Fuzzy Stochastic Goal Programming Problems

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Abstract

In this paper, we present a model to measure attainment value of fuzzy stochastic goals. Then, the new measure is used to de-randomize and de-fuzzify the fuzzy stochastic goal programming problem and obtain a standard linear program (LP). A numerical example is provided to illustrate the proposed method.

Key words: fuzzy random variables, fuzzy stochastic goal programming, attainment value.

1. Introduction

Solving fuzzy stochastic optimization problems has attracted more attention in recent years. Stochastic and fuzzy aspects are combined to provide an efficient tool to describe real-life problems where uncertainty and imprecision of information co-occur. However, this kind of combination creates a great challenge for the researcher to find an efficient solution method to deal with both fuzzy and stochastic terms. The general strategy is to de-fuzzify and/or de-randomize fuzzy random variables to convert the problem into a deterministic problem. The first direction is to perform the conversions (de-fuzzify, de-randomize) in a sequential manner (Luhandjula, 1996; Luhandjula and Gupta, 1996; Luhandjula, 2004). The second way is to perform both actions at the same time by calculating the expected value of fuzzy random variables. (Liu 2001a, 2001b; Liu and Liu, 2002; Liu and Liu 2003). The obtained LP from Luhandjula’s approaches can be solved directly by the traditional optimization packages such as LINGO, CPLEX. However, in some applications one has to deal with more than one objective function. In addition, the discrete process of fuzzy sets via α-levels in Luhandjula’s approaches creates a quite large number of constraints and variables. Although Liu, and Mohan and Nguyen (2001) have considered the case of fuzzy stochastic multiple objectives problems, the obtained results are still limited, especially for the case of fuzzy stochastic goal programming problems. Although the works of Liu proposed a simple expected value model of fuzzy random variables, the computation process of these expected values is quite complex and time consuming. The mix fuzzy stochastic programming (MFSP) problem, where fuzzy objectives/constraints and stochastic constraints are separated, is taken into account by Mohan and Nguyen (2001). Mohan and Nguyen’s approach converts the objectives and constraints into fuzzy sets to formulate the MFSP problem as a max-min-type deterministic single-objective optimization problem using Bellman-Zadeh’s min operator. The obtained deterministic is solved by an interactive procedure.
The more complex case of fuzzy random variables of fuzzy stochastic optimization problems has been neglected.

In this paper, we present a method to solve the fuzzy stochastic goal programming problem in which the objectives/constraints coefficients and goals are fuzzy random variables. In real-life, fuzzy stochastic goal programming arises in several situations. Goals depend on decision-makers’ perspective and vary with time due to related factors. They are often stated as “some what larger than goal $g_1$ with $p_1$ of achievement”, “substantially lesser than goal $g_2$ with probability of $p_2$” or “around goal $g_3$ with $p_3$% of time”. These are fuzzy stochastic goals. The other parameters, the right-hand-sides (RHSs) and coefficients of objectives and constraints, could also be fuzzy random variables due to the fact that they depend on many factors. Thus, they are difficult to determine at exact values. Moreover, the factors are fluctuating due to uncertain environment make these parameters varying. These circumstances often happen in long-term planning, development strategies (Luhandjula and Gupta, 1996); engineering design (Shih and WangsaWidjaja, 1996), and financial modeling (Zmeskal, 2005), in which the described conditions (goals, objectives, constraints, coefficients) cannot be determined precisely and certainly. An illustrated example of fuzzy stochastic goal programming could be the case of the production planning problem. Two objectives of minimizing total cost and of production output should be achieved at least at an estimated level. The goals of these objectives may be stated as “substantially lesser than $100,000$ with probability of $90\%$” and “some what larger than $20,000$ units with $95\%$ of achievement”. These goals can be expressed as fuzzy stochastic variables because total cost includes costs due to inventory holding, materials, and operation cost; and production output depends on process parameters (cutting speeds, feed rates, etc.) and equipment running time which are fluctuating and hard to estimate precisely. In addition, resources available, demand, and constraints’ coefficients can be modeled as fuzzy random variables because the vague perceptions with hard statistical data in several environment conditions such as seasonal factors, market prices, and suppliers. Another example is in the case of preventive maintenance. Equipment breaks down from time to time, causing a loss in production output. To reduce the breakdowns, preventive maintenance can be performed. Preventive works include inspection, repair, and/or replacement of components if necessary. This work costs money in terms of materials, wages, and loss of production due to downtime for preventive work. This downtime is uncertain due to the complexity of inspection, repair, and replacement jobs. The problem is to determine the preventive frequency which minimizes the downtime due to breakdowns and down time due to preventive maintenance and their associated costs. Here, the running time of the equipment is also an uncertainty in real situations. Therefore, we would rather consider these times and their associated costs as fuzzy random variables. Hence, the goals set for such objectives can also be expressed as fuzzy random variables, say, “downtime due to breakdown is about $2$ hours within $95\%$ of planning horizon”, “downtime due to preventive maintenance is some what less than $0.5$ hours within $95\%$ of planning horizon”. These are the examples that motivate us to propose a new model for solving fuzzy stochastic goal problems in this paper by expressing the attainment values of fuzzy stochastic goals. The paper is organized as follows. First, some important results of fuzzy random variable are summarized as a foundation of our development. Then, the
attainment model is developed in Section 3. Section 4 will utilize the attainment value to convert the fuzzy stochastic goal programming problem into the traditional LP. A numerical example is also provided to illustrate the proposed method. Finally, the paper is concluded in Section 5.

2. Fuzzy random variable

In this section, we will summarize some important concepts and results of fuzzy random variables as a basis for our development. There are several types of definition of fuzzy random variable. Here, we restrict our attention to the results of Luhandjula (1996) for the definition of fuzzy random variable and its characteristics.

**Definition 1** (Luhandjula, 1996): Consider a probability space \((\Omega, \mathcal{F}, P)\), A fuzzy random variable on this space is a fuzzy set \(-valued mapping:\)

\[
\tilde{X} : \Omega \rightarrow F_0(\mathbb{R})
\]

\[w \mapsto \tilde{X}_w\]

such that for any Borel set \(B\) and for every \(\alpha \in (0,1)\)

\[
\tilde{X}_\alpha^{-1}(B) = \{w \in \Omega | \tilde{X}_w^\alpha \subset B\} \in \mathcal{F}
\]

where \(F_0(\mathbb{R})\) and \(\tilde{X}_w^\alpha\) stand for the set of fuzzy numbers with compact supports and the \(\alpha\)-level set of the fuzzy set \(\tilde{X}_w\), respectively.

**Theorem 1** (Luhandjula, 1996): \(\tilde{X}\) is a fuzzy random variable if and only if given \(w \in \Omega\), \(\tilde{X}_w^\alpha\) is a random interval \(\forall \alpha \in (0,1]\).

3. Fuzzy random variable goal attainment values

**Definition 2:** The lower-side attainment index of fuzzy random variable \(\tilde{P}\) to fuzzy random variable \(\tilde{Q}\), \(\tilde{P} \leq \tilde{Q}\) is defined as

\[
D(\tilde{P}, \tilde{Q}) = \int_0^1 \max \left\{ 0, \sup_{s \in \mathbb{R}} \{ \tilde{P}_w(s) \geq \alpha \} - \inf_{r \in \mathbb{R}} \{ \tilde{Q}_w(r) \geq \alpha \} \right\} d\alpha
\]  

(2)

The lower-side attainment index \(D(\tilde{P}, \tilde{Q})\) is measured as the areas of fuzzy random variable \(\tilde{P}\) overlapping to fuzzy random variable \(\tilde{Q}\), if \(\tilde{P} \leq \tilde{Q}\), see Figure 1. This area is a nonnegative value. In this area, \(\tilde{P} = \tilde{Q}\). An extension of \(D(\tilde{P}, \tilde{Q})\) measure is the new concept of both-side attainment value \(V\) is introduced in the following definition.

**Definition 3:** The both-side attainment index of fuzzy random variable \(\tilde{P}\) to fuzzy random variable \(\tilde{Q}\) is defined as follows.

\[
V(\tilde{P}, \tilde{Q}) = \max \{ 0, \min(D(\tilde{P}, \tilde{Q}), D(\tilde{Q}, \tilde{P})) \}
\]  

(3)

Here, \(V\) could be understood as the matching degree between two fuzzy numbers. If \(\tilde{P} \leq \tilde{Q}\), then we have \(\sup_{s \in \mathbb{R}} \{ \tilde{P}_w(s) \geq \alpha \} < \sup_{r \in \mathbb{R}} \{ \tilde{Q}_w(r) \geq \alpha \}\) and
Thus, \( D(\tilde{P}, \tilde{Q}) < D(\tilde{Q}, \tilde{P}) \) and
\( V(\tilde{P}, \tilde{Q}) = D(\tilde{P}, \tilde{Q}) \). Otherwise, if \( \tilde{P} \geq \tilde{Q} \), \( D(\tilde{P}, \tilde{Q}) > D(\tilde{Q}, \tilde{P}) \) and \( V(\tilde{P}, \tilde{Q}) = D(\tilde{Q}, \tilde{P}) \).

To facilitate our presentation, let \( T \) be a collection of triangular fuzzy numbers (\( T \)-numbers) with the following membership function

\[
T = \{ \tilde{t}; \tilde{t} = (t, a, b), a, b \geq 0 \} \text{ and } \mu_t(x) = \begin{cases} 
\max \left( 0, 1 - \frac{t - x}{a} \right), & \text{if } x \leq t \\
1, & \text{if } a = 0, b = 0, t = x \\
\max \left( 0, 1 - \frac{x - t}{b} \right), & \text{if } x > t \\
0, & \text{otherwise}
\end{cases}
\]

where the scalars \( a, b \geq 0 \) \((a, b \in \mathbb{R})\) are called the left and right spreads, respectively.

For any \( \alpha \in (0, 1] \), let
\[
P^\alpha_w(x) = \inf \{ x \in \mathbb{R} \mid \tilde{P}_w(x) \geq \alpha \} \text{ and } P^\alpha_w(w) = \sup \{ x \in \mathbb{R} \mid \tilde{P}_w(x) \geq \alpha \}
\]
and
\[
Q^\alpha_w(x) = \inf \{ x \in \mathbb{R} \mid \tilde{Q}_w(x) \geq \alpha \} \text{ and } Q^\alpha_w(w) = \sup \{ x \in \mathbb{R} \mid \tilde{Q}_w(x) \geq \alpha \}
\]

It is clear that the \( \alpha \)-cut of the fuzzy sets \( \tilde{P}_w, \tilde{Q}_w \) are:

\[
\tilde{P}_w^\alpha = \left[ P^\alpha_w(w), P^\alpha_w(w) \right] \quad \tilde{Q}_w^\alpha = \left[ Q^\alpha_w(w), Q^\alpha_w(w) \right]
\]

By Theorem 1, these intervals are random intervals. If \( \tilde{P}_w, \tilde{Q}_w \) are \( T \)-numbers that can be represented as \( \tilde{P}_w = (u(w), a(w), b(w)), \tilde{Q}_w = (v(w), c(w), d(w)) \in T \) , the \( \alpha \)-cut of \( \tilde{P}_w, \tilde{Q}_w \) and the lower-side attainment index of \( \tilde{P} \) to \( \tilde{Q} \) at \( \alpha \)-level are (see Figure 1 and definition of \( T \)-numbers)

\[
\tilde{P}_w^\alpha = \left[ P^\alpha_w(w), P^\alpha_w(w) \right] = [u(w) - a(w)(1 - \alpha), u(w) + b(w)(1 - \alpha)] \\
\tilde{Q}_w^\alpha = \left[ Q^\alpha_w(w), Q^\alpha_w(w) \right] = [v(w) - c(w)(1 - \alpha), v(w) + c(w)(1 - \alpha)]
\]

\[
D(\tilde{P}, \tilde{Q})_\alpha = \max \left\{ 0, P^\alpha_w(w) - Q^\alpha_w(w) \right\}
\]

From these definitions, we have the following result:

**Proposition 1.** Consider two triangular fuzzy random numbers \( \tilde{P}, \tilde{Q} \) such that \( \tilde{P} \leq \tilde{Q} \), the average lower-side attainment index of \( \tilde{P} \) to \( \tilde{Q} \) is

\[
\overline{D}(\tilde{P}, \tilde{Q}) = \frac{u(w) - v(w) + b(w) + c(w)}{2}
\]
Figure 1. Lower-side attainment degree of $\tilde{P}$ to $\tilde{Q}$ at $\alpha$-level

$$D(\tilde{P}, \tilde{Q}) = \max \{0, P^w_\alpha(w) - Q^i_\alpha(w)\} = \max \{0, (u(w) - v(w)) + (b(w) + c(w))(1 - \alpha)\}$$

$D(\tilde{P}, \tilde{Q}) = \frac{1}{\lambda} \max \{0, (u(w) - v(w)) + (b(w) + c(w))(1 - \alpha)\} d\alpha$

$D(\tilde{P}, \tilde{Q}) = \frac{1}{\lambda} \int_0^{\lambda} [(u(w) - v(w)) + (b(w) + c(w))(1 - \alpha)] d\alpha$

$D(\tilde{P}, \tilde{Q}) = \frac{1}{\lambda} \int_0^{\lambda} [(u(w) - v(w)) + (b(w) + c(w))(1 - \alpha)] d\alpha$

From (3) and (4), the average both-side attainment index of $\tilde{P}$ to $\tilde{Q}$ is defined as

$$\bar{V}(\tilde{P}, \tilde{Q}) = \max \left\{0, \min \left\{ \frac{u(w) - v(w) + b(w) + c(w)}{2}; \frac{v(w) - u(w) + a(w) + d(w)}{2} \right\} \right\}$$

From (5) we derive the following result

Proposition 2

Let $\tilde{P}, \tilde{Q}, 0 < \lambda(w), w \in \Omega$. Then

$$\bar{V}(\tilde{P}, \tilde{Q}) \geq \lambda(w)$$

iff

$$\bar{D}(\tilde{P}, \tilde{Q}) = \frac{u(w) - v(w) + b(w) + c(w)}{2} \geq \lambda(w)$$

(7a)

$$\bar{D}(\tilde{Q}, \tilde{P}) = \frac{v(w) - u(w) + a(w) + d(w)}{2} \geq \lambda(w)$$

(7b)

Proof. The proof follows immediately from (5). □
4. Fuzzy Stochastic Goal Programming

Consider the following fuzzy stochastic goal program

(P1): \[ (\tilde{c}_k)_w x_i, (\tilde{g}_k)_w, k = 1, l \]  

Subject to

\[ \sum_{j=1}^{n} (\tilde{a}_{ij})_w x_j \leq (\tilde{h})_w \] \hspace{1cm} (9)
\[ x_j \geq 0; w \in \Omega; i = 1, 2, ..., m; j = 1, 2, ..., n; k = 1, 2, ..., l \]

where \[ A, b \] are \[ m \times n \] and \[ m \times l \] matrices of constraint coefficients, \[ (\tilde{c}_k)_w \] is \[ 1 \times n \] matrix of fuzzy random coefficients, and \[ (\tilde{g}_k)_w \] are given fuzzy random goals required to maximally satisfy from both sides, i.e. if \[ (\tilde{c}_k)_w x \leq (\tilde{g}_k)_w \], the lower attainment values should be maximized; otherwise, if \[ (\tilde{c}_k)_w x \geq (\tilde{g}_k)_w \], the upper attainment values should be maximized. From such meaning, the problem (P1) is reformulated as follows.

\[ \text{Max } \lambda_t (w) \]
\[ \text{s.t.} \]

\[ (P2) \]
\[ V \left[ \sum_{j=1}^{n} (\tilde{c}_{jk})_w x_j, (\tilde{g}_k)_w \right] \geq \lambda_t (w) \] \hspace{1cm} (10)
\[ \sum_{j=1}^{n} (\tilde{a}_{ij})_w x_j \leq (\tilde{h})_w \] \hspace{1cm} (11)
\[ x_j \geq 0; w \in \Omega; i = 1, 2, ..., m; j = 1, 2, ..., n; k = 1, 2, ..., l \]

Here, the additional set of constraints (10) expresses that the average both-side attainment should be maximized. We continue to apply the lower-side attainment index to the constructive constraints because we always want to minimize the achievement of the Left-Hand-Side (LHS) to the Right-Hand-Side (RHS) to avoid any violation of constructive constraints (11). Applying the results of Proposition 2, we have:
The stochastic programming problem (P3) can be solved by many techniques. One of the recommended methods is the flexible programming approach reviewed by Luhandjula and Gupta (1996). To simplify our presentation, we continue our stream of transformations to convert (P3) to a deterministic linear program. Here the objective function of (P3) is put into the form

\[ \lambda_1(w) - \lambda_2(w) \geq \lambda_0 \]

Then, the (P3) is converted to the following form:

\[
\begin{aligned}
\text{Max} & \quad \lambda_0 \\
\text{s.t.} & \quad \lambda_1(w) - \lambda_2(w) \geq \lambda_0 \\
(P4) & \quad \bar{D} \left( \sum_{j=1}^{n} (\tilde{c}_{jk}) w x_j, (\tilde{g}_k) w \right) \geq \lambda_1(w) \\
& \quad \bar{D} \left( (\tilde{g}_k) w, \sum_{j=1}^{n} (\tilde{c}_{jk}) w x_j \right) \geq \lambda_1(w) \\
& \quad \bar{D} \left( \sum_{j=1}^{n} (\tilde{a}_{ij}) w x_j, (\tilde{b}_j) w \right) \leq \lambda_2(w) \\
x_j \geq 0; w \in \Omega; i = 1, 2, \ldots, m; j = 1, 2, \ldots, n; k = 1, 2, \ldots, l
\end{aligned}
\]

The two-stage programming approach is often used to solve (P4) by assigning a penalty cost for any violation of inequalities. Let \( p_0 \), \( p_k \) and \( p_i \) be unit penalty cost of the violation between LHS and RHS of (15), (16), (17) and (18) constraints, respectively. (P4) is equivalent to the following problem
\[
\begin{aligned}
\text{Max } & \left\{ \lambda_0 - E[p_0 y_0(w)] - E \left[ \sum_{k=1}^{l} p_k y_k^+(w) + \sum_{k=1}^{l} p_k y_k^-(w) + \sum_{i=1}^{m} p_i u_i(w) \right] \right\} \\
\text{s.t.} & \\
y_0(w) = \lambda_0 - [\lambda_1(w) - \lambda_2(w)] \quad (19) \\
y_k^+(w) = \lambda_1(w) - D \left( \sum_{j=1}^{n} (\tilde{c}_{jk})_w x_j, (\tilde{g}_k)_w \right) \quad (20) \\
y_k^-(w) = \lambda_1(w) - D \left( (\tilde{g}_k)_w, \sum_{j=1}^{n} (\tilde{c}_{jk})_w x_j \right) \quad (21) \\
u_i(w) = D \left( \sum_{j=1}^{n} (\tilde{a}_{ij})_w x_j, (\tilde{b}_i)_w \right) - \lambda_2(w), i = 1, m \quad (22) \\
x_j, y_0(w), y_k(w), u_i(w), \lambda_1(w), \lambda_2(w) \geq 0; \\
w \in \Omega; i = 1, 2, ..., m; j = 1, 2, ..., n; k = 1, 2, ..., l \\
\end{aligned}
\]

where \( E \) denotes the mathematical expectation.

The remaining issue of (P5) is the size of the obtained problem. Actually, the size of the problem depends on \( m, n, l \) and the size of \( \Omega \). Since the number of goals \( l \) is countable in practice, thus, the main concern is the size of \( \Omega \). If \( \Omega \) includes countable discrete events \( w \), the size of the problem does not increase so much. Otherwise, if the size of \( \Omega \) is large, the expected values are used to estimate the violation in constraints (19) - (22). In this case, the size of the problem is reasonable. Therefore, the size issue is handled.

**Example:**
Consider the following fuzzy stochastic goal programming problem
\[
\begin{align*}
\tilde{z}_1(w) &= \tilde{c}_{11}(w)x_1 + \tilde{c}_{21}(w)x_2; \tilde{g}_1(w) \\
\tilde{z}_2(w) &= \tilde{c}_{12}(w)x_1 + \tilde{c}_{22}(w)x_2; \tilde{g}_2(w)
\end{align*}
\]

Subject to
\[
\begin{align*}
\tilde{A}x &\leq \tilde{b} \\
x &\geq 0, w \in \Omega
\end{align*}
\]

where

\[
(\tilde{c}, \tilde{g}, \tilde{A}, \tilde{b}) = \begin{cases}
(\tilde{c}_{w1}, \tilde{g}_{w1}, \tilde{A}_{w1}, \tilde{b}_{w1}) = \left\{ \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 10 \\ 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix} \right\} \\
(\tilde{c}_{w2}, \tilde{g}_{w2}, \tilde{A}_{w2}, \tilde{b}_{w2}) = \left\{ \begin{pmatrix} 8 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} 10 \\ 8 \\ 11 \end{pmatrix}, \begin{pmatrix} 5 \\ 2 \\ 3 \end{pmatrix} \right\}
\end{cases}
\]

\[
p(w_1) = 0.25, p(w_2) = 0.75; \Omega = (w_1, w_2)
\]

and \( \tilde{m} \) denotes a triangular fuzzy number with the following membership function
Suppose $p_0 = p_k = 2$, $p_i = 3$, the program (P5) corresponding to this example is then

\[
\begin{align*}
\text{Max} \quad & \left[ \lambda_0 - \left[ 0.5y_{o_1} + 1.5y_{o_2} \right] \right] - \left[ (0.5y_{i_1}^- + 0.5y_{i_2}^- + 1.5y_{i_2}^- + 1.5y_{i_2}^-) \right] + \left[ (0.5y_{i_1}^+ + 0.5y_{i_2}^+ + 1.5y_{i_2}^+ + 1.5y_{i_2}^+) \right] + \left[ (0.75u_{i_1} + 0.75u_{i_2} + 2.25u_{i_2} + 2.25u_{i_2}) \right] \\
\text{s.t.} \\
& y_{o_1} = \lambda_0 - (\lambda_{i_1} - \lambda_{i_2}) \\
& y_{o_2} = \lambda_0 - (\lambda_{i_2} - \lambda_{i_2}) \\
& y_{i_1}^- = \lambda_{i_1} - D\left(3x_1 + 4x_2, \tilde{10}\right) \\
& y_{i_2}^- = \lambda_{i_2} - D\left(2x_1 + 3x_2, \tilde{9}\right) \\
& y_{i_2}^- = \lambda_{i_2} - D\left(2x_1 + 1x_2, \tilde{9}\right) \\
& y_{i_2}^- = \lambda_{i_2} - D\left(4x_1 + 3x_2, \tilde{11}\right) \\
& y_{i_2}^+ = \lambda_{i_1} - D\left(10, 3x_1 + 4x_2\right) \\
& y_{i_1}^+ = \lambda_{i_1} - D\left(9, 2x_1 + 3x_2\right) \\
& y_{i_2}^+ = \lambda_{i_2} - D\left(8, 2x_1 + 1x_2\right) \\
& y_{i_2}^+ = \lambda_{i_2} - D\left(11, 4x_1 + 3x_2\right) \\
& u_{i_1} = D\left(1x_1 + 1x_2, \tilde{3}\right) - \lambda_{i_1} \\
& u_{i_2} = D\left(2x_1 + 1x_2, \tilde{4}\right) - \lambda_{i_2} \\
& u_{i_2} = D\left(1x_1 + 3x_2, \tilde{5}\right) - \lambda_{i_2} \\
& u_{i_2} = D\left(1x_1 + 2x_2, \tilde{4}\right) - \lambda_{i_2} \\
& x_f, y_{oh} = y_0(w_h), y_{i_1}^+ = y_{i_1}^+(w_h), u_{i_1} = u_i(w_h), \lambda_{i_1} = \lambda_i(w_h), \lambda_{i_2} = \lambda_2(w_h) \geq 0 \\
& i = 1, 2; j = 1, 2; k = 1, 2; h = 1, 2 \]

Applying (4) we have
\[
\begin{align*}
\text{Max} \quad & \lambda_0 - \left[0.5y_{01} + 1.5y_{02}\right] - \left[(0.5y_{1i}^- + 0.5y_{2i}^- + 0.5y_{1i}^+ + 1.5y_{2i}^- + 1.5y_{2i}^+) + (0.75u_{1i} + 0.75u_{2i} + 2.25u_{1i} + 2.25u_{2i})\right] \\
\text{s.t.} \quad & y_{01} = \lambda_0 - (\lambda_{11} - \lambda_{21}) \\
& y_{02} = \lambda_0 - (\lambda_{12} - \lambda_{22}) \\
& y_{1i}^- = \lambda_{11} - \frac{1}{2}(3x_i + 4x_2 - 10 + 4x_i + 5x_2 + 9) \\
& y_{1i}^+ = \lambda_{11} - \frac{1}{2}(10 - 3x_i - 4x_2 + 11 + 2x_i + 3x_2) \\
& y_{2i}^- = \lambda_{11} - \frac{1}{2}(2x_i + 3x_2 - 9 + 3x_i + 4x_2 + 10) \\
& y_{2i}^+ = \lambda_{11} - \frac{1}{2}(9 - 2x_i - 3x_2 + 10 + 3x_i + 4x_2) \\
& y_{12}^- = \lambda_{12} - \frac{1}{2}(2x_1 + 1x_2 - 8 + 3x_1 + 2x_2 + 7) \\
& y_{12}^+ = \lambda_{12} - \frac{1}{2}(8 - 2x_1 - 1x_2 + 9 + x_1 + 0) \\
& y_{22}^- = \lambda_{12} - \frac{1}{2}(4x_1 + 3x_2 - 11 + 5x_1 + 4x_2 + 10) \\
& y_{22}^+ = \lambda_{12} - \frac{1}{2}(11 - 4x_1 - 3x_2 + 12 + 3x_1 + 2x_2) \\
& u_{11} = \frac{1}{2}(x_1 + x_2 - 3 + 2x_1 + 2x_2 + 2) - \lambda_{21} \\
& u_{21} = \frac{1}{2}(2x_1 + x_2 - 4 + 3x_1 + 2x_2 + 3) - \lambda_{21} \\
& u_{12} = \frac{1}{2}(x_1 + 3x_2 - 5 + 2x_1 + 4x_2 + 4) - \lambda_{22} \\
& u_{22} = \frac{1}{2}(x_1 + 2x_2 - 4 + 2x_1 + 3x_2 + 3) - \lambda_{22} \\
& x_j, y_{0h} = y_{0}(w_h), y_{1h}^- = y_{1}^-(w_h), y_{1h}^+ = y_{1}^+(w_h), u_{ih} = u_i(w_h), \lambda_{1h} = \lambda_1(w_h), \lambda_{2h} = \lambda_2(w_h) \geq 0 \\
& i = 1, 2; j = 1, 2; k = 1, 2; h = 1, 2 \\
& \text{The solution for this problem is } x_1 = 2.4, x_2 = 0
\end{align*}
\]

5. Conclusion

In this paper, we consider a general case of the fuzzy stochastic goal programming problem where objective/constraint coefficients and goals are fuzzy random variables. A new model to express the attainment values of fuzzy random variables has been proposed to convert the problem into an LP that is easily solved by the standard optimization packages. Our consideration has extended Luhandjula’s work to the goal programming.
approach of fuzzy stochastic optimization streamline by developing a different approach. The proposed approach is also more efficient than the existing approaches because our obtained model is the LP model (compared with the complexity of computation of expected value of Liu’s models) with smaller number of constraints and variables (compared with the LP model of Luhandjula).

References