Interference Relay Channels with precoded Dynamic Decode and Forward protocols

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Abstract—In this paper, a full duplex relay using a Dynamic Decode and Forward (DDF) protocol is considered for improving the performance of multiple source-destination pairs interfering one on each other. A relay precoder is optimized as a function of the symbols correctly decoded by the relay in order to improve the channel capacity. Furthermore, a DDF-Patching technique allows for increasing the number of precoded symbols by the relay and providing highly improved performance in interference-limited scenarios.

I. INTRODUCTION

When interference occurs during concurrent communications, the performance can be improved by the help of a relay. For example, [1] considers the combination of dirty paper coding and beamforming under the assumption of non causal knowledge of the messages at the relay. In [2], interference forwarding is proposed in order to improve the successive interference cancellation at the receiver. Interference neutralization is mentioned by S.Mohajer et al. [3], [4], where the signals sent by the relay cancel the interference over the air. The aim of this work is first to improve the achievable capacity on each source-destination link by the combination of a relay precoder optimization and an in-band reception/transmission [5] DDF protocol. The performance is further improved by a Patching technique originally proposed to maximize the achievable macro and micro diversity orders for the relay channel [6]. The paper is organized as follows: the system model is presented in Section II. In Section III, we describe the precoder optimization. The DDF protocol and Patching technique are presented in Section IV. Simulations results are presented in Section V.

II. SYSTEM MODEL AND PARAMETERS

We consider $n$ sources transmitting data on the same physical resource to $n$ destinations carrying $N_r$ reception antennas, and a full-duplex relay with $N_f$ antennas shared by all the sources. The Interference Relay Channel (IRC) is presented in Fig.1. The full-duplex relay can stay in reception mode of the DDF protocol for a subset of source-to-destination links while being in transmission mode for the others. We also consider the half-duplex as an adaptation of the full-duplex case. Sources are relay-unaware, i.e., they do not know the relay’s presence in the system [7], [8], [9] and no control signal is needed between the sources and the relay. We assume that the relay has a perfect channel state information (CSI) knowledge of all transmission links. Each source information word is built from the concatenation of a message and Cyclic Redundancy Check (CRC) bits. The information bits are then encoded and sent to the input of Quadrature Amplitude Modulation (QAM). The resulting codeword is transmitted during $T$ symbol times, or time-slots. The relay jointly decode the received messages. After the correct decoding of one of the messages, which is detected thanks to the embedded CRC bits, the relay perfectly knows the information bits. The relay can perfectly re-build the codeword sent by this source and deduce its past and future symbols. By definition, the $k$-th correct decoding at the relay occurs after having correctly decoded the message sent by the $k$-th source $S_k$ to the destination $D_k$. Phase $k$ ends after the $k$-th correct decoding. Thus, there might be at most $n+1$ phases. During the $k+1$-th phase, the relay can generate the symbols sent by sources 1 to $k$. A vector of $k$ non-null symbols and $n-k$ null symbols is then precoded by $P_{k+1}$ of size $N_f \times n$, and sent by the relay on the same resource as the sources. The precoder at the relay is optimized so as to achieve a trade-off between boosting the destination useful signal and reducing the interference as proposed in III. The $k$-th destination tries to decode $S_k$’s message with a low complexity reception scheme, i.e., does not perform a joint decoding of the sources messages. If a Patching technique is used, the according low-complexity reception scheme must be used at the destination [6].

We consider quasi static flat-fading channels remaining constant during the transmission of one codeword, and independent from one transmission to another. We denote $H_m$ the matrix of fading coefficients between all the sources and the $m$-th destination, and $F_m$ the matrix of fading coefficients between the relay and the $m$-th destination. They estimate the channel coefficients and forwards it to the relay. Each fading coefficient is complex Gaussian distributed with a variance equal to the average receive power on the considered link. At each receive antenna, the additive complex white Gaussian

![Fig. 1. Interference Relay Channel with n pairs. Only the interference from $S_1$ are illustrated (dashed lines).](image-url)
noise has zero mean and variance $2N_0$. The Signal to Noise Ratio (SNR) and Signal to Interference plus Noise Ratio (SINR) are defined as usual by taking into account the path gain between the transmitter and the receiver, interferers and noise power. We denote $s_{m,k,i}$ the symbol sent by the source $S_m$ and $y_{m,k,i}$ the signal received by the destination $D_m$ during the $i$-th time-slot of the Phase $k$. Bold upper case letters denote matrices, bold lower case letters denote vectors, $\dagger$ is the transpose operator, $\bar{\cdot}$ is the transpose conjugate operator and $\ast$ is the conjugate operator. The trace and determinant of the matrix $M$ are noted $\textrm{Tr}(M)$ and $|M|$, respectively.

III. PRECODER OPTIMIZATION AT THE RELAY

During Phase $k$, a linear precoder $P_k$ of size $N_t \times n$ is applied at the relay in order to improve the capacity obtained at each destination. The precoder $P_k$ is recomputed as soon as the relay has correctly decoded $S_k$’s message. For instance, during Phase $1$, full interference is experienced at the destinations, during Phase $k$, the symbols $S_1$ to $S_{k-1}$ are precoder altogether by $P_k$, and in Phase $n+1$, all symbols are precoder altogether at the relay. The $m$-th destination receives the vector $y_{m,k,i}$ during the $i$-th time-slot of Phase $k$

$$y_{m,k,i} = M_m(P_k)s_{k,i} + n_{m,k,i}$$

where

$$M_m(P_k) = H_m + F_m P_k \Delta_k$$

and $s_{k,i} = [s_{1,k,i}, \ldots, s_{n,k,i}]$ and $n_{m,k,i}$ is the noise vector. We note $\Delta_k$ as a $n \times n$ matrix whom non-null entries are the first $k-1$ diagonal coefficients, equal to one. As a result, the $N_t \times n$ precoder matrix $P_k$ is only applied to the $k-1$ first symbols.

This channel model is specific to IRC channels with DDF. Thus, the following precoder optimization is different from existing precoders mostly derived for AF channels [11][10] or for a joint-source/relay precoding (e.g., [12]), as we assume relay-unaware sources. The precoder matrix $P_k$ is computed according to channel capacity metrics. We define the capacity at the $m$-th destination during Phase $k$ as

$$C_{m,k}(P_k) = \log_2 \left| M_m(P_k) \right| + \log_2 \left| M_m(P_k) \Delta_m \right| + 2N_0I$$

where $D_m$ is a $n \times n$ matrix with a single non-null entry at the $m$-th position on the diagonal, and $D_m = I - D_m$. We define $C_k(P_k)$ as the generalized mean of the capacities $C_{m,k}(P_k)$

$$C_k(P_k) = \left( \frac{1}{n} \sum_{m=1}^{n} C_{m,k}(P_k)^p \right)^{1/p}$$

where $p$ is the parameter of the generalized mean.

In order to reach fairness between the non-cooperating destinations, we maximize the minimal capacity by considering $p \to -\infty$, or $p$ negative and sufficiently low. One could also maximize the sum-capacity by choosing $p = 1$. One can optimize $C_k(P_k)$ under a total maximal power constraint $h(P_k) = \text{Tr} \left( P_k \Delta_k \right) - 1 \leq 0$ at the relay, which leads to the following Lagrange multipliers system

$$\frac{\partial C_k(P_k)}{\partial P_k} = \lambda \frac{\partial h(P_k)}{\partial P_k}$$

and $h(P_k) \leq 0$

where

$$\frac{\partial C_k(P_k)}{\partial P_k} = \frac{C_k(P_k)^{1-p}}{n} \sum_{m=1}^{n} C_{m,k}(P_k)^{p-1} \frac{\partial C_{m,k}(P_k)}{\partial P_k}$$

and by using the matrix differentiation tools [13],

$$\frac{\partial C_{m,k}(P_k)}{\partial P_k} = 1 - \frac{1}{\ln(2)} F_k \left[ (M_m(P_k) M_m(P_k)^{\dagger} + 2N_0I)^{-1} \right] M_m(P_k) - (M_m(P_k) \Delta_m M_m(P_k)^{\dagger} + 2N_0I)^{-1} M_m(P_k) \Delta_m \right] \Delta_k$$

No closed form expression can be derived from (5), a gradient descent iteratively optimizes the precoder $P_k$ instead:

$$P_k \leftarrow \left( P_k + \mu \frac{\partial C_k(P_k)}{\partial P_k} \right) / \min \left( \text{Tr} \left( \Delta_k^\dagger P_k^\dagger \Delta_k \right), 1 \right)$$

where $\mu$ is the gradient descent parameter, and the denominator allows for projecting the estimated precoder matrix on the set of solutions satisfying the constraint $h(P_k)$. It is preferable to choose a moderately high value for the geometric mean parameter such as $p = -5$ in order to find a good trade-off between the approximation of the min function and the good convergence of the system.

We have presented in this section an optimization of the precoder applied at the shared relay in order to reach fairness between the destinations. The relay transmit power is shared between useful signal boosting and interference reduction. The proposed optimization can be applied for any number of sources, transmit antennas at the relay and receive antennas at the destination. In the following, we illustrate how to combine the precoder optimization at the relay with a DDF relaying protocol.

IV. A DDF PROTOCOL DESIGN FOR IRCs

A. Implementation of the DDF protocol for IRCs

The precoder is optimized at each phase end, according to $\Delta_k$ which characterizes the set of symbols correctly decoded at the relay. The precoder optimization requires full CSI knowledge at the relay, which is forwarded by the destinations or obtained from channel reciprocity when possible, allowing the sources to be relay-unaware.

An early decoding of a symbol at the relay will drastically improve the destination’s performance if the transmission link does not suffer from a high interference level. Equivalently, if a source whom message has been decoded by the relay generates a high interference level on an another transmission link, the later will highly take benefit from interference reduction from the relay precoder. Unfortunately, for late correct decoding
events, the relay does not bring much gain on the system. In the next section, we use a Patching technique similar to the one presented in [6] for a different target: the Patching technique is used in order to virtually apply the precoding matrix on symbols sent in previous phases of the DDF protocol, i.e. symbols transmitted before the correct decoding of the related messages by the relay.

B. Patching principle

A Patching technique aims at increasing the number of source symbols taking benefit from the relay-destination link and from the relay precoder, while keeping a low complexity decoding at the destination side. A Patching technique is composed of two steps. The first one is done at the relay which, after having correctly decoded a message, transmits a linear combination of symbols previously sent by the source and of symbols the source is going to send. The combination must be chosen so as to allow a low-complexity decoding at the destination. For instance, one can build a 4QAM symbol by combining $q$ QPSK symbols $s_i$ using the expression:

$$ s = a(q) \sum_{i=1}^{q} 2^{i-1} s_i \quad \text{with} \quad a(q) = \sqrt{\frac{3}{4^q - 1}} \quad (9) $$

where the weighting coefficients are chosen in order to keep the unity energy property of the constellation, and the Patching order is by definition equal to $q$. The second step is performed by the destination which combines the received signals according to the combination at the relay. After this combination, the combined symbols appear to be transmitted through both the source-destination and the relay-destination links. A coding gain loss is observed due to the constellation expansion and the attenuation of the relay received power.

C. Patching for n-pair IRC

In this section, we present a generalization of the Patching to the n-pair IRC. The Patching technique combines symbols of different phases in order to maximize the number of symbols virtually sent through the precoder. The precoder cannot be adapted to a given phase and is chosen as $P_{n+1}$ which is designed for the full precoded case. The Patching technique for DDF on IRC channel is a trade-off between the interference reduction and power boosting, the coding gain loss inherent to the Patching technique, and a performance loss due to the use of $P_{n+1}$ for Phase $k$ instead of $P_k$.

For the sake of clarity, we present the simplest version of the Patching algorithm, where Patching is only applied on the symbols of Phase 1, experiencing and generating interference. We denote $L_1$ the number of time-slots in the first phase of the DDF protocol. At the beginning of a new transmission, a vector $v$ storing the indexes of the last patched symbol of Phase 1 for each source is initialized to zero. Then, the relay executes Algorithm 1 for each time slot $t$ of each Phase $k+1$ ($2 \leq k+1 \leq n+1$, for each source $m \leq k$ and for a Patching order $q$.

### Algorithm 1 Generation of $m$-th source symbols by the relay for the $i$-th time slot of Phase $k+1$

1. $s_{m,k,i} \leftarrow 0$
2. $j_{k,i} \leftarrow 1$
3. while $v(m) + j_{k,i} \leq L_1$ and $j_{k,i} < q$ do
   4. $s_{m,k,i} \leftarrow s_{m,k,i} + 2^{j_{k,i}-1} s_{m,k,i}$
   5. $j_{k,i} \leftarrow j_{k,i} + 1$
4. end while
7. $s_{m,k,i} \leftarrow a(j_{k,i})(\tilde{s}_{m,k,i} + 2^{j_{k,i}-1} s_{m,k,i})$
8. $v(m) \leftarrow v(m) + j_{k,i}$

As a remark, as soon as all symbols from Phase 1 have been precoded, the system behaves as the DDF protocol for IRC. The $m$-th destination receives

$$ y_{m,k,i} = H_m s_{k,i} + F_m P_{n+1} \Delta_k \times (s_{1,k,i}, \ldots, s_{k-1,k,i},0,\ldots,0)^t + n_{k,i} \quad (10) $$

By knowing the instant of correct decoding at the relay, each destination $m$ applies Algorithm 1 for combining the received vectors $y_{m,k,i}$ instead of the symbols $s_{m,k,i}$, which leads to the following equivalent channel model:

$$ \tilde{y}_{m,k,i} = (H_m + F_m P_{n+1} \Delta_k a(j_{k,i})2^{j_{k,i}-1}) s_{k,i} + n_{k,i} \quad (11) $$

During the $i$-th time slot of Phase $k+1$, the symbols after patching are $4^q$-QAM symbols sent on the equivalent channel with power boosting and interference reduction of the symbol decoded at the relay, i.e. from sources 1 to $k$. If Phase 1 is longer than the cumulated length of the other phases, unprecoded symbols remain. When all sources are decoded at the same time, if Phase 1 is shorter than the last Phase, all symbols are precoded, some of them being patched, the other being transmitted through the relay-destination link using the DDF protocol. An adaptation of this protocol to half-duplex relays implies that the relay waits for all messages to be correctly decoded before switching into the transmission mode.

V. SIMULATION RESULTS

In this Section, we consider a two sources case ($n=2$) where the wireless links are symmetric, i.e., the long term signal to noise ratios (SNRs) and signal to noise plus interference ratios (SNIRs) of the two pairs without relay are equal. The mutual information $\chi_{m,k}$ observed by the $m$-th destination during Phase $k$ equals

$$ \chi_{m,k} = E_{s_{m,k},P_k} \left[ \log_2 \left( \frac{E_{y_{m,k},s_k} [p(y_{m,k}|s_k)]}{E_{y_{m,k},s_k} [p(y_{m,k}|s_k)]} \right) \right] \quad (12) $$

where $p(y_{m,k}|s_k) \propto e^{-\|y_{m,k}-M_m(s_k)\|^2/2N_0}$.

We focus on the mutual information achieved by one of the two pairs in the system. The transmitting node of this pair is called source $S$, and the other transmitter is called interferer $I$. The nodes are assumed to transmit QPSK symbols. We define as $P_R/P_S$ the ratio of the average power received by the destination from the relay and from the source. We consider
that the destination has a single receive antenna \( N_r = 1 \) and that the relay has two transmit antennas \( N_t = 2 \).

Fig. 3 illustrates the gain brought by the precoder optimization based on the capacity metric as presented in Sec. III. It can be shown that this precoder optimization provides almost equal performance as a precoder specifically optimized for maximizing the discrete input mutual information, with a lower computational complexity. We consider that the relay knows the symbols sent from the source and interferer, and we focus on the improvement of the average QPSK discrete input mutual information observed during the phase in which the relay transmits signal from both sources. When no precoder is used, the relay transmits each symbol of each source from its two transmit antennas. We can observe that for high SINR values, when the interference becomes negligible with respect to the noise level, the relay without precoder introduces interference in the system which drastically degrades the discrete input mutual information. By using the full CSI knowledge at the relay in order to optimize the precoder, we see that the relay transmit power is efficiently used to remove the interference and boost the useful signal. It has to be noted that, since the precoder is not distributed among the sources and relay, the transmit beamforming or zero forcing approach do not show such high performance. When the SINR is close to zero, we see how the relay improves the performance by a factor up to 200%. The performance can be further improved by increasing the number of transmit antennas at the relay \( N_t \).

In the following, we assume that the power received from the relay is 6dB higher than the power received from the source. We consider a particular codeword segmentation, such as for HARQ-OFDM systems, which restricts the possible instants of correct decoding to the set \( \{ \frac{5T}{7}, \frac{3T}{7}, \frac{7T}{9}, \frac{5T}{9}, T \} \), where \( T \) is the total number of time slots for a given codeword transmission. We denote \( M_S \) and \( M_I \) the index of the segment after which the relay correctly decodes the source message and the interferer message, respectively. Note that the case with no relay activation corresponds to \( M_S = M_I = 5 \). All the results are presented according to the SINR observed at \( D \) for a SNR between the source and destination of 30dB.

In Fig. 3, the averaged mutual information achieved for \( (M_S, M_I) \in \{ (1,1), (1,5), (5,1) \} \) are presented. The relay uses the DDF protocol with a precoder optimized for each phase. The performance obtained when the relay uses the DDF protocol outperforms the case without relay whatever the considered couples \( (M_S, M_I) \). The sooner the relay correctly decodes the source message, the higher the performance, which is due to the power boosting by the relay. Furthermore, the sooner the instant of correct decoding of the interferer, the higher the performance, which is due to the interference reduction by the relay.

In Fig. 4 and Fig. 5, the averaged mutual information obtained using the DDF protocol or the Patched DDF protocol, is presented for \( M_S = M_I \), which is also the performance obtained for a half duplex relay (beginning to transmit after the correct decoding of both messages). One can see that, whatever the considered instant of correct decoding, the Patched DDF protocol outperforms the DDF protocol thanks to the increased number of precoded symbols by the Patching technique.

In Fig. 6, the averaged mutual information is presented for \( M_I = 1 \), and \( M_S \in \{ 1, 3, 5 \} \), when the relay uses the DDF protocol or the Patched DDF protocol. For the very low SINR regime, the Patched DDF protocol is outperformed by the DDF protocol which benefits from a precoder optimized for each phase and always use QPSK modulation symbols. In the following, we consider the higher and more practical SINR regime. For the \( M_S = 1 \) case, all transmitted symbols can be precoded by the relay using the Patched DDF protocol. The transmission results in \( 3T/7 \) precoded 16QAM symbols and \( T/7 \) precoded QPSK symbols with full knowledge of the messages at the relay. When the relay uses the DDF protocol, the destination receives \( 3T/7 \) unprecoded QPSK symbols, and \( 4T/7 \) precoded QPSK symbols. The Patched DDF protocol achieves higher performance than the DDF protocol, the gain obtained from more symbols taking benefit from interference reduction.
reduction and power boosting is higher than the coding gain loss inherent to the Patching technique. For the $M_S = 3$ case, the transmission with Patched DDF protocol results in the transmission of $2T/7$ precoded 16QAM symbols only knowing the interferer message at the relay, $T/7$ precoded 16QAM symbols and $T/7$ precoded QPSK symbols with full knowledge of the messages at the relay. For the $M_S = 5$ case, the relay only allows to decrease the interference experienced at the destination side. When using the Patched DDF protocol, the transmission results in $3T/7$ precoded 16QAM symbols and $T/7$ precoded QPSK symbols. Only $4T/7$ QPSK symbols are precoded with the DDF protocol.

VI. CONCLUSION

We have studied an interference channel with a relay using the DDF protocol. The channel capacity is maximized by transmitting a precoded version of the symbols sent by the different sources. This precoding step and the choice of the precoding matrix are optimized under a power constraint at the relay which leads to a practical scheme. The precoded DDF technique brings performance improvement to the IRC channel, which can be further increased by using Patching techniques. The precoder optimization allows for obtaining good performance even when the destination only has $N_r = 1$ receive antenna.

REFERENCES


