

## Sliding mode control of a dual clutch during launch

V.N.Tran<sup>a,b</sup>, J. Lauber<sup>b</sup> and M.Dambrine<sup>b</sup>

<sup>a</sup> University of Transport and Communication, Hanoi, Vietnam, [nhutrv@gmail.com](mailto:nhutrv@gmail.com)

<sup>b</sup>LAMIH – University of Valenciennes and Haunait Cambresis, Le Mont Houy, 59300 Valenciennes,  
France

{[vannhu.tran](mailto:vannhu.tran), [jimmy.lauber](mailto:jimmy.lauber), [michel.dambrine](mailto:michel.dambrine)}@univ-valenciennes.fr

---

### Abstract

The Automated Manual Transmission was introduced in vehicles to improve driving comfort, performances and fuel efficiency compared with manual transmissions. In this system, the management of the clutch is a key point especially when considering driving comfort. In this paper, we propose a control law for clutch engagement based on sliding mode control. The goal is to ensure a smooth clutch engagement while limiting slip clutch and avoiding engine no-stall. To achieve this goal, the speed of clutch slip and engine speed are controlled in order to track reference trajectories. In addition, some parametric variations of the model are also considered. Several simulations are provided to show the effectiveness of proposed control law.

*Key Words: Dual clutch, Clutch slip control, Non-linear models, Trajectory tracking, Sliding control*

---

### 1. Introduction

Recently, with the increasing use of Automated Manual Transmissions (AMT), the control of a clutch has become an important challenge. Nowadays, there exist two technologies for automated lay-shaft gearing transmissions. One uses a single clutch and is basically a manual transmission with an added-on control unit that automates the clutch and shift operations. The other one, using a Dual Clutch Transmission (DCT), consists of two independents sub-boxes, each one activated by separate clutches: on-coming clutch and off-going clutch. A shift process involves the

engagement of the on-coming clutch and the release of the off-going clutch to ensure a shift without traction interruption.

The problems associated with AMT in literature, are the engagement of the clutch, the strategy of gearshift, and also the control of the actuator. The goal is to reduce the jerk in standing start and gearshift and thus ensure a good driveability and also reduce fuel consumption and emission of  $CO_2$ . Specifically, the dry clutch engagement must be controlled to satisfy conflicting objectives such as minimizing the slip energy and preservation of driving comfort.

To achieve these goals, many different approaches based on an optimal problem can

be found in the literature. For example in (Dolcini 2006), (Dolcini, Canudas de Wit, and Bechart 2007), (Dolcini, Canudas de Wit, and Bechart 2008), (Garofalo et al. 2002), (Wu et al. 2010), they use an optimal control of dry clutch engagement in the standing start, or in (Heijden et al. 2007), (Lucente, Montanari, and Rossi 2007) they use a hybrid optimal control of dry clutch engagement. The cost function takes into account the clutch slip speed and the jerk, to minimize the jerk and the dissipated energy.

Another family of approach is based on the use of tracking reference for the clutch slip speed (Ni, Lu, and Zhang 2009), (Gao et al. 2009), (Dassen 2003), (Amari, Alamir, and Tona 2008), and for engine speed (Ni, Lu, and Zhang 2009), (Dassen 2003), or for the vehicle acceleration (Lucente, Montanari, and Rossi 2007), or for the position of the clutch pedal (Horna et al. 2003), or finally a reference trajectory for the clutch pressure. Optimized clutch pressure profiles have been created for the best possible shift quality based on model simulation (Kulkarni, Shim, and Zhang 2007). The reference trajectory for clutch slip speed is pre-defined to satisfy the conditions no-lurch (Glielmo and Vasca 2000).

Based on sliding mode (Kim and Choi 2010) proposing the control architecture that consists of the speed control for synchronization and the output torque control for reducing jerk during launch.

The aim of this paper is to develop a control law based on the second family of approaches, i.e., using tracking trajectories for the clutch slip speed and the engine speed during the standing start. The trajectory reference is pre-defined to reduce the jerk and the oscillations during and after the synchronization, and to avoid the dead zone of the engine. Thus, a robust control law coming from sliding control methodology is proposed to ensure the desired tracking. Some simulations are provided to show the efficiency of the proposed controller.

## 2. Vehicle powertrain dynamic model

If we neglect the motion of the engine on its suspension and supposing that the drive train is symmetric, the powertrain model is one-dimensional mechanical system in which each element is a lumped mass model and a spring-damper model as show in Figure 1.

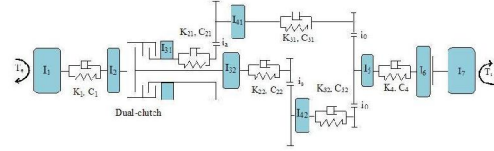


Figure 1. Dynamic models of powertrain with Dual clutch

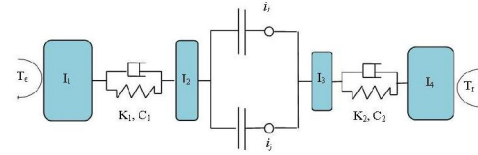


Figure 2. Simplified model of powertrain with Dual clutch

where:  $I_1, I_2, I_{31}, I_{32}, I_{41}, I_{42}, I_5, I_6, I_7$  are the mass moment of inertia of the engine and flywheel, of the dual clutch drum, of the clutch disc, the gears and the input shaft of two sub-gearboxes, of the gears and the output shaft of two sub-gearboxes, of the final drive, of the half-shaft and wheels and of the vehicle mass, respectively;  $K_1, C_1, K_{2i}, C_{2i}, K_{3i}, C_{3i}, K_4, C_4, (i=1, 2)$  are the stiffness and damping coefficient of the flywheel, of the input shaft of two sub-gearboxes, of the output shaft and of the half-shaft, respectively;  $i_a, i_s$  are the gear ratios of the current and next rapport involved in the shift, respectively;  $i_0$  is the final drive ratio;  $T_e$  is the engine torque;  $T_r$  is the load torque.

Considering the following assumptions: the tires have a perfect adherence and no transitory effects on tire-ground contact; the

input shafts and output shafts of the two sub-gearboxes are infinitely rigid, then a simplified model with four states is obtained as shown Figure 2, with

$$\begin{aligned} I_3 &= I_5 + (I_{41} + I_{42})i_0^2 + I_{31}(i_0i_a)^2 + I_{32}(i_0i_s)^2, \\ I_4 &= I_6 + I_7 = n_w I_w + m_v r_w^2, \\ i_i &= i_0 i_a, \quad i_j = i_0 i_s. \end{aligned} \quad (1)$$

Where:  $n_w$  is the number of wheels;  $I_w$  is the mass moment of inertia of one wheel;  $m_v$  is the vehicle mass;  $r_w$  is the radius of the wheel.

The differential equations describing the dynamics of the simplified model are given by

$$I_1 \dot{\omega}_1 = T_e - K_1(\theta_1 - \theta_2) - C_1(\omega_1 - \omega_2), \quad (2)$$

$$\begin{aligned} I_2 \dot{\omega}_2 &= K_1(\theta_1 - \theta_2) + C_1(\omega_1 - \omega_2) \\ &\quad - T_{c1}(\cdot) - T_{c2}(\cdot), \end{aligned} \quad (3)$$

$$\begin{aligned} I_3 \dot{\omega}_3 &= i_i T_{c1}(\cdot) + i_j T_{c2}(\cdot) - K_2(\theta_3 - \theta_4) \\ &\quad - C_2(\omega_3 - \omega_4), \end{aligned} \quad (4)$$

$$I_4 \dot{\omega}_4 = K_2(\theta_3 - \theta_4) + C_2(\omega_3 - \omega_4) - T_r, \quad (5)$$

$$\dot{\theta}_i = \omega_i i, \quad (i = 1..4). \quad (6)$$

Where:  $\omega_i, \theta_i$  are the angular velocities and angular displacements of the engine crankshaft, clutch drum, final drive and wheel, respectively;  $T_{c1}, T_{c2}$  are the clutch off-going and clutch on-coming torque, respectively.

When standing start, using only a single clutch is used, we assume that it is the first clutch, so  $T_{c2} = 0$ . Now we note  $T_c$  instead of  $T_{c1}$ .

### 2.1 Engine model

This part concerns the simplified model of the engine. As an assumption, the engine is modeled as a mean value torque generator that does not include the engine transients. Engine output torque is considered as a

function of engine speed  $\omega_1$  and throttle position  $p$

$$T_e = T_e(\omega_1, p).$$

Engine output torque is interpolated corresponding to engine speed and throttle position from an engine map modeled as a look-up table.

### 2.2 Clutch model

The torque transferred by the clutch in slip phase is the friction torque. In the literature, most of the models use classical friction models, such as static friction, Coulomb friction (dynamic friction), viscous friction, Stribeck friction (with Stribeck effect). Another model is developed by Carlos de Wit et al (Canudas de Wit et al. 1995). This solution provides a dynamic continuous model of the friction. In our study, a static model with Stribeck effect is considered as

$$T_c = 2n_d r_c \mu(\Delta\omega) \text{sign}(\Delta\omega) F_n. \quad (7)$$

Where:  $n_d$  is the number of clutch discs;  $r_c$  is the friction radius of the clutch disc;  $F_n$  is the normal force applied on clutch face and  $\mu(\Delta\omega)$  is the coefficient of friction that can be formulated as a function of clutch slip

$$\mu(\Delta\omega) = \mu_c + (\mu_s - \mu_c) e^{-\left(\frac{\Delta\omega}{\omega_s}\right)^2}.$$

Where:  $\mu_c, \mu_s$  are the Coulomb friction and the Stribeck friction;  $\omega_s$  is the Stribeck angular velocity;  $\Delta\omega$  is the sliding speed,

$$\Delta\omega = \omega_2 - i_i \omega_3.$$

When the vehicle is running in a particular speed with the clutch fully closed,  $\omega_2 = i_i \omega_3$  and  $\dot{\omega}_2 = i_i \dot{\omega}_3$ , the torque applied on the clutch is obtained by combining the differential equations (3) and (4),

$$T_c = (T_{in} I_3 + T_{out} i_i I_2) / (I_2 i_i^2 + I_3).$$

Where:

$$T_{in} = K_1(\theta_1 - \theta_2) + C_1(\omega_1 - \omega_2);$$

$$T_{out} = K_2(\theta_3 - \theta_4) + C_2(\omega_3 - \omega_4).$$

### 2.3 Vehicle resistance

The vehicle resistance force includes rolling resistance force  $F_r$ , the aerodynamic resistance force  $F_a$ , and the uphill driving force caused by gravity when driving on non-horizontal roads  $F_g$ . The aerodynamic resistance force is approximated by

$$F_a = 0.5\rho_a A_f C_a (v_v + v_a)^2.$$

Where:  $A_f$  is the frontal area of vehicle;  $C_a$  is the aerodynamic drag coefficient;  $v_v$  is the vehicle speed;  $v_a$  is the wind speed;  $\rho_a$  is the density of the ambient air.

The rolling resistance force is often modeled as

$$F_r = m_v g \cos(\beta) c_r(\cdot).$$

Where:  $m_v$  is the vehicle mass;  $g$  is the acceleration due to gravity;  $c_r$  is the rolling friction coefficient;  $\beta$  is the slope angle of road.

Uphill driving force induced by gravity when driving on a non-horizontal road

$$F_g = m_v g \sin(\beta).$$

The torque of resistance to forward is

$$T_r(\cdot) = (F_a + F_r + F_g) r_w.$$

### 3. Dual clutch actuator model

The simplified diagram of a dual clutch actuator is show Figure 3. It consists in the pump (1), the three-way pressure controlled servo valve (2), the accumulator (3), the clutch actuator (4), the disc springs (5) and the dual clutch (6). The gearbox selectors and pressure modulating valves are not shown.

The simplified model of servo valve presented by (Owen 2001)

$$\dot{x}_v = -\frac{1}{\tau_v} x_v + \frac{K_v}{\tau_v} V. \quad (8)$$

Where:  $K_v$  is the valve position gain;  $\tau_v$  is the time constant of the valve; and  $V$  is the control voltage.

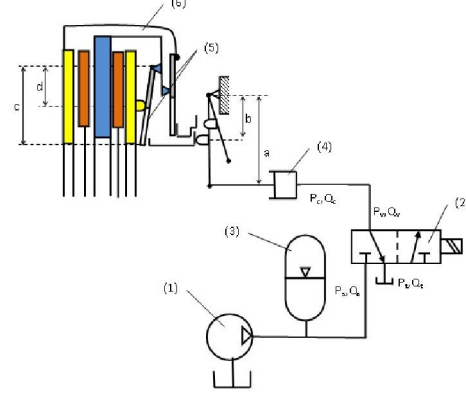


Figure 3. Simplified diagram of a dual clutch actuator

The following equations determine the flow rate through the servo valve

$$Q_v = Q_s - Q_t.$$

Where,  $Q_s, Q_t$  are flow rate in and out, respectively. They can be written according to the Bernoulli equation

$$Q_s = C_d \sqrt{\frac{2}{\rho}} A_f(x_v) \sqrt{|p_s - p_v|} \text{sign}(p_s - p_v),$$

$$Q_t = C_d \sqrt{\frac{2}{\rho}} A_d(x_v) \sqrt{|p_v - p_t|} \text{sign}(p_v - p_t).$$

Where:  $\rho$  is the oil density;  $C_d$  is the discharge coefficient;  $A_f, A_d$  are the filling and dumping orifice areas la surface;  $p_s, p_t$  are the oil pressures of the power supply and of the reservoir, they are assumed constant; and  $p_v$  is the oil pressures at the outlet.

$$A_f = \arccos\left(1 - \frac{x_v - L_d}{r_o}\right) r_o^2 - (r_o - x_v + L_d) \sqrt{r_o^2 - (r_o - x_v + L_d)^2}$$

if  $x_v \in [L_d, 2r_o + L_d]$ , else  $A_f = 0$ .

$$A_d = \arccos\left(1 + \frac{x_v + L_d}{r_o}\right) r_o^2 + (r_o + x_v + L_d) \sqrt{r_o^2 - (r_o + x_v + L_d)^2}$$

if  $x_v \in [-L_d, -2r_o - L_d]$ , else  $A_d = 0$ .

Where:  $r_o$  is the radius of the orifice;  $L_d$  the amplitude of the dead-zone;

$$x_v \in [-2r_o - L_d, 2r_o + L_d].$$

If we neglect the loss of pressure in the pipe. The pressure dynamic equation is written as follow

$$\dot{p}_c = \dot{p}_v = \frac{\beta}{V_0 + x_p A_p} (Q_v - \dot{x}_p A_p).$$

Where:  $\beta$  is the bulk modulus of the oil;  $V_0$  is the minimum volume of the chamber;  $x_p$  is the actuator position; and  $A_p$  is the actuator cross-sectional area.

Finally, the dynamic equation of the actuator position is written as follow

$$\ddot{x}_p = \frac{1}{m_p} \left( p_c A_p - F_f(\dot{x}_p) - \frac{1}{i_1 i_2} F_n(x_p) \right).$$

Where:  $m_p$  is the equivalent mass of the actuator and the pressure plate;  $i_1 = b/a$ ,  $i_2 = d/c$  are the leverage ratios;  $F_n$  is the disc spring force (normal force on clutch face); and  $F_f$  is the force friction. In our study, a model of classical friction force with Stribeck effect is considered

$$F_f(\dot{x}_p) = \left( F_c + (F_s - F_c) e^{-\left(\frac{\dot{x}_p}{v_s}\right)^2} \right) \text{sign}(\dot{x}_p).$$

Where:  $F_c, F_s$  is the Coulomb and Stribeck friction force, respectively;  $v_s$  is the Stribeck velocity.

The disc spring force  $F_n$  is determined by the formula of Almen and Láaszló (Almen and Laszlo 1936)

$$F_n(s) = \frac{4E}{(1-\mu^2)KD_e^2} s \left( (h-s)(h-\frac{s}{2})t + t^3 \right).$$

With

$$K = \frac{1}{\pi} \frac{\left(\frac{\delta-1}{\delta}\right)^2}{\delta-1 - \ln \delta}, \quad \delta = \frac{D_e}{D_w}.$$

Where:  $E$  is the Young's modulus, [N/m<sup>2</sup>];  $D_e, D_w$  are the outside diameter and the diameter at the root of slots, [mm];  $h$  is the cone height, [mm];  $t$  is the thickness of individual, [mm];  $\mu$  is the Poisson's ratio;  $s$  is the axial deformation of the cone respective to its unconstrained height, [mm]. If we neglect the deformation of the lever

$$s = 1000 \frac{1}{i_1 i_2} x_p \text{ [mm]}.$$

## 4. Launch control design

In this section, the control law for clutch during engagement is developed based on sliding control methodology (Slotine and Li 1991). The goal of the control is to minimize the jerk, the oscillations during and after synchronization and to avoid the dead zone of the engine, while limiting the clutch slip time. In addition, the control law allows to take into account both road conditions and model parametric variations.

### 4.1 Reference trajectory of engine speed

The normal force applied to the clutch disk, and consequently the engine speed could drop too severely resulting in engine stall. To avoid the dead zone of the engine, the engine speed must be controlled. The reference trajectory of the engine during clutch engagement is presented in some works (Ni, Lu, and Zhang 2009), (Dassen 2003), (Amari, Alamir, and Tona 2008). Dassen et al (Dassen 2003) define the reference trajectory for the engine speed as a

decreasing linear function satisfying the no-kill condition,  $\omega_1 > \omega_1^{idle}$ .

In the standing start and gear shifting phase, Amari et al (Amari, Alamir, and Tona 2008) define the reference speed of the engine at least at the idle speed set-point, unless the desired torque exceeds the maximum torque. This ensures that the engine is no-kill during standing start.

To avoid sudden change of engine speed, the initial values of the reference trajectory for engine speed and its differential must be equal to the real one

$$\omega_1^{ref}(t_0) = \omega_1(t_0), \quad (9)$$

$$\dot{\omega}_1^{ref}(t_0) = \dot{\omega}_1(t_0). \quad (10)$$

Where,  $t_0$  is the time to begin launch.

The clutch slipping time is limited to a sufficiently short time, we propose the hypothesis that during the engagement, the throttle position  $p(t)$ , which is controlled by the driver, increases linearly. With this hypothesis, we can determine the throttle position at the time of the lock-up point  $t_f$

$$p(t_f) = p(t_0) + \dot{p}(t_0)(t_f - t_0).$$

The engine speed at that time  $\omega_1^{ref}(t_f)$  is chosen so that the torque is stored for the maximum possible acceleration and so that the reference trajectory for the engine speed is always above the engine idle speed  $\omega_1^{idle}$

$$\omega_1^{ref}(t_f) = \left\{ \begin{array}{l} T_e^{-1} \left( \max \{ T_e(\omega_1, p(t_f)) \}, p(t_f) \right) \\ \left| \min \{ \omega_1^{ref}(t) \} > \omega_1^{idle} \right. \end{array} \right\}. \quad (11)$$

Ideally, at the time of lock-up point, there are no difference of acceleration between the flywheel and the clutch drum, and between the final drive and the wheel

$$\dot{\omega}_1(t_f) = \dot{\omega}_2(t_f) = i_i \dot{\omega}_3(t_f) = i_i \dot{\omega}_4(t_f). \quad (12)$$

Combining equations (2), (3), (4), (5) and (12) with the engine torque control  $T_e^c(t_f) = 0$ , we have

$$\dot{\omega}_1^{ref}(t_f) = \frac{T_e^d(t_f) - T_r(t_f)}{I_1 + I_2 + (I_3 + I_4)/i_i^2}. \quad (13)$$

Where,  $T_e^d(t_f)$  is the engine torque coming from the driver request

$$T_e^d(t_f) = T_e(\omega_1(t_f), p(t_f)).$$

To pre-calculate the trajectory for the engine speed, the resistant torque  $T_r$  and the equivalent moment of inertia of the vehicle  $I_4$  are needed. The resistant torque is a nonlinear function which depends on a priori unknown parameters such as the number of passengers, vehicle speed, tire pressure, and road surface conditions.

In normal conditions, those coefficients are bounded

$$c_r^{min} \leq c_r \leq c_r^{max},$$

thus,

$$T_r^{min} \leq T_r \leq T_r^{max}.$$

With

$$T_r^{min} = 0.5 \rho_a A_f C_d (v_v + v_a)^2 r_w + m_v^{min} g \cos(\beta) c_r^{min} r_w + m_v^{min} g \sin(\beta) r_w,$$

$$T_r^{max} = 0.5 \rho_a A_f C_d (v_v + v_a)^2 r_w + m_v^{max} g \cos(\beta) c_r^{max} r_w + m_v^{max} g \sin(\beta) r_w.$$

We consider, the average value defined by

$$\hat{T}_r = 0.5 (T_r^{min} + T_r^{max}).$$

On the other hand, the moment of inertia of the vehicle  $I_4$  depends on the vehicle mass. The vehicle mass is an uncertain parameter but also bounded

$$m_v^{min} \leq m_v \leq m_v^{max},$$

thus

$$I_4^{min} \leq I_4 \leq I_4^{max}.$$

With

$$I_4^{min} = n_w I_w + m_v^{min} r_w^2,$$

$$I_4^{max} = n_w I_w + m_v^{max} r_w^2.$$

As previously, we define the average moment of inertia of the vehicle as

$$\hat{I}_4 = 0.5(I_4^{max} + I_4^{min}).$$

Then,  $\hat{T}_r$  and  $\hat{I}_4$  instead of  $T_r$  and  $I_4$  are used to compute  $\dot{\omega}_1^{ref}(t_f)$

$$\dot{\omega}_1^{ref}(t_f) = \frac{T_e^d(t_f) - \hat{T}_r(t_f)}{I_1 + I_2 + (I_3 + \hat{I}_4)/i_i^2}. \quad (14)$$

Different choices are possible to define a trajectory satisfying the constraints (9), (10), (11) and (14). In this study, a polynomial of degree 3 is chosen to deal with the requirements.

#### 4.2 Reference trajectory of the clutch slip speed

The clutch engagement process should ensure driving comfort and minimize the dissipated friction energy. In general, if engagement duration is limited in a suitable short time, the dissipated energy will not be too important. As to the driving comfort, because the sudden change of torque due to clutch lock-up tends to cause an undesired vehicle jerk just after the synchronization of clutch, the clutch engagement should satisfy the so called no-lurch condition introduced by Glielmo and Vasca (Glielmo and Vasca 2000). This condition can be characterized as the rotational acceleration of clutch input shaft should be equal to the one of the output shaft at the lock-up point,  $\dot{\omega}_2 - i_i \dot{\omega}_3$ .

Therefore, the reference trajectory for the clutch slip speed must satisfy the following requirements: the clutch slipping should be finished after the selected time  $\Delta t = t_f - t_0$

$$\Delta \omega_{ref}(t_f) = \omega_2(t_f) - i_i \omega_3(t_f) = 0;$$

at the lock up point, sliding acceleration must equal to zero

$$\Delta \dot{\omega}_{ref}(t_f) = \dot{\omega}_2(t_f) - i_i \dot{\omega}_3(t_f) = 0;$$

and under the initial conditions

$$\Delta \omega_{ref}(t_0) = \omega_2(t_0) - i_i \omega_3(t_0) = \omega_2(t_0),$$

$$\Delta \dot{\omega}_{ref}(t_0) = \dot{\omega}_2(t_0) - i_i \dot{\omega}_3(t_0) = \dot{\omega}_2(t_0).$$

Similarly to above, a polynomial of degree 3 is chosen.

#### 4.3 Engine speed control

The engine torque is divided into two parts, the first part which is the one coming from the driver request  $T_e^d$  and the second part which is considered as a control input  $T_e^c$ . Rewriting the dynamic equation of the engine speed, (equation (2)), the following equation is obtained

$$\dot{x}_1 = f_1(\cdot) + g_1 u_1. \quad (15)$$

With

$$x_1 = \omega_1, \quad g_1 = \frac{1}{I_1}, \quad u_1 = T_e^c,$$

$$f_1(\cdot) = \frac{1}{I_1} (T_e^d - K_1(\theta_1 - \theta_2) - C_1(\omega_1 - \omega_2)).$$

In order for the engine speed to track the reference trajectory  $x_{1r}$ , a sliding surface  $S_1$  is defined by the following integral structure

$$\begin{aligned} S_1(t) &= \left( \frac{d}{dt} + \lambda \right) \int_0^t e_1(\tau) d\tau \\ &= e_1(t) + \lambda \int_0^t e_1(\tau) d\tau. \end{aligned}$$

Where:  $\lambda$  is a strictly positive constant;  $e_1$  is the tracking error,  $e_1(t) = x_1(t) - x_{1r}(t)$ .

The derivative of the sliding surface is given by

$$\begin{aligned} \dot{S}_1(t) &= \dot{e}_1(t) + \lambda e_1(t) \\ &= f_1(\cdot) + g_1 u_1(t) - \dot{x}_{1r} + \lambda e_1(t). \end{aligned}$$

If the sliding regime is perfect, the operating point moves on the sliding surface  $S_1 = 0$  and satisfies  $\dot{S}_1 = 0$  as the function  $S_1$  is constant. The best approximation  $\hat{u}_1$  a continuous control law that would achieve  $\dot{S}_1 = 0$  is thus,

$$\hat{u}_1(t) = g_1^{-1} (-f_1(\cdot) + \dot{x}_{1r} - \lambda e_1(t)).$$

We add to  $\hat{u}_1$  a term discontinuous across the surface  $S_1 = 0$ . The control law is defined as

$$u_1(t) = \hat{u}_1(t) - g_1^{-1} k \text{sign}(S_1).$$

Where,  $k$  is a strictly positive constant.

#### 4.4 Clutch slip control

By combining equations (3) and (4), the dynamic equation for the clutch slip control is

$$\dot{x}_2 = \frac{1}{I_2} T_{in}(\cdot) + \frac{i_i}{I_3} T_{out}(\cdot) - \left( \frac{1}{I_2} + \frac{i_i^2}{I_3} \right) T_c. \quad (16)$$

Where,  $x_2 = \Delta\omega$ .

By combining equations (7) and (16), we have

$$\begin{aligned} \dot{x}_2 &= \frac{1}{I_2} T_{in}(\cdot) + \frac{i_i}{I_3} T_{out}(\cdot) \\ &- 2n_d r_c \left( \frac{1}{I_2} + \frac{i_i^2}{I_3} \right) \text{sign}(x_2) F_n(x_p) \mu(x_2). \end{aligned}$$

To develop a control law, the dynamic model of the valve is simplified. The time constant of the valve ( $\tau_v$ ) is very small compared with the time constants of other components of the control system. Therefore, the dynamic of the valve is neglected, the dynamic equation (8) becomes the algebraic equation,

$$x_v = K_v V. \quad (17)$$

The main differential equations to develop the control law are

$$\begin{aligned} \dot{x}_2 &= \frac{1}{I_2} T_{in}(\cdot) + \frac{i_i}{I_3} T_{out}(\cdot) \\ &- 2n_d r_c \left( \frac{1}{I_2} + \frac{i_i^2}{I_3} \right) \text{sign}(x_2) F_n(x_p) \mu(x_2), \end{aligned} \quad (18)$$

$$\ddot{x}_p = \frac{1}{m_p} \left( p_c A_p - F_f(\dot{x}_p) - \frac{1}{i_1 i_2} F_n(x_p) \right), \quad (19)$$

$$\dot{p}_c = \frac{\beta}{V_0 + x_p A_p} (Q_v(V, p_v) - \dot{x}_p A_p). \quad (20)$$

We derive the equation (18) three times, then the control  $u_2 = V$  appear on the dynamic equation of the clutch slip.

So neglecting the dead zone of the valve ( $2L_d = 0$ ) and limiting  $x_v \in [-2r_o, 2r_o]$ , the function  $Q_v = Q(V, p_v)$  is monotonically increasing function of  $V$ . So, it exists an inverse function  $V = Q^{-1}(Q_v, p_v)$ . The dynamic equation of the clutch slip can be rewritten as follow

$$x_2 = f_2(\cdot) + g_2(\cdot) v. \quad (21)$$

With  $v$  is the new input,

$$v = Q_v(V, p_v).$$

In order to have the engine speed track the reference trajectory  $x_{2r}$ , a sliding surface  $S_2$  is defined by

$$\begin{aligned} S_2(t) &= \left( \frac{d}{dt} + \lambda \right)^{(3)} e_2(t) \\ &= e_2(t) + 3\lambda \dot{e}_2(t) + 3\lambda^2 \ddot{e}_2(t) + \lambda^3 e_2(t). \end{aligned}$$

Where:  $\lambda$  is a strictly positive constant;  $e_2$  is the tracking error  $x_2$ ,

$$e_2(t) = x_2(t) - x_{2r}(t).$$

The dynamic of the sliding surface is

$$\dot{S}_2(t) = e_2 + 3\lambda \dot{e}_2 + 3\lambda^2 \ddot{e}_2 + \lambda^3 \dot{e}_2.$$

As previously, a control law of that form is introduced

$$v(t) = \hat{v}(t) - g(\cdot)^{-1} k \text{sign}(S_2).$$

With

$$\hat{v}(t) = g(\cdot)^{-1} \begin{pmatrix} x_{2r}(t) - f_2(\cdot) - 3\lambda \dot{e}_2(t) \\ -3\lambda^2 \ddot{e}_2(t) - \lambda^3 \dot{e}_2(t) \end{pmatrix}.$$

Where  $k$  is a strictly positive constant.

$$u_2(t) = V(t) = Q^{-1}(v(t), p_v(t)).$$

#### 4.5 Control strategy

The reference trajectory for the engine speed is calculated with the previous



hypotheses and taking into account parameter uncertainties. Therefore, the engine torque control is not zero at the lock-up point. To ensure that the driver take control of the vehicle, after closing, it is controlled to zero by another strategy. After synchronization, the normal force is also controlled by a strategy to a set value. The set value ensures the complete closure of the clutch and allows avoiding an over load in the powertrain.

To show the effectiveness of the proposed control law in the engagement phase, the dynamic model of the powertrain is implemented in Matlab/Simulink and simulated. Different tests with parametric  $m_v$  variations have been realized and are presented in Figure 4 to Figure 8. Begin of standing starts at 1 second and duration is limited to 2.5 seconds. The simulation results show that the engine speed is well tracking the reference trajectory (Figure 4) while the parameters are variables (9.4% of the vehicle mass), the clutch slip time about 2.5 second (Figure 7). Figure 6 show the behavior of the engine with and without engine controller. In the case without engine controller, the engine speed drops quickly to the dead zone. The vehicle jerk is shown in Figure 8, it depends on the engagement time but does not depend on driver behavior during the engagement.

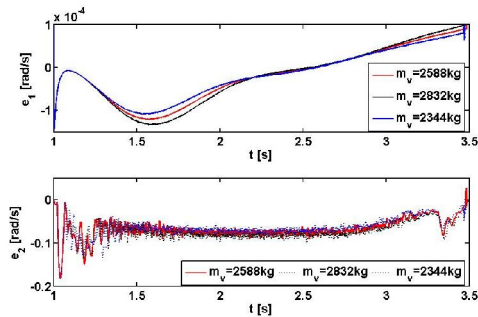


Figure 4. Trajectory tracking error  $e_1$  and  $e_2$

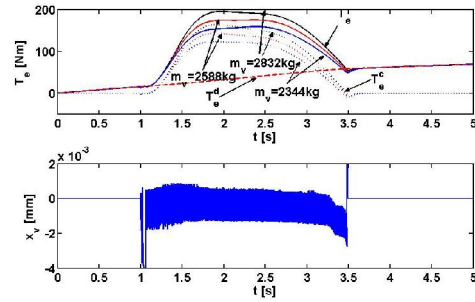


Figure 5. Engine torque driving  $T_e^d$ , engine torque control  $T_e^c$  and position of valve  $x_v$

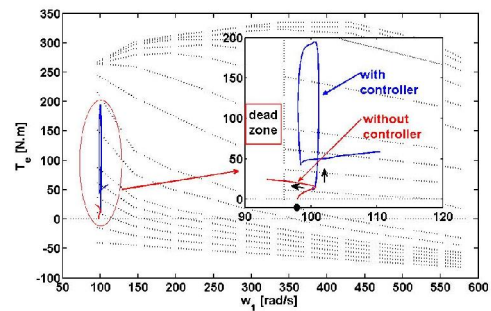


Figure 6. Engine behavior with engine control and without engine control

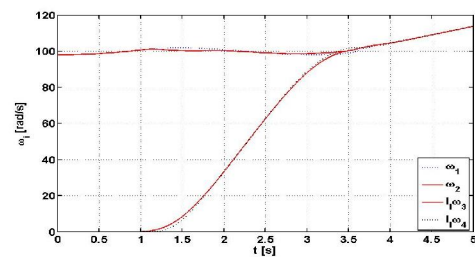


Figure 7 Angular speed of each element of the simplified model

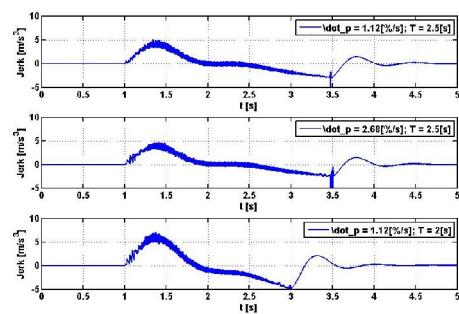


Figure 8. Vehicle jerk

## 5. Conclusion

A nonlinear model of the powertrain has been proposed. From the simplified model, a control law based on the sliding mode control methodology to ensure robust with respect to model parametric variations and road condition, has been developed. Finally, the simulation results applied in the case of standing start showed the efficiency of the proposed approach.

## References

- Almen, J.O., and A. Laszlo. 1936. "The Uniform-Section Disk-Spring." *Transactions of the American Society of Mechanical Engineers* 58, S: 305/314.
- Amari, R., M. Alamir, and P. Tona. 2008. "Unified MPC Strategy for Idle-speed Control, Vehicle Start-up and Gearing Applied to an Automated Manual Transmission." In *Proceedings of the 17th World Congress, The International Federation of Automatic Control, Seoul, Korea, July 6-11*.
- Canudas de Wit, Carlos, H.Olsson, K.J.Astrom, and P.Lischinsky. 1995. "A New Model for Control of Systems with Friction." *IEEE Transactions on Automatic Control* 40: 419–424.
- Dassen, M.H.M. 2003. *Modelling and Control of Automotive Clutch Systems*. Department of mechanical engineering TU/e Eindhoven.
- Dolcini, Pietro. 2006. "Contribution Au Confort De L'embrayage". Universite de Grenoble.
- Dolcini, Pietro, Carlos Canudas de Wit, and H Bechart. 2007. "Observer-Based Optimal Control of Dry Clutch Engagement." *Oil & Gas Science and Technology* 62: 615–621.
- Dolcini, Pietro, Carlos Canudas de Wit, and Hubert Bechart. 2008. "Lurch Avoidance Strategy and Its Implementation in AMT Vehicles." *Mechatronics* 18: 289–300.
- Gao, Bingzhao, Hong chen, Yan Ma, and Kazushi Sanada. 2009. "Clutch Slip Control of Automatic Transmission Using Nonlinear Method." In *Joint 48th IEEE Conference on Decision and Control And*.
- Garofalo, Franco, Luigi Glielmo, Luigi Iannelli, and Francesco Vasca. 2002. "Optimal Tracking for Automotive Dry Clutch Engagement." In *IFAC, 15th Triennial World Congress, Barcelona, Spain*.
- Glielmo, L., and F. Vasca. 2000. "Optimal Control of Dry Engagement." *SAE 2000-01-0837*.
- Heijden, A. C. Van Der, A. F. A. Serrarens, M. K. Camlibel, and H. Nijmeijer. 2007. "Hybrid Optimal Control of Dry Clutch Engagement." *International Journal of Control* 80: 1717–1728.
- Horna, Joachim, Joachim Bamberger, Peter Michau, and Stephan Pindl. 2003. "Flatness-based Clutch Control for Automated Manual Transmissions." *Control Engineering Practice* 11: 1353–1359.
- Kim, Jinsung, and Seibum B. Choi. 2010. "Control of Dry Clutch Engagement for Vehicle Launches via a Shaft Torque Observer." In *American Control Conference, Marriott Waterfront, Baltimore, MD, USA*.
- Kulkarni, Manish, Taehyun Shim, and Yi Zhang. 2007. "Shift Dynamics and Control of Dual-clutch Transmissions." *Mechanism and Machine Theory* 42: 168–182.
- Lucente, G., M. Montanari, and C. Rossi. 2007. "Hybrid Optimal Control of an Automated Manual Transmission System." In *Seventh IFAC Symposium on Nonlinear Control Systems*.
- Ni, Chunsheng, Tongli Lu, and Jianwu Zhang. 2009. "Gearshift Control for Dry Dual-clutch Transmissions." *Issn* 8: 1109–2777.
- Owen, William Scott. 2001. "An Investigation into the Reduction of Stick-slip Friction in Hydraulic Actuators". The University of British Columbia.
- Slotine, Jean-Jacques E., and Weiping Li. 1991. *Applied Nonlinear Control*. Printice Hall, Inc.
- Wu, M. X., J. W. Zhang, T. L. Lu, and C.S. Ni. 2010. "Research on Optimal Control for Dry Dual-clutch Engagement During Launch." *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering* 224: 749–763.