ transmits the current data packet, and all the other EHSs remain idle, so that no collision occurs and the transmission is successful. Moreover, we define the average long-term aggregate importance of the reported data (from now on referred to as network utility for brevity) as

$$R_{\eta}(e_0) = \sum_{u=1}^{U} R_{\eta}^{(u)}(e_0).$$

(6)

In this paper, we design control policies $\eta$ which maximize the network utility, i.e.,

$$\eta^* = \arg \max_{\eta} R_{\eta}(e_0).$$

(7)

We consider the set of admissible policies $\mathcal{U}$ that result in an average reward independent of the initial state $e_0$, as defined below.

**Definition 1** The set of admissible policies is defined as

$$\mathcal{U} = \{ \eta : \eta(0) = 0, \eta(e_{\max}) \in (0, 1), \eta(e) \in (0, 1), e \neq 0, e_{\max} \}.$$  

It can be shown that the Markov chain $\{E_k\}$ under the aggregate policy $\eta \in \mathcal{U}^U$ is irreducible. Hence, there exists a unique steady-state distribution, $\pi_{\eta}(e), e \in E^U$, independent of $e_0$ [19]. From (5), we thus obtain

$$R_{\eta}^{(u)} = \sum_{e \in E^U} \pi_{\eta}(e) g(\eta_u(e_u)) \prod_{i \neq u} (1 - \eta_i(e_i)).$$

(8)

Moreover, since the action $Q_{u,k}$ is based only on $(E_{u,k}, V_{u,k})$ and does not depend on $(E_{i,k}, V_{i,k}), i \neq u$, and harvesting is i.i.d. across EHSs, the energy level of EHS $u$ is independent of the energy levels of all the other EHSs, so that we can write $\pi_{\eta}(e) = \sum_{u=1}^{U} \pi_{\eta_u}(e_u)$, where $\pi_{\eta_u}(e_u)$ is the steady state probability of the energy level of EHS $u$, $(E_{u,k})$. Letting $G(\eta_u) = \sum_{e=1}^{e_{\max}} \pi_{\eta_u}(e) g(\eta_u(e))$ and $P(\eta_u) = \sum_{e=1}^{e_{\max}} \pi_{\eta_u}(e) \eta_u(e)$, we can rewrite (8) as

$$R_{\eta}^{(u)} = G(\eta_u) \prod_{i \neq u} (1 - P(\eta_i)).$$

(9)

Eq. (9) can be interpreted as follows. $G(\eta_u)$ is the average reward of EHS $u$, assuming that all the other EHSs remain idle, so that no collisions occur. $P(\eta_i)$ is the average long-term transmission probability of EHS $i$, so that $\prod_{i \neq u} (1 - P(\eta_i))$ is the steady-state probability that all the EHSs, except $u$, remain idle. From (6), the network utility under the aggregate policy $\eta = (\eta_1, \eta_2, \ldots, \eta_U)$ then becomes

$$R_{\eta} = \sum_{u=1}^{U} G(\eta_u) \prod_{i \neq u} (1 - P(\eta_i)).$$

(10)

In order to guarantee fairness among the EHSs in the network, we consider only symmetric control policies, i.e., all the EHSs employ the same policy $\eta_u = \eta, \forall u$. From (10), we thus obtain

$$R_{\eta} = U G(\eta)(1 - P(\eta))^U - 1.$$

The optimization problem (7) over the class of admissible and symmetric policies is stated as

$$\eta^* = \arg \max_{\eta \in \mathcal{U}} U G(\eta)(1 - P(\eta))^U - 1.$$

(12)

IV. Optimization and Analysis

It can be shown that, since $g(x)$ is strictly concave, the optimal symmetric control policy $\eta^*$ is unique and belongs to $\mathcal{U}$. The optimization problem (12) when $U = 1$ has been studied in detail in [4] and can be solved using the Policy Iteration Algorithm (PIA) [20]. However, in general, when $U > 1$, (12) cannot be recast as a convex optimization problem, hence we resort to approximate solutions. In particular, in order to determine a local optimum of (12), we use a mathematical artifice based on a game theoretic formulation of the multiaccess problem considered in this paper: we model the optimization problem as a game, where it is assumed that each EHS, say $u$, is a player which attempts to maximize the common payoff $10$ with respect to its own policy $\eta_u$.  

We proceed as follows. We first characterize the general Nash Equilibrium (NE). Then, we study the existence of the Symmetric NE (SNE) for this game, i.e., such that all EHSs employ the same policy $\eta_u = \eta^*, \forall u$, and have no incentive to deviate from it. In Theorem 1, we show that the SNE is unique, and we also provide Algorithm 1 to compute it. In Theorem 2, we prove that the SNE, and thus the policy returned by Algorithm 1, represents a local optimum of the original optimization problem (12).

If a NE exists for this game (not necessarily symmetric), defined by the policy profile $\eta^* = (\eta_1^*, \eta_2^*, \ldots, \eta_U^*)$, then it solves, $\forall u$,

$$\eta_u^* = \arg \max_{\eta_u \in \mathcal{U}} \left[ G(\eta_u) \prod_{i \neq u} (1 - P(\eta_i^*)) + (1 - P(\eta_i^*)) \sum_{n \neq u} G(\eta_n^*) \prod_{i \neq n, u} (1 - P(\eta_i^*)) \right]$$

$$= \arg \max_{\eta_u \in \mathcal{U}} \left[ G(\eta_u) - P(\eta_u) \sum_{n \neq u} \frac{G(\eta_n^*)}{1 - P(\eta_n^*)} \right],$$

where, in the last step, we have removed positive multiplicative factors and additive terms independent of $\eta_u$, which do not affect the optimization problem. In particular, we are interested

3We point out that this formulation is only a mathematical artifice to determine the optimal policy, which is then followed by all EHSs (which are not assumed to behave strategically).
in characterizing the SNE. Then, by further imposing \( \eta^* = e^* \), \( \forall u \), in (13), we obtain
\[
\eta^* = \arg \max_{\eta \in \mathcal{U}} \left[ G(\eta) - \Lambda(\eta^*)P(\eta) \right], \tag{14}
\]
where we have defined
\[
\Lambda(\eta) = (U - 1) \frac{G(\eta)}{1 - P(\eta)}. \tag{15}
\]
Note that \( \eta^* \) defined in (14) is simultaneously optimal for all the EHSs, \( \text{i.e.} \), any unilateral deviation of a single EHS from the SNE \( \eta^* \) yields a smaller network utility \( R_{\eta} \). The interpretation of (14) is as follows. \( G(\eta) \) is the reward when the network contains only one user, so that the unique EHS has no constraint on the collisions caused to the other users in the network. The term \( \Lambda(\eta^*) \) is interpreted as a Lagrange multiplier constant associated to a constraint on the transmission probability of each EHS, so as to limit the collisions to the other EHSs in the network. The overall objective function is thus interpreted as the maximization of the individual reward of each user, with constraint on the average transmission probability to limit collisions, which are deleterious to network performance. Interestingly, the Lagrange multiplier (15) increases with the number of EHSs \( U \), so that, the larger the network size, the more stringent the constraint on the average transmission probability of each EHS, as expected.

In order to carry out (14), we solve the more general optimization problem, for \( \lambda \geq 0 \),
\[
\eta^{(\lambda)} = \arg \max_{\eta \in \mathcal{U}} \left[ G(\eta) - \lambda P(\eta) \right]. \tag{16}
\]
The following properties of \( \eta^{(\lambda)} \) can be proved, which follow from the fact that \( g(x) \) is a strictly concave function of \( x \):

**Proposition 1**  
1) \( \eta^{(\lambda)} \) is uniquely defined, \( \text{i.e.} \),
\[
G(\eta^{(\lambda)}) - \lambda P(\eta^{(\lambda)}) > G(\eta) - \lambda P(\eta), \quad \forall \eta \neq \eta^{(\lambda)}; \tag{17}
\]
2) \( \eta^{(\lambda)} \) is continuous in \( \lambda \);  
3) \( \eta^{(\lambda)} \in \text{int}(\mathcal{U}) \), where \( \text{int}(\mathcal{U}) \) denotes the interior of \( \mathcal{U} \);  
4) \( 0 < P(\eta^{(\lambda)}) \leq b, \quad 0 < G(\eta^{(\lambda)}) < g(P(\eta^{(\lambda)})) \leq g(b) \).

**Remark:** The first property is a consequence of the fact that (16) can be recast as a convex optimization problem, where the objective function is strictly concave. The second property follows from the strict concavity of \( g(x) \). The third property follows from the fact that, for \( \lambda = 0 \), \( \eta^{(0)}(e) \in (0, 1), \quad \forall e \neq 0 \), as proved in [4]. This holds also for \( \lambda > 0 \), as can be proved following similar arguments as in [4]. The last property is a consequence of the fact that, for any policy \( \eta \in \mathcal{U} \), the average long-term transmission probability cannot be larger than the average harvesting rate \( b \), since the transmission of one packet requires the expenditure of one energy quantum. Moreover, from the strict concavity of \( g(x) \), we obtain \( G(\eta) < g(P(\eta)) \leq g(b) \), since \( g(x) \) is an increasing function of \( x \).

By comparing (14) and (16), \( \eta^* \) is optimal for (14) if and only if \( \eta^* = \eta^{(\lambda^*)} \), for \( \lambda^* \geq 0 \) and \( \Lambda(\eta^{(\lambda^*)}) = \lambda^* \).
The following lemmas are functional to the main Theorem 1, which proves the existence and uniqueness of the solution of (14):

**Lemma 2**  
\( P(\eta^{(\lambda)}) \) is a non-increasing function of \( \lambda \) for \( \lambda \geq 0 \), with limits \( P(\eta^{(0)}) > 0 \) and \( \lim_{\lambda \to \infty} P(\eta^{(\lambda)}) = 0 \).

**Lemma 3**  
\( \Lambda(\eta^{(\lambda)}) \) is a continuous, non-increasing function of \( \lambda \), for \( \lambda \geq 0 \), with limits \( \Lambda(\eta^{(0)}) > 0 \) and \( \lim_{\lambda \to \infty} \Lambda(\eta^{(\lambda)}) = 0 \).

Their proof is omitted due to space limitations.

**Theorem 1**  
There exists a unique \( \eta^* \in \mathcal{U} \) solution of (14), \( \text{i.e.} \), \( \exists! \eta^* \in \mathcal{U} \) such that
\[
G(\eta^*) - \Lambda(\eta^*)P(\eta^*) > G(\eta) - \Lambda(\eta^*)P(\eta), \quad \forall \eta \neq \eta^*, \quad \eta \in \mathcal{U}.
\]
Moreover, \( P(\eta^*) \leq \min \{\tilde{b}, \frac{1}{\lambda^*}\} \).

**Proof:** The existence and uniqueness of \( \eta^* \) solution of (14) is proved by using Lemma 3. In fact, \( h(\lambda) = \Lambda(\eta^{(\lambda)}) - \lambda \) is a continuous decreasing function of \( \lambda \) (since \( \Lambda(\eta^{(\lambda)}) \) is continuous non-increasing), with limits \( h(0) = \Lambda(\eta^{(0)}) > 0 \) and \( \lim_{\lambda \to \infty} h(\lambda) = -\infty \), hence there exists a unique \( \lambda^* \in (0, \infty) \) such that \( h(\lambda^*) = 0 \), \( \text{i.e.} \), \( \Lambda(\eta^{(\lambda^*)}) = \lambda^* \), which guarantees that \( \eta^{(\lambda^*)} \) is optimal for (14).

We now prove that \( P(\eta^{(\lambda^*)}) \leq \min \{\tilde{b}, \frac{1}{\lambda^*}\} \). From Prop. 1, we have \( P(\eta^{(\lambda^*)}) \leq \tilde{b} \), hence it is sufficient to prove that \( P(\eta^{(\lambda^*)}) \leq \tilde{b} \). This is trivially true if \( P(\eta^{(0)}) \leq \frac{1}{\lambda^*} \), since \( P(\eta^{(\lambda)}) \) is a non-increasing function of \( \lambda \) (Lemma 2). Now, assume that \( P(\eta^{(0)}) > \frac{1}{\lambda^*} \). Then, since \( \lim_{\lambda \to \infty} P(\eta^{(\lambda)}) = 0 \), there exists \( \lambda \in (0, \infty) \) such that \( P(\eta^{(\lambda)}) = \frac{1}{\lambda} \). For such \( \lambda \), from (16) we have
\[
G(\eta^{(\lambda)}) - \lambda \frac{1}{U} = G(\eta^{(\lambda)}) - \lambda P(\eta^{(\lambda)}) = \max_{\eta \in \mathcal{U}} \left[ G(\eta) - \lambda P(\eta) \right] > G(0) - \lambda P(0) = 0, \tag{18}
\]
and, using (15) and the fact that \( P(\eta^{(\lambda)}) = \frac{1}{\lambda} \),
\[
UG(\eta^{(\lambda)}) - \lambda = (U - 1) \frac{G(\eta^{(\lambda)})}{1 - 1/U} - \lambda = \Lambda(\eta^{(\lambda)}) - \lambda > 0.
\]
Therefore, we obtain \( \Lambda(\eta^{(\lambda)}) > \lambda \). Since \( \Lambda(\eta^{(\lambda)}) - \lambda \) is a decreasing function of \( \lambda \) (Lemma 3) and \( \Lambda(\eta^{(\lambda)}) - \lambda^* = 0 \), necessarily \( \lambda < \lambda^* \). Finally, using Lemma 2, we obtain \( P(\eta^{(\lambda)}) = \frac{1}{\lambda} \geq P(\eta^{(\lambda^*)}) \), since \( P(\eta^{(\lambda)}) \) is a non-increasing function of \( \lambda \). The theorem is thus proved.

We also have the following result.

**Theorem 2**  
The SNE \( \eta^* \) in (14) is a local optimum for (12).

**Proof:** Since \( \eta^* \) is globally optimal for the optimization problem (14) and \( \eta^* \in \text{int}(\mathcal{U}) \) from Prop. 1, then the gradient with respect to \( \eta \), \( \Delta_{\eta}(\cdot) \), of the objective function in (14), computed in \( \eta^* \), is equal to zero, and its Hessian with respect to \( \eta \), \( H_{\eta}(\cdot) \), computed in \( \eta^* \), is semidefinite negative. More precisely, since \( g(x) \) is a strictly concave function of \( x \), it can be proved that the Hessian of the objective function in (14), computed in \( \eta^* \), is negative definite, \( \text{i.e.} \), for the SNE \( \eta^* \) we have
\[
[\Delta_{\eta}(G(\eta)) - \Lambda(\eta^*) \Delta_{\eta}(P(\eta))]_{\eta=\eta^*} = 0, \tag{19}
\]
\[
[H_{\eta}(G(\eta)) - \Lambda(\eta^*) H_{\eta}(P(\eta))]_{\eta=\eta^*} < 0. \tag{20}
\]
On the other hand, the gradient of (11) is given by

\[ \Delta_\eta (R_\eta) = U(1 - P(\eta))^{U-1} \Delta_\eta (G(\eta)) - U(U - 1)G(\eta)(1 - P(\eta))^{U-2} \Delta_\eta (P(\eta)). \]  

(21)

The Hessian matrix of (11) is then obtained by further computing the gradient of each component of (21), yielding

\[ H_\eta (R_\eta) = U(1 - P(\eta))^{U-1} H_\eta (G(\eta)) - U(U - 1)(1 - P(\eta))^{U-2} \Delta_\eta (G(\eta)) \Delta_\eta (P(\eta))^T \]

\[ - U(U - 1)(1 - P(\eta))^{U-2} \Delta_\eta (P(\eta)) \Delta_\eta (G(\eta))^T \]

\[ + U(U - 1)(U - 2)G(\eta)(1 - P(\eta))^{U-3} \Delta_\eta (P(\eta)) \Delta_\eta (G(\eta))^T \]

\[ - U(U - 1)G(\eta)(1 - P(\eta))^{U-2} H_\eta (P(\eta)). \]

(22)

By computing (21) under the SNE \( \eta^* \), and by using (15) and substituting (19) in (21), we then obtain \( \Delta_\eta (R_\eta)_{\eta=\eta^*} = 0 \). Moreover, since \( H_\eta (G(\eta))_{\eta=\eta^*} < \Lambda(\eta^*) \), we have used the fact that the product of the column vector \( \Delta_\eta (P(\eta)) \) by its transpose is semidefinite positive. Therefore, \( H_\eta (R_\eta)_{\eta=\eta^*} \leq 0 \), hence \( \eta^* \) is a local optimum for (12).

To conclude, we present an algorithm to determine the SNE \( \eta^* \) in (14), hence, from Theorem 2, a local optimum of (12). In particular, we employ the bisection method to compute the unique \( \lambda^* \) such that \( h(\lambda^*) = 0 \), where we have defined \( h(\lambda) = \Lambda(\eta^*) - \lambda \), which determines the SNE \( \eta^* \) as \( \eta^* = \eta(\lambda^*) \). We use the fact that \( h(\lambda) \) is a continuous decreasing function of \( \lambda \), with \( h(0) > 0 \) and \( \lim_{\lambda \to \infty} h(\lambda) = -\infty \), so that, if \( h(\lambda) > 0 \) (respectively, \( h(\lambda) < 0 \) for some \( \lambda \), then necessarily \( \lambda < \lambda^* \) (\( \lambda > \lambda^* \)). We need upper and lower bounds to \( \lambda^* \), denoted as \( \lambda_{\text{max}} \) and \( \lambda_{\text{min}} \), respectively, so that \( \lambda_{\text{min}} \leq \lambda^* \leq \lambda_{\text{max}} \). These bounds are then iteratively updated and refined, by testing the sign of \( h(\lambda) \) for the new \( \lambda = (\lambda_{\text{min}} + \lambda_{\text{max}})/2 \), until the desired accuracy is attained. The initialization of the lower bound is chosen as \( \lambda_{\text{min}} = 0 \). As to the upper bound, note that \( P(\eta^*) \leq \min\{b, 1/T\} \) from Prop. 1, hence \( G(\eta^*) \leq g(P(\eta^*)) \leq g(0) \). Therefore, from (15) we obtain

\[ \lambda^* = \Lambda(\eta(\lambda^*)) < \min\left\{ \frac{U - 1}{1 - b}, Ug\left( \frac{1}{U} \right) \right\} = \lambda_{\text{max}}. \]

(23)

**Algorithm 1 (Bisection method)**

1) INIT: accuracy \( \epsilon > 0 \), \( \lambda_{\text{min}} = 0 \) and \( \lambda_{\text{max}} \) as in (23);
2) MAIN: \( \lambda := (\lambda_{\text{min}} + \lambda_{\text{max}})/2 \);
   DETERMINE \( \eta(\lambda) \) using the PLA [20];
   COMPUTE \( h(\lambda) = \Delta(\eta(\lambda)) - \lambda \);
   IF \( |h(\lambda)| < \epsilon \), RETURN optimal policy \( \eta^* = \eta(\lambda^*) \);
   IF \( h(\lambda) > \epsilon \), UPDATE \( \lambda_{\text{min}} := \lambda \) and \( \lambda_{\text{max}} := \min\{\lambda_{\text{max}}, \Lambda(\eta(\lambda))\} \);
   IF \( h(\lambda) < -\epsilon \), UPDATE \( \lambda_{\text{max}} := \lambda \) and \( \lambda_{\text{min}} := \max\{\lambda_{\text{min}}, \Lambda(\eta(\lambda))\} \);
   REPEAT MAIN;

**Remark:** Note that the UPDATE step updates both \( \lambda_{\text{min}} \) and \( \lambda_{\text{max}} \). This is because \( h(\lambda) \) is a decreasing function of \( \lambda \) and \( \Lambda(\eta(\lambda)) \) is a non-increasing function of \( \lambda \) (Lemma 3), hence, if \( h(\lambda) > 0 \), then \( \lambda < \lambda^* = \Lambda(\eta(\lambda)) \leq \Lambda(\eta(\lambda^*)) \). Similarly, if \( h(\lambda) < 0 \), then \( \lambda > \lambda^* = \Lambda(\eta(\lambda)) \geq \Lambda(\eta(\lambda^*)) \).

V. NUMERICAL RESULTS

In this section, we present some numerical results. We model \( V_{u,k} \) as an exponential random variable with unit mean, with pdf \( f_V(v) = e^{-v}, v \geq 0 \). From (3) and (4), we obtain \( g(x) = x(1 - \ln x) \). In Fig. 2, we plot the network utility (6) under the policy (14), computed using Algorithm 1. We consider different scenarios varying in the battery capacity \( e_{\text{max}} \in \{1, 10\} \) and the EH rate \( b \in \{1/U, 0.1, 0.01\} \), as a function of the number of EHSs in the network \( U \). In particular, when \( b = 1/U \) the total expected energy harvested by the network in one time-slot is 1. Interestingly, the network utility increases with the number of EHSs \( U \). This behavior is due to the strict concavity of \( g(x) \), such that a diminishing return is associated to a larger transmission probability \( x \). Therefore, the smaller the number of EHSs \( U \), the more the transmission opportunities for each EHS, but the smaller the marginal gain, so that the network utility decreases. Note that, for \( U < 10 \), the best performance is attained in the case \( b = 1/U \), since more energy is available to the EHSs. In contrast, for \( U > 10 \) and \( e_{\text{max}} = 10 \), the best performance is attained in both cases \( b = 1/U \) and \( b = 0.1 \), despite a larger energy availability in the latter case. This is due to the fact that, as proved in Theorem 1, under the optimal policy \( \eta^* \), \( P(\eta^*) \leq \min\{b, 1/T\} = 1/U \), hence the performance bottleneck is due to the number of EHSs in the network, rather than to the energy availability. In the case \( b = 0.1 \), a large amount of energy cannot be employed for data transmission and is lost via overflow, in order to limit the collisions. This amounts to \( b - P(\eta^*) \geq b - 1/U \). In contrast, when \( b = 0.01 \), we have \( P(\eta^*) \leq \min\{b, 1/U\} = b \) for all values of \( U \) considered, hence the performance bottleneck is energy availability.

A different trend is observed when \( e_{\text{max}} = 1 \). In this case, for \( U > 10 \), the scenario \( b = 0.1 \) outperforms \( b = 1/U \). This is a consequence of the fact that, when \( e_{\text{max}} = 1 \), whenever
studied the problem of designing random access policies so as to maximize the overall network utility, defined as the average unilaterally maximizes the network utility, and characterized the optimization problem as a game, where each EHS unilaterally maximizes its own utility. We have derived an algorithm to compute it. Finally, we have presented some numerical results.

VI. CONCLUSIONS
In this paper, we have considered a WSN of EHSs which randomly access a collision channel, to transmit packets of random importance to a common fusion center. We have studied the problem of designing random access policies so as to maximize the overall network utility, defined as the average long-term aggregate network importance of the data packets successfully reported to the fusion center. We have formulated the optimization problem as a game, where each EHS unilaterally maximizes the network utility, and characterized the Symmetric Nash Equilibrium (SNE). We have proved the existence and uniqueness of the SNE, showing that it is a local optimum of the original optimization problem, and we have derived an algorithm to compute it. Finally, we have presented some numerical results.

REFERENCES