Constraint of Contacting Points in Cooperative Handling

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Abstract

We are going to analyze on the constraint among contacting points between robots and an object that is manipulated by the robot group. Constraint can be obtained by solving pseudo inverse of matrix filled by force data measured by all robots supporting one object. The result provides additional information for obtaining a relative positioning information to the object to ordinary constraint based on geometrical reasons. This analysis on the constraint has been verified by sensing system. The model explained here can be used for estimating contacting points, fitting coordinate system among robots, and making active sensing strategy for detecting the change of environment.

1 Introduction

Nowadays the demand for autonomous transporting system is getting larger. Especially the people working in a construction site and a distribution center are waiting for systems that can change manipulation strategy according to dynamic environment and various kinds of objects (Figure 1).

![Figure 1: Handling System in Cooperative Robots](image)

"Centralized or Distributed?" -- This is one of the typical problems for robotic researchers. For the cooperative transportation, it is just the same. Kosuge and his companies have developed the transporting system by distributed control to avoid excessive communication rate among the transporting robots [1][2]. Ahmadabadi have presented a transporting system based on the distributed architecture [3]. However, distributed system cannot warrant the completion of their task generally. Both distributed systems can stop the transporting task when they collide with a wall or something else, but they cannot do any more plans until the walls get removed. Since we consider that estimating a change of environment and selecting another strategy are essential functions in an autonomous robot system, we accept data communication on a centralized system.

For these demands we have presented the strategy to obtain the property of the object and to decide the optimal transporting state to realize a safe transportation. [4] has presented the criterion of contacting points that equalizes load distribution to each robot. [5] has presented the strategy to make an arrangement of contacting points that realizes safest state transportation before touching an object. [7] has revealed the concept of "Unknown Parameter Area" and shown the criteria in [6] visually by using this concept.

Mobile robots have a merit of large workspace, but they are also known that they generate large positioning error. Therefore, some transportation tasks may not be successful in general. The most important problems in this system is to adjust coordinate systems belonging to all robots and world coordinates. All variables have to be expressed in the same coordinate system, but the force sensed by each robot is obtained on its own coordinates. Data obtained from an object are largely affected by the position/posture error of robots.

Position and posture error may cause the excessive
internal force, or dropping an object from their contacting state. To prevent the effect of these errors in the minimum level, the robots need to follow the information of the points where the robots are touching the object. This kind of information should be given by the state that the robots are touching the same object and they are sharing the load according to a law of physical stability. This paper gives the constraint on the contacting points when the robots are sharing load in a certain distribution. The result of this analysis is going to be used for obtaining a relative positioning information to the object.

2 Structure of the Problem

2.1 Definition of Problem, Variables and Coordinates

Based on the background we have mentioned before, we assume that a shape of an object can be obtained before/during manipulation. To simplify the problem, all contacting states are described by point contact with friction. Manipulation is assumed to be in a quasi-static state, therefore the object does not get any acceleration during manipulation. Now we will show the model of coordinate systems and variables in Figure 2, where

- $\sum_{(W,O,i)}$: coordinate system fixed on world($W$), object($O$), and $i(1,\ldots,n)$th robot($i$)
- $Wb_i$: base position of robot $i$ on $\sum_w$
- $WR_i$: rotation matrix of robot $i$ refered to $\sum_w$
- $Wp_i$: $i$th contacting point on $\sum_w$
- $f_i$: force applied to robot $i$th robot on $\sum_i$
- $Wp_O$: position of base point of an object on $\sum_w$
- $WR_O$: rotational matrix of an object with reference to $\sum_w$
- $WP_E$: the position where external force (including gravitational force) is applied on $\sum_w$
- $WF_E$: external force applied to an object on $\sum_w$

The robots can obtain $Wb_i$, $WR_i$ and $f_i$ by dead-reckoning and force sensing system. Our current goal is to estimate the relative positions among the robots $p_i$ by geometrical constraint and physical constraint.

2.2 Dependence among Variables

Dependence among these variables are drawn in Figure 3. Variables with mesh ($4,8,9$) are given by the estimation based on least-square method or other optimizing calculation, and the other variables are given by a simple transformational calculation. Thick arrows represent the new constraint to estimate the relative positions between contacting points supporting a common object. This constraint is given by load distribution and position/posture of an object.

$Wp_i$ and $WF_i$, drawn as "1" and "2", are obtained by a simple transformation. $Wp_i$ indicates a position of contacting point, which is always constant.

\begin{align}
Wp_i &= Wb_i + WR_i f_i \\
WF_i &= WR_i f_i
\end{align}

$Wu$, which is drawn as "3", is defined as the sum of sensed force/moment data of group robots. This value represents force and moment that is applied to an object on the world-coordinate system. If the robots are
doing a stable lifting/pushing task to an object, this variable indicates the center of gravity/friction.

\[ w_u \equiv w_P w_f \]  
\[ W_R \text{ and } W_O \text{ are drawn as "S" are estimated from previous data of } W_P, W_R, O_P, \text{ and } W_P. \]

\[ \sum_{i=1}^{n} \| W_P - W_P' \| \rightarrow \text{min.} \]  

Figure 4: Geometric Constraints

By this constraint we can easily obtain the contacting points on \( \sum_{i} \) that is drawn as "5":

\[ o_{p_{i \text{geo}}} = w_R^{-1} (w_{p_i} - w_P) \]  
\[ O_u \text{ drawn as "6" is given by sensing data } w_u, w_P \text{ and } w_R. \]

\[ O_u = - \left( o_{p_E} \times I_3 \right) o_f \]  

\[ O_{p_E} = w_R^{-1}(w_{p_E} - w_P) \]  
\[ O_f = w_R^{-1} w_f \]

Safeness of transportation and optimal load distribution can be provided by \( O_u \) [4] [6]. Now we call \( O_u \) “Unknown Parameter” of an object. This variable represents the current force applied to an object, which is estimated by the integrated sensor data of all robots. The strategy, performance and the merit of estimating this parameter are described in previous paper (frictional force for [8], and gravity force for [6] and [7]). Plural number of sensing by changing the posture of an object enables the robots to obtain the further information of the center of gravity/friction [7]. This is why we need force-sensing data in the coordinate-system fixed on the object. In this paper, the parameter \( u \) is assumed to keep constant during the stable cooperative manipulation.

\[ O_{f_i} \text{ drawn as "7" is transformed from } W_f: \]

\[ O_{f_i} = w_R^{-1} W_f \]  

Detail of calculations “8” and “9” which are not based on a simple transformation will be explained in the next section.

3 Physical Constraint and Estimating Contact Points

3.1 Constraint by Load Distribution

Our main approach in this paper is in “8” and “9”, as we say “What kind of constraints the load distribution gives?” \( o_{p_{\text{phys}}} \) drawn as “8” gives the constraint between contacting points. This constraint is produced by physical reasons; force and moment applied to an object \( O_u \) and force distribution applied to each contact point \( O_{f_i} \). To satisfy this demand, we are going to transform Eqn.(14) into Eqn.(15):

\[ O_F o_f = O_u \]  
\[ O_F o_p = O_u_m \]  

where

\[ O_F = ( o_{f_1} \times I_3 \ldots o_{f_n} \times I_3 ) \]  
\[ o_p = ( o_{p_1}^T, \ldots, o_{p_n}^T )^T \]  
\[ o_{u_m} = - ( o_{p_E} \times I_3 ) o_{f_E} \]

Eqn.(15) can give us the constraint of contacting points as shown in Eqn.(19).

\[ o_{p_{\text{phys}}} = - O_F o_{u_m} + V\lambda \]  
\[ = - O_F o_{u_m} + \sum_{j=1}^{r} v_i \lambda_i \]
where $^OF^+$ is a pseudo inverse matrix of $^OF$, $V = (v_1, \cdots, v_r)$ is a matrix which consists of linearly independent component of matrix $I_{3n} - ^OF^+^OF$, $r$ represents $\text{rank}(I_{3n} - ^OF^+^OF)$, and $\lambda = (\lambda_1, \cdots, \lambda_r)$ is a vector that consists of any real numbers. This result indicates that the contact points bearing load in a certain distribution can be constrained on the geometrical figure in 3n-dimensional space (Figure 5).

![Figure 5: Constraints of Contacting Points](image)

Throughout these analyses, we can obtain relative positions of each contacting points, and we can also feedback these data into the robot system in order to keep transportation from some failure occurred by uneven motion errors of mobile robots.

### 3.2 Numerical Example (1): Pusing by Two Robots

Now we are going to show an example. Two robots are pushing an object to a given direction in two-dimensional world (Figure 6). We assume that robots have obtained the parameters of the object at the beginning of manipulation, and now the robots can measure load distribution during the manipulation as follows:

$$^op = (60, 25), \quad ^of = (0, -5),$$
$$^of_1 = (0, 2), \quad ^of_2 = (0, 3)$$

From these data we can obtain the constraint as shown in Eqn. (21).

$$\begin{pmatrix}
^op_{1x} \\
^op_{1y} \\
^op_{2x} \\
^op_{2y}
\end{pmatrix} = \begin{pmatrix}
\frac{500}{13} \\
0 \\
\frac{750}{13} \\
0
\end{pmatrix} + \begin{pmatrix}
3 & 0 & 0 \\
0 & 1 & 0 \\
-2 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix} \lambda
$$

The relationship between both $^op_{1x}$ of two robots is described in Figure 7. The solution that gives minimum norm appears at $^OF^+^nu_m$ in this graph, and redundant line is drawn as a solid line. All arrangements of contact points on this solid line satisfy the constraint of physical status. This result shows that the system can detect the positioning error of $^op_{1x}$ and $^op_{2x}$ in direction $A$, but it is difficult to modify the errors in direction $B$.

![Figure 7: Detectable Direction and Undetectable Direction](image)

### 3.3 Numerical Example (2): Lifting by Four Robots

Second example is about the constraint in lifting by four robots (Figure 8).

$$^op_G = (40, 30, 25), \quad ^of_G = (0, 0, -30),$$
$$^of_1 = (0, 0, 10), \quad ^of_2 = (0, 0, 10),$$
$$^of_3 = (0, 0, 10), \quad ^of_4 = (0, 0, 10),$$

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From these data we can obtain the constraint as shown in Eqn. (22) and Eqn. (23).

\[-O_F^+ \cdot u_m = (40, 30, 0, 40, 30, 0, 40, 30, 0, 40, 30, 0)^T\]

\[V = \begin{pmatrix}
3 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 3 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 3 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & -1 & 0 & 0 & 3 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & -1 & 0 & 0 & 3 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
-1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}\]  

This result says that the system can detect the positioning error in the direction \(d\) that make \(d \cdot v_i = 0\) for all \(i\), and it is hard to modify the errors in that direction parallel to \(v_i\).

3.4 Combination of Two Constraints

\(O_p\) drawn as "9" in Figure 3 is obtained by geometrical and physical constraints, in order to satisfy both constraints as close as possible. We adopt \(O_p = (O_{p1}, \ldots, O_{p_n})\) as the solution \(O_{p_{est}}\) that minimizes an answer of Eqn. (24). In a mathematical sense, Eqn. (24) represents the distance between geometrical constraint \(O_p\) and physical constraint (Eqn. (19)) in a 3\(n\)-dimensional space (Figure 9).

\[\text{dist.} = \| O_{p_{geo}} + O_F^+ \cdot u - V \lambda \| \rightarrow \text{min.} \]

4 Verification by Real Sensing

The method we have mentioned above has to be verified and modified corresponding to real world, by obtaining sensing data. In this section we are going to check the correspondence between real contacting point and constraints which controls the arrangement of contacting points by static sensing. We used one object and two robots that can push it as you can see in Figure 10. Dimensions of this system are on Figure 11. This robot has a sensing and lifting/pushing unit in front of the robot.

The system measures load distribution and the position of contacting point at the beginning of manipulation. After obtaining load distribution, the system solves the constraint of their contacting points. This measurement is done under the assumption that the center of friction of the object can be obtained at the beginning of this sensing session. Consistency with the constraints is evaluated by the distance \(d\) between the actual contacting point and obtained constraint space in 2x2-dimensional space like a numerical example in Section 3.2. The outline of the object is given to the system as a set of 50 point-data like \((O_{p_{obj1}}, \ldots, O_{p_{obj50}})\).

The result of estimation is shown in Figure 12. To compare the dead-reckoning values and result of estimation easier, all variables are described in the world coordinate system. Although the read-reckoning positions are not on the trajectory of the object exactly, the contacting points that are on the trajectory and satisfy the ratio of load-distribution.

5 Conclusion and Current Work

In this paper we have discussed constraint when the property of an object and load distribution to the robots are obtained. The result can lead the robots to improve position estimation by the robots themselves, by bringing a physical constraint to obtained data measured on a different coordinate systems. The accuracy of estimation depends on the data of an object, but the strategy we have introduced here can be
used for the environment that the landmark or any external position-recognising system are installed. Now we are applying this strategy to more practical situations, such as dynamical situation with control, or estimating environmental force and position with sensing by active behavior of the robots.

References