A Markovian model for the hybrid manufacturing planning and control method 'Double Speed Single Production Line'

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Abstract: In this paper, the hybrid production planning & control method Double Speed Single Production Line (DSSPL) is presented, modeled and its performances evaluated and compared to classical Production Planning and Control methods (PPC). DSSPL combines JIT/kanban and Material Requirement Planning for the production of different classes of products (A- and B-items based on a market or customer oriented analysis) on one single production line. By the use of a Markovian birth-death queuing model of a single-stage, two-product production system, the performance and the behavior of the basic DSSPL concept are analyzed and compared to the classical MRP concepts. Its capability to cope with limited resources is illustrated with an industrial case study where DSSPL has been implemented to solve coordination problems between a plastic molding feeder shop and the final assembly line of a micromotor producer.

Keywords: Hybrid manufacturing planning & control method; Markov processes; Industrial case study; Double Speed Single Production Line
1. Introduction

An appropriate choice of the manufacturing planning & control (MPC) system is an important success factor for any manufacturing firm. The characteristics of the chosen MPC system must not only meet the requirements of the focused market, but also those of the manufacturing process design (Vollmann et al. 1997). In the field of business strategy research, the issue of optimal configurations of typical MPC design options for market and manufacturing design requirements is addressed by the development of manufacturing strategy frameworks (Bozarth and McDermott, 1998). Two of the most representative frameworks have been developed by Hayes and Wheelwright (1979, 1984) and Berry and Hill (1992). Hayes and Wheelwright describe in their framework the relationship between the product characteristics and the manufacturing process type. Berry and Hill refined this framework by adding links from product and process characteristics to strategic MPC design options ranging from time-phased Material Requirement Planning (MRP) to rate-based Just-in-Time (JIT). Generally, MRP and JIT are considered as the classical MPC concepts. MRP (resp. JIT) is also often termed as push (resp. pull)-based production system due to the use of production orders (resp. Kanbans) to trigger production. These frameworks give not only a valuable insight into the characteristics of optimal configurations of manufacturing systems, but they also illustrate the fact that the two classical MPC design options MRP and JIT fit unambiguously manufacturing environments with opposite characteristics (MRP to low-volume, high-mix, job shop environments, JIT to high-volume, low-mix, line flow environments). For (intermediate) manufacturing environments not having such (extreme) characteristics however, the optimal MPC strategy is less obvious. This issue has therefore generated much research, focused on either extending the application domain of classical MPC concepts (MRP and JIT) or on developing new (hybrid) MPC concepts.

In contrast to the first research direction, the second abandons the view of mutually exclusive MPC concepts and considers particularly the advantages and flaws of the MRP and JIT concepts as complementary. This consensus about the benefits of hybrid MPC designs, combining the advantages of MRP and JIT, is based on the outcome of a debate between proponents of pure JIT or MRP solutions that emerged after the introduction of the JIT philosophy in western countries. Impressed by the obvious success of JIT in Japan (Monden 1983), some authors have seen JIT as the only MPC system design able to meet world-class manufacturing standards. As one of the most influential advocates of JIT, Schonberger (1986) claimed in opposition to the concepts of the focused factory (Skinner 1974) and the theory of performance trade-off (Porter 1980, 1985) that the adoption of JIT enables firms to excel simultaneously in all important types of manufacturing performance. Other concepts such as the Zero Inventory management introduced by Hall (1983) supported this optimistic view.

In opposition, advocates of MRP tried to prove that JIT techniques are less beneficial in manufacturing environments. Most of these studies are based on the two-step research structure (Krajewski et al. 1987, Rees et al. 1989, Sarker and Fitzsimmons 1989); In a first step, MRP and JIT systems are analyzed separately in manufacturing environments judged as typical for the corresponding MPC concept (low setups, small lot sizes and reduced variability for the JIT and big lot sizes, high setups and strong variability for the MRP manufacturing environment). In a second step, the MRP system is analyzed in the manufacturing environment defined for the analysis of the JIT concept. The principal findings of these studies are similar. JIT systems outperform MRP systems when analyzed in their corresponding manufacturing environment. However, the MRP system exhibits comparable or even better results when analyzed in manufacturing environments used for the JIT system.

Both opposing views have in common that they do not take into account capacity constraints found in practice. The success of typical JIT implementations is uncontested but surveys of implementation cases show that successful JIT implementations depend basically on two conditions (Crawford et al.
1998, Gilbert 1990). First, the company product characteristics and manufacturing process design should be as close as possible to typical JIT configurations. Second, successful JIT implementations are generally accompanied by a reorganization of the whole manufacturing and planning process. However, both conditions cannot be met by a wide range of companies. Particularly, in the case of small and medium-sized companies with limited resources, the effort of a complete redesign is difficult to perform (White et al. 1999).

Similarly, studies in favour of the MRP technique underestimate the difficulties found in practice related to the underlying flaws of the MRP concept. In fact, in a manufacturing environment with accurate lead times and forecasts and infinite capacity, MRP is effectively the optimal PPC concept. However, if these conditions are not met, MRP leads to inefficient and unrealistic production plans (Hopp and Spearman 2000).

As a conclusion and as shown in the following survey of existing hybrid MPC concepts, the general strategy for the integration of JIT and MRP is based on the strengths of both concepts. MRP is superior to JIT in its capability for long term planning and to handle lumpy demand. On the other hand, JIT offers the simplest solution for the execution of the production process.

The purpose of this paper is twofold. The first purpose is the presentation of the hybrid MPC strategy Double Speed Single Production Line (DSSPL) (Stagno et al. 2000) on the basis of Markovian analyses. Its novel aspects and performance compared to MPR systems are also explored. The second purpose is to validate the DSSPL concept with an industrial case study. It consists of an implementation of DSSPL in an internal supply unit for final assembly lines. This case is particularly appropriate for the validation of the DSSPL conceptual design even though it must be pointed out that DSSPL is developed for a much wider application domain.

This work is an extension of the original results by the same authors (Hachen et al. 2003). One of the novel aspect in this paper is the development of analytical models for different PPC strategies, aiming at the development of benchmarking for a more comprehensive work related to DSSPL. The industrial case gives more practical information and feedback on the benefits of DSSPL implementation.

This paper is organized into three sections. In section 2, after a review of existing hybrid MPC concepts, the key elements of DSSPL and its novel aspects are presented. The basic mechanics and the performances of DSSPL compared to MRP systems are illustrated by the use of Markovian models in section 3. The industrial case study is presented in section 4 where emphasis is put on a description of the manufacturing environment and the implementation procedure. Finally, concluding remarks with directions for future research are given in section 5.

2. The concept of Double Speed Single Production Line (DSSPL)

2.1. Hybrid MPC concepts

The wide variety of existing hybrid MPC concepts shows that a great amount of research effort has been focused on how best to combine the different MPC approaches. These hybrid concepts can be broadly classified into three classes that are characterized according to the combination and integration typology of the different MPC concepts. In vertically integrated hybrid production systems (VIHPS) the JIT concept is exclusively applied at the shop floor level whereas MRP is used to generate the production plans. Horizontally integrated hybrid production systems (HIHPS) are characterized by the use of either the MRP or JIT concept for the management of the different production stages. In parallel integrated hybrid production systems (PIHPS), production can be triggered by more than one MPC concept that are applied in parallel.

Most of the literature related to VIHPS presents strategies and conditions for a successful embedding of JIT in a MRP environment. Flapper et al. (1991) present a three-step implementation framework, which makes use of the MRP backflushing techniques and phantom items. Based on simulation studies, Huq and Huq (1994) identify load imbalance and machine breakdowns as the most critical
factors affecting a successful implementation of the JIT/kanban technique into an MRP environment. The problem of the choice of the typical JIT design parameters (number and size of kanbans) implemented in an MRP environment is addressed by Gupta and Brennan (1993) and Nagendra and Das (1999). Gupta and Brennan propose a knowledge-based simulation system, which allows the simulation and dimensioning of such systems. Nagendra and Das on the other hand, propose a concept for additional modules for the MRP system, which perform the task of dimensioning the JIT design parameters.

The problem of the optimal integration of different production control methods and master production schedule approaches within a production line is addressed by researchers who develop and analyze HIHPS. Cochran and Kim (1998) develop a methodology to determine the optimal configuration of a HIHPS applied to serial production lines. They assume that the first production stages are managed by the push (MRP) approach up to a production stage called a “junction point”. The intermediate stock for the intermediate items manufactured by the junction point production stage serves as the input buffer for the succeeding production stages managed by the JIT/Kanban method. The optimal solution for the location and safety stock level of the junction point, and the number of kanbans with respect to minimized inventory carrying and shortage costs were found by applying the simulated annealing optimization technique on a simulation model. In the HIHPS described by Lackes (1994), no restriction is made to the location of the production stages controlled by the pull concept. For the case where production stages controlled by the JIT/Kanban method are followed by push controlled production stages, Lackes develops an algorithm that smooths the demand for the pull controlled production stages by minimizing inventory and instability costs. Instability costs are defined as costs caused by unstable demand in a Kanban-controlled production system. Other research work of Hodgson and Wang (1991a, b), Huang and Kusiak (1998) and Olhager and Östlund (1990) are mainly concerned with the optimal choice of the push or pull concept for individual production stages in an HIHPS. By using a Markov decision process model of a production system with converging network structure, Hodgson and Wang (1991a, b) find that the HIHPS concept is superior in terms of total costs to a pure push or pull solution. In their configuration of the HIHPS, the push concept has been applied to the first production stages and the pull concept to the last production (assembly) stages.

In a similar study, Huang and Kusiak (1998) develop a set of rules to determine the production control method (push or pull) for every individual production stage. According to these rules, the pull concept is used for all production stages up to bottleneck or assembly stages and the pull concept is applied at the final stages. By using a simulation model with a converging network structure, they prove that their HIHPS configuration outperforms, in terms of total costs, a pure push solution. Finally, Olhager and Östlund (1990) present a set of decision rules for the configuration of HIHPS. They propose to integrate the push and pull method according to the order penetration point, the bottleneck resources or the product structure. Additionally, they present a case study from the packaging industry where they implement successfully a HIHPS.

Existing PIHPS can be grouped according to the use of product specific or generic kanbans. In the case of PIHPS with product specific kanbans, MRP is primarily used to widen the application domain of the kanban concept to production environments with an unstable demand. In the concept proposed by Deleersnyder et al. (1992), production for one item is triggered either by a kanban or a production order. In the SynchoMRP concept presented by Hall (1983), production at a stage is only allowed in the presence of a kanban or a corresponding production order. Another hybrid approach has been presented by Lackes (1994) where the information provided by the MRP system about changes in demand is transmitted to all stages. This allows the production stages to anticipate future fluctuations in demand and to begin production even before the presence of a Kanban. The most typical PIHPS is developed by Spearman et al. (1990). As in the other PIHPS using generic kanbans, CONWIP (CONstant Work In Process) uses MRP to generate the production schedule. CONWIP can be considered as a generalized pull concept, since cards attached to production orders traverse a
circuit that includes the entire production line. Since the cards are (1) not assigned to a particular item and (2) limited in number, they represent an efficient way to limit the work in process. A similar PIHPS called POLCA (Paired-cell Overlapping Loops of Cards with Authorization) has been presented by Suri (1998). The combined use of short CONWIP loops and MRP allows efficient allocation of the capacity at every production stage.

Several work have focused on the benchmark of hybrid MPC concepts and on the combination of these different MPC in order to take advantages of every concept within its optimal environment (i.e. Takahashi et al. 2005, Geraghty and Heavey 2004, Bonvik et al. 1997).

2.2. DSSPL problem statement

Rare are the production systems where the production capacity is in balance with the external demand. The reason for such overloaded and consequently inefficient production systems is not only a presumably increased and/or fluctuating demand but also evolving market requirements, common management practices and the wide use of the MRP concept.

Today, the market requirements evolve towards highly customized products combined with minimal delivery times. However, for a given production system, the resulting increased product variety does not only increases variety-related costs (Stalk 1988) but also the overall production lead times (Hopp and Spearman 2000). The design and configuration of production systems able to respond to these (contradicting) requirements is addressed by the agile manufacturing concept. But as a survey of Gunasekaran and Yusuf (2002) shows, most related research activities are rather focused on issues like virtual enterprise formation, new information technologies and rapid prototyping techniques than on new MPC concepts. As pointed out by Suri (1998), the most common management practices resulting in overloaded production systems are scale- and cost-based strategies. Typically, production resources are scheduled to run close to 100% utilization in order to minimize the amortization period.

Finally, as already mentioned, flaws of the MRP concept lead to production plans that do not respect the limited capacity of production systems. Even capacity requirement planning (CRP) modules available for MRPII systems cannot solve adequately this problem, since their logic is based on fixed lead times (Hopp and Spearman, 2000).

The above described situation corresponds to the first hypothesis of the conceptual DSSPL framework that is stated as follows:

**H1**: Production systems tend to be in overloaded states.

The problems related to the situation stated in the first hypothesis can be solved by just increasing the production capacity and/or by an implementation of a new MPC approach such as JIT or lean manufacturing. However, most of these approaches have in common that considerable financial, organizational and technical resources are required. As already mentioned in the first section, such redesigns are particularly difficult to perform when the product structure and manufacturing processes do not fit to typical JIT requirements (linear production flow, low variety,...). The second hypothesis is stated therefore as follows.

**H2**: For companies with complex production systems, significant improvement of the logistic performances are difficult to achieve when financial, technical and organizational resources are limited.

The third hypothesis concerns the demand typology and characteristics of the products of a company that can be determined by a multiple-criteria ABC analysis (Flores and Whybark 1986).

**H3**: The results of a multiple-criteria ABC analysis of the products of a company follow approximately Pareto’s Law.

Typical criteria used are those describing the demand typology (annual cost volume usage, demand regularity), product characteristics (substitutability, impact of running out) and marketing objectives (product market share, number of clients).
2.3. DSSPL framework
Derived from the previous hypothesis and problem statement, the DSSPL framework is based on the following key concepts:

- By applying a multiple-criteria ABC analysis (demand typology, product characteristics, marketing objectives), products are divided into two (A- and B-)product groups. A-products are generally characterized by a high and stable demand and their strategic importance is such that a running out is very damaging and must be avoided. B-products are characterized mainly by a lower and unstable demand and reduced strategic importance;
- According to the characteristics of the demand typology of the product groups, the JIT/kanban concept is applied for the management of the A-products and MRP (or Inventory Control) for the B-products;
- Local scheduling at production stages with A- and B-product flows is governed by specific sequencing rules that handle the different priorities. In the simplest case, priority is always given to A- over B-products.

The division of the products into different product groups allows companies to concentrate their limited resources on the most important products without the need for additional production resources. Furthermore, by applying JIT techniques to the limited number of A-products, the JIT implementation efforts can be better focused. In addition, local scheduling at every production stage is simplified due to the transparent allocation of priorities to the different product groups which is particularly important in cases where a wide variety of products is produced.

Compared to existing MPC concepts, DSSPL can be distinguished by three characteristics. First, DSSPL relays upon a strategic reasoning; the goal is the optimal allocation of limited resources in order to obtain the maximum impact in terms of customer service and satisfaction. Second, DSSPL chooses the different MPC methods (JIT/kanban, MRP or Inventory control) according to a multi-criteria analysis taking into account strategic aspects (marketing objectives) as well as operational ones (demand typology and product characteristics). Third, in contrast to existing hybrid MPC concepts, DSSPL is not limited in its capacity to handle a wide variety of products. In fact, except for the CONWIP and POLCA concepts, all reviewed approaches use the JIT/kanban method for all products for some or all production stages. Since only a limited number of products can be managed efficiently by product specific kanbans at one production stage, their introduction in manufacturing environments with a wide variety of products is difficult. As already mentioned in section 1, the introduction of the kanban control concept in such environments requires additional production resources and/or the introduction of techniques like production cell technology. The application domain of the CONWIP or POLCA concept is not limited by the number of products or the stability of the demand. However, in contrast to DSSPL, MRP is used to generate the production schedule for all products. Even though the JIT system is used in CONWIP and POLCA in the form of generic kanbans (more stable lead times by limiting work in process), the material flow is still pushed according to an MPS that is generally affected by large uncertainties. In contrast, the A-products of DSSPL are only pulled, according to the effective consumption, by the product-specific kanbans.

3. Performance of DSSPL
The performances of DSSPL are analyzed and compared to those of classical MRP concept through the use of a Markovian birth-death queuing model of a single-stage, two-product production system. The use of a Markovian model allows the capture of the basic mechanisms of DSSPL when applied to a random manufacturing environment and gives a closed form of the performance measures with respect to the parameters of the system. The service levels $SL$ and inventory holding costs $IC$ are chosen as principal performance measures.
3.1. Model description

As shown in Figure 1, the modelled manufacturing environment consists of a single-stage (manufacturing center $MC$), two-product (A- and B-products) production system, that is managed either by MRP, MRPprior (MRP with priority to A-products) or DSSPL concept.

In the MRP model, the production of both A- and B-products is initiated by production orders waiting in the production order queue $PO_{AB}$. At this stage, no priority rule is applied to either of the two types of products. The incoming production orders have mean arrival rates: $\lambda_A = \frac{1}{E[t_A]}$ and $\lambda_B = \frac{1}{E[t_B]}$ where $t_A$ (respectively $t_B$) is the exponentially distributed time between two arrivals of A-products (resp. B-products). Items are processed according to the FCFS (first-come, first-served) rule. The mean service rates are defined by $\mu_A = \frac{1}{E[s_A]}$ and $\mu_B = \frac{1}{E[s_B]}$ where $s$ corresponds to the exponentially distributed service time per job. After completion, the finished A- and B-jobs are sent to the intermediate inventory $II$, from where they are removed after a predefined lead time $lt$ (counted from arrival time in the production order queue) for further use or delivery to the client.

In the MRP model with priorities (MRPprior), production orders of A-products (in $PO_A$) are served ahead of production orders of B-products (in $PO_B$) without preemption. The MRPprior model is a first transition from the classical MRP to the DSSPL concept.

In the DSSPL concept, a priority rule combined with the kanban technique is also used instead of the MRP concept for the management of the A-products. As a characteristic element of the JIT concept, the kanban technique is used to initiate production only when an effective demand has occurred. As shown in Figure 1, the demands for A-products arrive at queue $BO$ and wait there if they cannot be satisfied immediately by finished A-jobs waiting in the queue $FKI$. That is, accumulations in queue $BO$ are considered as back orders. If finished A-jobs are available, the attached kanban is removed and sent back to the queue for kanbans $QK$ in order to initiate production of A-products. Non-preemptive priority is allocated to the kanbans over the production orders for B-products depending on a predefined priority threshold level $thl$ ($1 \leq thl \leq nmbk$, where $nmbk =$ number of kanbans). In fact, priority is allocated to the kanbans only if the number of waiting kanbans in $QK$ is bigger or equal to $thl$. This represents a simplified case of the more general B-waiting time versus A kanban level competition-based priority rule of the generic DSSPL concept (Stagno et al. 2000). Consequently, kanbans always have higher priority than production orders for B-products if $thl$ is set to one.

3.1.1. Definition of system states

The system states of the different models are represented by vectors that describe the state of the corresponding queues.

In the MRP model, presented in figure 1, a state is defined by the sequence of production orders of A- and B-products waiting in $PO_{AB}$. Let $n$ be the state vector representing the system under study and $M$ and $N$ be the number of A- and B-production orders waiting in $PO_{AB}$ respectively. For the particular case where $M=N=2$, there are six possible states considered in the system: (A, A, B, B), (A, B, A, B), (A, B, B, A), (B, A, B, A), (B, B, A, A) or (B, A, A, B), where the last position of the vector corresponds to the production order in service.
In the MRPprior model in figure 1, where priority is given to MRP orders in \( P_O^A \), a state is defined by \( m \): the number of A-production orders waiting in \( P_O^A \), \( n \): the number of B-production orders waiting in \( P_O^B \) and \( r \): the type of the product in service (\( r = A \) or \( B \)). Let \( M \) and \( N \) be the capacity of \( P_O^A \) and \( P_O^B \) queues for A- and B-production orders respectively. The different states are illustrated by the vector \( \mathbf{n} = (m, n, r) \). Relying upon figure 1, the two distinct queues in the MRPprior model can exclusively handle either A- or B-products. It comes that there are \( M \) possible places in \( P_O^A \) combined with \( N+1 \) possible states of B-products in the system, if the considered product in service is of type B. Otherwise, if an A-product is in service, then the number of possible states in \( P_O^B \), \( N \), is combined with the possible total number of A-products in the system, namely \( M+1 \). Based on these possible cases, the total number of states is calculated using equation 1

\[
\Omega_{MRPprior} = (N + 1)M + (M + 1)N
\]

(1)

on the other hand, a state of the DSSPL model, as presented in figure 1, is described by the state vector \( \mathbf{n} = (b, m, n, r) \) where \( b \): demands back ordered in \( BO \), \( m \): waiting kanbans in \( QK \), \( n \): production orders in \( P_O^B \) and \( r \): the type of product in service. Let \( B \), \( M \) and \( N \) be the capacities of respectively \( BO \) for back ordered demands, \( P_O^A \) for A-production orders and \( P_O^B \) for B-production orders. For simplicity reasons, we assume that the demand rate \( \lambda_A \), constituted of different orders of A-products is considered as a rate of A-production orders. Similarly to the MRPprior case, the total number of states in the DSSPL model can be calculated with respect to the order in service. If this latter is of type A, the maximum number of A-production orders in the system is then \( (M+1+B) \), considering the fact that they could be only found in \( P_O^B \), in \( BO \) or in the machining center. On the other hand, the maximum number of B-production orders in the system is \( N \). If the order in service is a B-type, then there will be \( N+1 \) B-production orders and \( M+B \) A-products at maximum. Consequently, the number of states of the DSSPL model is

\[
\Omega_{DSSPL} = thl + (N + 1)(M + B)
\]

\[
+ (M + B + 1)N
\]

(2)

The steady state balance equations defining the state transitions and their probabilities for the three MPC concept models are listed in the appendix of this paper.

3.1.2. Performance measures

The inventory holding costs consist of the costs of the raw material required for the production quantity demanded by the production orders or kanbans and of finished products waiting in \( II \) or \( FKI \) until they are removed for further use. In addition, it is assumed that the waiting time of finished products in \( II \) is equal to the difference between a predefined acceptable lead time \( lt \) and the average time in the system if a production order; A final product is removed from the inventory \( II \) after time \( lt \), counted from the moment when the corresponding production order is sent to the system. If, however, the average time in the system is longer than \( lt \), the fulfilled production orders leave the system without passing by the inventory \( II \). Thus, if the inventory holding cost for raw material \( C_i \) is set to one, the inventory holding costs for a product managed by MRP become
This is a preprint of an article submitted for consideration in Computers and Industrial Engineering 2009

\[ IC_{MRP} = \sum_{i}^{\Omega} p_i s_i (m_i + (C_0/C_1)\lambda d) \]

where

\[ d = \max \left( lt - \frac{m_i}{\lambda}; 0 \right) \]

with

- \( \Omega \) = number of system states
- \( p_i \) = steady-state probability of state \( i \)
- \( s_i \) = order size proportional to service time per job for any type of product
- \( m_i \) = number of production orders in the system (in queue and in service) for state \( i \)
- \( lt \) = predefined lead time for all products
- \( C_0 \) = inventory holding cost for finished product
- \( \lambda \) = mean arrival rate of production orders
- \( d \) = difference between the lead time \( lt \) and the time in the system of a production order \( \frac{m_i}{\lambda} \) (\( m \) for A-products and \( n \) for B-products)

Similarly, the inventory holding costs for the A-product managed by the kanban technique become

\[ IC_{Kanban}^A = \sum_{i}^{\Omega} p_i s_A (m_i + (C_0/C_1)(M - m_i)) \]

with

- \( \Omega \) = number of system states
- \( p_i \) = steady-state probability of state \( i \)
- \( s_A \) = order size proportional to service time per job
- \( m_i \) = number of waiting kanbans for state \( i \)
- \( M \) = total number of kanbans.

Consequently, the total inventory costs for the MRP, MRP prior and DSSPL model become

\[ IC_{MRP(prior)} = IC_{MRP}^A + IC_{MRP}^B \]

and

\[ IC_{DSSPL} = IC_{Kanban}^A + IC_{MRP}^B \]

where \( IC_{MRP}^A \) (respectively \( IC_{MRP}^B \)) is the inventory costs of the A-products (resp. B-products), managed by the MRP strategy, calculated using equation 3. Equation 6 differentiates the costs of the A-products and the B-products in the DSSPL model since they are managed by kanbans and MRP respectively. \( IC_{Kanban} \) represents the inventory costs of the A-production orders controlled by Kanbans in the DSSPL model.

Since the production schedule of all products in MRP based systems is generated based on predefined lead times, a job is considered as fulfilled if its time in the system (queue and service) is
shorter or equal to the predefined lead time \( l_t \). Consequently, the service level for the MRP and MRP\(_{\text{prior}}\) concepts becomes
\[
SL_{\text{MRP(prior)}} = \frac{L'_{A} \rho_{A}s_{A} + L'_{B} \rho_{B}s_{B}}{L_{A} \rho_{A}s_{A} + L_{B} \rho_{B}s_{B}}
\]  
(7)

where
\[
L' = \sum_{i} \rho_{i} \neq \sum_{i} \rho_{i} \neq
\]
and
\[\Omega\]
with
\[
\Omega = \text{number of system states where the time in system } (\frac{m_{i}}{\lambda_{A}} \text{ or } \frac{n_{i}}{\lambda_{B}}) \text{ is smaller or equal to the predefined lead time } (l_{tA} \text{ or } l_{tB})
\]
\[
m_{i} = \text{number of production orders } (m \text{ for A-products, } n \text{ for B-products}) \text{ in system (in queue and in service) for state } i
\]
\[
\rho_{A} = \frac{\lambda_{A}}{\mu_{A}}
\]
\[
\rho_{B} = \frac{\lambda_{B}}{\mu_{B}}
\]

The determination of the service level for products managed by the kanban technique is not based on the effective and predefined lead time but on the capacity to satisfy a demand immediately from finished kanban jobs.

We define:
\[
b_{i} = \text{number of back ordered jobs for state } i \text{ in BO}
\]
\[
\Omega^{K} = \text{system states where no back orders occur } (b_{i} = 0)
\]

The service level for the DSSPL concept becomes therefore
\[
SL_{\text{DSSPL}} = \frac{L'_{K} \rho_{A}s_{A} + L'_{B} \rho_{B}s_{B}}{L_{K} \rho_{A}s_{A} + L_{B} \rho_{B}s_{B}}
\]  
(8)

where
\[
L'_{K} = \sum_{i} \rho_{i} \neq \sum_{i} \rho_{i} \neq
\]
and
\[\Omega^{K}\]

3.1.3. Experimental design

Beside the different MPC concepts, the modelled manufacturing environment is further characterized by the following parameters:
\[
\text{stp} = \text{set-up time in case of a change from A- to B-jobs and vice-versa}
\]
\[
\text{Ratio}_{AB} = \text{ratio between loads generated by A- and B-jobs defined by } \text{Ratio}_{AB} = \frac{\rho_{A}}{\rho_{B}}
\]
\[
\text{Ratio}_{S} = \text{ratio between the size of A- and B-jobs defined by } \text{Ratio}_{S} = \frac{s_{A}}{s_{B}}
\]
\[
\text{Cost}_{Ratio} = \text{ratio between the inventory holding cost for the raw material } (C_{1}) \text{ and the finished product } (C_{0}) \text{ defined by } \text{Cost}_{Ratio} = \frac{C_{0}}{C_{1}}
\]
In the case that set-ups are required, the mean service rates are therefore modified to 
\[
\mu_{As} = \frac{1}{E[s_A + stp]} \quad \text{and} \quad \mu_{Bs} = \frac{1}{E[s_B + stp]},
\]

The experimental design is based on a two-step approach. In a first step, a configuration with a service level \( SL = 0.95 \) for every MPC concept is determined for standard parameter values and for a load level \( \rho = \rho_A + \rho_B = 0.4 \). The lead times \( lt \) and the number of kanbans \( nmbk \) are therefore adjusted in order to reach the targeted service level. In the second step, the models are evaluated with the parameter values indicated in Table 1. A reduction of the lot size expressed by \( RatioS \) is only applied to the A-products in the DSSPL concept, since it is assumed that significant lot size reductions are only applicable with the kanban technique. The variation of \( lt \) represents the forecast error due to the fact that MRP systems have to determine the production orders before the effective demand has occurred. The low and high value of \( lt \) represent therefore an under- or overestimation of the real demand.

The capacities of the queues defining the standard configuration of the analyzed MPC models are summarized in Table 2. The results are reported and analysed on different tables.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>stp</td>
<td>0, 0.5</td>
</tr>
<tr>
<td>RatioAB</td>
<td>1, 4</td>
</tr>
<tr>
<td>RatioS</td>
<td>1, 0.4 (only for A-products in DSSPL)</td>
</tr>
<tr>
<td>CostRatio</td>
<td>1, 5</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.4</td>
</tr>
<tr>
<td>( thl )</td>
<td>1, 2</td>
</tr>
<tr>
<td>( s_A, s_B )</td>
<td>1</td>
</tr>
<tr>
<td>( lt_A, lt_B )</td>
<td>10, 15, 20</td>
</tr>
</tbody>
</table>

Table 1  Experimental design (standard values in bold face)

<table>
<thead>
<tr>
<th>Capacity of queues</th>
<th>MRP</th>
<th>MRPprior</th>
<th>DSSPL</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M )</td>
<td>8</td>
<td>8</td>
<td>4 (nmbk)</td>
</tr>
<tr>
<td>( N )</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>( B )</td>
<td>-</td>
<td>-</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 2  Standard configuration for a service level of 0.95
### 3.2. Computational results

Three configurations of the DSSPL model are used: DSSPL.1.1, DSSPL.1.04, and DSSPL.2.04. They are termed according to the syntax DSSPL.thl.RatioS (thl=1 RatioS=1; thl=1 RatioS=0.4; thl=2 RatioS=0.4). The results shown in Table 3 are obtained by solving with an iterative Gauss-Seidel procedure the stationary equations:

\[
\begin{align*}
0 &= pQ, \\
1 &= pe,
\end{align*}
\]

where \( p \) is the steady-state probability vector, \( Q \) is the infinitesimal generator of the continuous-time Markov chains, and \( e \) is the unitary vector (Gross and Harris 1998). The results have been compared to those of simulation, performed with a discrete-event simulator based on the ARENA simulation package (simulation results are in italic font on doubled lines in Table 3). The computational results explain the impact of the DSSPL key concepts and assumptions on the chosen performance metrics and allow the determination of the optimal parameter settings and application domain of DSSPL. Table 3 reports also the performance measures of MRP and MRPprior in comparison to the three configurations of DSSPL for various values of \( Ratio_{AB} \), \( CostRatio \) and \( stp \), and for lead times \( lt = 15 \).

The graphs shown in Figure 2 are obtained by dividing the results from the MRPprior and DSSPL model by those from the MRP model. On each of the four graphs of the figure 2, and for each configuration, 2 test series have been performed with \( stp = 0 \) (represented on the figure by an underscore ‘_’) and \( stp = 0.5 \).
Consequently, service level ratios greater than one indicate a better service level compared to MRP. Similarly, inventory cost ratios smaller than one indicate lower inventory costs compared to MRP.

The following observations can be made concerning the obtained results:

- The service level performance of DSSPL.1.1 is better than those of MRP particularly for high values of RatioAB and stp, however at significant higher inventory levels;
- The inventory costs of DSSPL.1.04 and DSSPL.2.04 are similar to those of MRP for high values of RatioAB for a similar or improved service level performance;
- From the proposed configurations, inventory costs are much more sensitive to the MPC method (variation up to 450%) than service levels (variation up to 8%);
- A relationship between the parameters thl, stp and RatioAB can be observed particularly for DSSPL.1.04. In fact, for low values of RatioAB and thl, an increase of stp results in a reduced service level compared to those of MRP.

The impact of a variation of the lead time $lt$ ($lt = 10$, represented on the figure by an underscore _, $lt = 15$ and $lt = 20$) on the performance of MRP, MRPPrior, DSSPL.1.04 and DSSPL.2.04 is shown in Figure 3 for two different test series on the systems (2 graphs) from which the following observations can be made:

- The variation of $lt$ (and thus forecast errors) has a significant impact on the logistic performance of both MRP and MRPPrior. High service levels are reached for high values of $lt$ ($lt = 20$), however with increased inventory levels. Low inventory levels can only be reached with low values of $lt$ ($lt = 10$) that result however in a poor service level performance;
- The variation of $lt$ has smaller impacts on the inventory cost performance of DSSPL than on MRP and MRPPrior, particularly for high values of RatioAB, since only the performance of the production of B-products is affected by variations of $lt$. In the case of DSSPL.2.04, the impact on the service level performance is also significantly smaller.

Generally, DSSPL exhibits satisfactory inventory level performance only when the size of kanbans related to the A-product can be decreased compared to the lot sizes used in an MRP system. This is due to the fact that the replenishment of the finished kanban jobs is triggered only when a demand occurs. Finished kanban jobs produced due to a past demand are thus waiting until a next demand occurs. If the capacity and number of kanbans and the added value (CostRatio) are high, JIT/kanban systems lead generally to increased inventory values compared to those of the MRP system (DSSPL.1.1), that triggers production only before a future demand will occur. However, as shown in Figure 3, the performance of the MRP (push) concept depends significantly on the accuracy of the forecast of the future demand.

The resulting uncertainties in a MRP system must be buffered by time and/or inventory that leads to increased lead times and inventory levels. In addition, by allocating a higher priority to the A-products having smaller lot sizes than B-products, DSSPL behaves like the SI dispatching rule (Shortest Imminent operation) which showed in a survey on dispatching rules good overall results (Blackstone et al. 1982). Beside the size of the kanbans for A-products, the priority threshold level thl is another important parameter. The results in Figures 2 and 3 show how the logistic performance of DSSPL.2.04 ($thl = 2$;
ratioS = 0.4) is improved compared to those of DSSPL.1.04 (thl = 1; ratioS = 0.4). A better service level is mainly due to a reduced number of setups that is achieved by a grouping of kanban jobs. The reduced inventory costs are mainly due to the lower level of waiting finished kanban jobs, since the replenishment is triggered only when more than one job has been consumed by the demand.

4. Industrial implementation of DSSPL

In order to improve the logistic performance of a Swiss micromotor producer, DSSPL has been implemented between a supply unit and the final assembly line. The presented case with its particular manufacturing environment characteristics serves well to justify and validate the design decisions taken for DSSPL. It illustrates how DSSPL copes with problems of limited resources and interface coordination problems occurring in firms operating in assemble-to-order (ATO) manufacturing environments.

4.1. Problem description

Firms that offer a wide variety of products such as the micromotor producer, typically choose an ATO master production schedule approach when delivery speed requirements and the changing product mix prevent the exclusive choice of the make-to-order (MTO) or make-to-stock (MTS) option, respectively (Vollmann et al. 1996). One condition for a successful implementation of the ATO concept is a certain level of modularization of the product structure where components and intermediate subassemblies are assembled in the last production steps to generate a wide variety of products. These final production or assembly steps are performed in an MTO environment. Thus, in order to further reduce lead times and to fulfill the requirement to react quickly to a changing customer demand, firms can choose from various PPC methods (JIT/Kanban, CONWIP, MRP, DSSPL, ...) for the management of these final assembly lines. However, an imperative condition for the performance of such final assembly lines is the availability of components and intermediate subassemblies supplied by internal or external supply units. In fact, the master production schedule of the supply units is partly or entirely derived from unreliable forecasts since the order penetration point (point in the production process where the product is assigned to a customer order) is defined typically at the beginning of the final assembly process.

In summary, the manufacturing environment of the selected DSSPL pilot line has the following characteristics:

- The production system of the micromotor manufacturer is structured according to the ATO concept. The final assembly line is organized according to a flow manufacturing technique, whereas the feeder shops including the plastic molding shop are managed by an MRP-system;
- Coordination problems occur frequently between the final assembly line and the different feeder shops. In fact, most production interruptions are provoked by a lack of subassemblies supplied by the plastic molding shop. The coordination problems are mainly caused by production schedules for the feeder shop inconsistent with the final assembly schedule;
- An increase of the production capacity of the feeder shops (particularly of the plastic molding shop) is difficult due to limited financial and human resources;
- An outsourcing of the plastic molding unit is difficult due to the high quality and technical requirements of the subassemblies.

The subassemblies manufactured by the plastic molding shop are mainly plastic molded parts with metallic inserts. Since the two subassemblies, collector and flange, show the biggest turnover within the plastic molding shop, it is decided to concentrate the analysis on these two types of parts. The approximately 100 variants of these two parts are manufactured on two work centers. The first work center producing the flange parts includes only one plastic molding machine whereas the work center producing the collector parts includes three plastic molding machines. All collector parts can be produced on one of the three plastic molding machines of the corresponding work center.
Every molding cycle is executed by an operator. Additionally, a mechanic is required to prepare the molding tools and adjust the molding process parameters in case of a setup. These setup can vary from 20 minutes to one day. The working period of the operators and mechanics is normally one shift. However, in the case of high workload and the availability of additional operators, a second shift can be added. Consequently, the main characteristics of this manufacturing process are the need for specialists for the handling of the expensive and sensitive molding tools and machines and the high setup times. An additional problem is the fact that the human resources (mainly the mechanics) are rarely available immediately when needed. This is due to the limited availability of qualified mechanics and operators and the fact that both mechanics and operators are involved in the production and the maintenance of other machines in the plastic molding shop.

4.2. Implementation procedure and results
The most important characteristics of the DSSPL pilot line are:
- Six out of the approximately 100 variants of the collector and flange parts have been identified as A-items. The criteria on which the classification is based are high relative volumes and a stable demand. The stability of the demand is measured by the coefficient of variation $CV$ of the size of the demand and the interarrival times of the weekly demand. According to the variability classification scheme developed by Hopp and Spearman (1996), a small variability ($0 < CV < 0.8$) of the demand is chosen as the first criterion for the A-items;
- The A-items are managed by a one-card Kanban loop, connecting the plastic molding shop directly with the final assembly line. In order to keep the MRP database consistent also for the A-items, a notice is sent from the plastic molding shop to central planning after the completion of each kanban;
- The simplest dispatching rule giving always priority to the A-items with respect to the B-items (non-preemptive) is used in the two work centers;
- The choice of the packaging for the delivered A-items and the quality control requirements has been unified and coordinated between the plastic molding shop and the final assembly line;
- The responsibility for releasing production quantities of the A-items have been shifted from central planning to the foremen of the plastic molding shop;
- Weekly meetings of all involved responsibles of the final assembly line, central planning and plastic molding shop for coordination issues are held. The most important discussion topics are quality problems and anticipated changes in demand and capacity planning.

The impact of DSSPL on the average inventory level of the A-items has been analyzed based on the inventory level data for a period of three months before and after the implementation. For five of the selected A-items, the average inventory level has been decreased (-19... -39%) without causing shortages of items. The average inventory level of one A-item (due to reduced demand shortly after implementation of DSSPL) and all B-items has not been changed significantly. In addition to the validation of the new MPC method based on data analysis, a qualitative evaluation of the perceived success of the implemented DSSPL method was done by interviewing the people concerned, six months after implementation. The main outcomes of this survey are as follows:
- The lead time and the availability of the A-items on the final assembly line has been considerably improved. In fact, no more stockout situations have been observed for the A-items;
- Even though priority is given to the production of A-items, the performance of the production for B-items has not decreased significantly. This is due to the fact that the production of the A-items is simplified and only triggered when really needed;
- Communication and coordination between the plastic molding shop, the final assembly line and the central planning have been improved;
- Further improvements of the performance of the plastic molding shop can only be achieved by either reducing the setup times or by increasing the availability of the operators and mechanics.
5. Conclusions
In this paper, the hybrid Manufacturing Planning and Control method Double Speed Single Production Line (DSSPL) is presented, modelled using Markov chains and validated with an industrial case study. The key concepts of DSSPL are derived from a strategic reasoning that seeks the optimal allocation of limited resources in order to obtain a maximum impact in terms of customer service and satisfaction. This is achieved by the fact that the most important products (A-products) can be delivered to the clients in a Just-in-Time manner. At the same time, the production planning is simplified thanks to a transparent allocation of priorities to the different product groups. Less perturbations due to forecast errors can be expected from DSSPL, since the products A managed by the JIT/kanban technique triggers production only when an effective demand occurs.

It has been shown that, beside an improved logistic performance in terms of service level and inventory costs, DSSPL has some important benefits over other MPC concepts from an operational point of view. The application domain of DSSPL is not limited by the variety of products or the structure of the production process. However, less significant improvements with respect to the MRP concept can be expected when important setups exclude the efficient use of the kanban technique or when the product mix is too unstable over a planning period.

A direction for future research on DSSPL is the extension of DSSPL to value-adding networks from the external supplier to the final assembly line. This subject is challenging due to the limitation of Markov chains as a tool for modelling large-scale systems. In addition, simulation studies with more representative should be performed to better understand the behaviour of DSSPL within its framework (in particular priority rules, criteria for the choice of A- and B-products and use of inventory control for the management of B-products) and to define more accurately its application domain compared to other hybrid MPC such as CONWIP or POLCA.

Acknowledgments
The authors wish to thank the Swiss Agency of Promotion and Innovation (CTI) as well as the industrial partners for their financial support of this work and for providing the opportunity to implement DSSPL.

Appendix
The steady-state balance equations are given for the MRP model for the case where both M and N are set to two:

\[ 0 = - (\lambda_A + \lambda_B) p_0 + \mu_A p_1 + \mu_B p_2, \]
\[ 0 = - (\lambda_A + \lambda_B) p_1 + \mu_B p_{12} + \mu_A p_{11} + \lambda_A p_0, \]
\[ 0 = - (\lambda_A + \lambda_B) p_2 + \mu_B p_{22} + \mu_A p_{21} + \lambda_B p_0, \]
\[ 0 = - (\lambda_A + \lambda_B + \mu_B) p_{12} + \mu_B p_{122} + \mu_A p_{121}, \]
\[ 0 = - (\lambda_A + \lambda_B + \mu_A) p_{21} + \mu_A p_{211} + \mu_B p_{212}, \]
\[ 0 = - (\lambda_B + \mu_A) p_{11} + \mu_B p_{112}, \]
\[ 0 = - (\lambda_A + \mu_B) p_{22} + \mu_A p_{221}, \]
\[ 0 = - (\lambda_A + \mu_B) p_{122} + \mu_A p_{1221}, \]
The steady-state balance equations for the MRPprior model are defined as follows:

\[
\begin{align*}
0 &= -(\lambda_A + \mu_A + \mu_{BS}) p_{212} + \mu_A p_{2121}, \\
0 &= -(\lambda_A + \mu_{BS}) p_{221} + \mu_A p_{2211}, \\
0 &= -(\lambda_B + \mu_{BS}) p_{112} + \mu_B p_{1122}, \\
0 &= -(\lambda_B + \mu_A) p_{121} + \mu_{BS} p_{1212}, \\
0 &= -(\lambda_B + \mu_A) p_{211} + \mu_B p_{2112},
\end{align*}
\]

with \(2 \leq m \leq M\),

\[
\begin{align*}
0 &= -(\lambda_A + \lambda_B + \mu_A) p_{0} + \mu_A p_{101} + \mu_B p_{012}, \\
0 &= -(\lambda_A + \lambda_B + \mu_{BS}) p_{101} + \mu_A p_{201} + \mu_B p_{112} + \lambda_A p_{0}, \\
0 &= -(\lambda_A + \lambda_B + \mu_B) p_{012} + \mu_A p_{212} + \mu_B p_{022} + \lambda_A p_{0}, \\
0 &= -(\lambda_A + \lambda_B + \mu_A) p_{m01} + \mu_{BS} p_{m12} + \mu_B p_{m12} + \lambda_A p_{m-1, 1, 2},
\end{align*}
\]

with \(2 \leq m \leq M\),

\[
\begin{align*}
0 &= -(\lambda_A + \lambda_B) p_{m01} + \lambda_A p_{M-1, 0, 1} + \mu_{BS} p_{M12}, \\
0 &= -(\lambda_A + \lambda_B + \mu_A) p_{101} + \mu_A p_{201} + \mu_B p_{112} + \lambda_A p_{0}, \\
0 &= -(\lambda_A + \lambda_B + \mu_{BS}) p_{m12} + \mu_B p_{m12} + \lambda_A p_{m-1, 1, 2},
\end{align*}
\]

with \(1 \leq m < N\),

\[
\begin{align*}
0 &= -(\lambda_A + \lambda_B + \mu_A) p_{m01} + \lambda_A p_{M-1, n, 1} + \mu_{BS} p_{M, n+1, 2} + \lambda_B p_{M, n-1, 1},
\end{align*}
\]

with \(1 \leq m < N\),

\[
\begin{align*}
0 &= -(\lambda_A + \lambda_B + \mu_A) p_{0n2} + \mu_A p_{1n1} + \mu_B p_{0, n+1, 2} + \lambda_B p_{0, n-1, 2},
\end{align*}
\]

with \(2 \leq n < N\),

\[
\begin{align*}
0 &= -(\lambda_A + \lambda_B + \mu_A) p_{mn1} + \mu_A p_{m+1, n, 1} + \mu_{BS} p_{m+1, n, 1} + \lambda_A p_{m-1, n, 1} + \lambda_A p_{m, n-1, 1},
\end{align*}
\]

with \(2 \leq m < M\) and \(1 \leq n < N\),
The steady-state balance equations for the DSSPL model are defined as follows:

0 = \left( \lambda_A + \mu_B \right) P_{0N2} + \lambda_B P_{0,N-1,2} + \mu_{AS} P_{1N1},

0 = \left( \lambda_A + \mu_A \right) P_{mN1} + \lambda_B P_{m,N-1,1} + \mu_{AP} p_{m+1,N,1} + \lambda_A p_{m-1,N,1}

with \( 2 \leq n < N \),

0 = -\left( \lambda_A + \mu_{AS} \right) P_{1N1} + \mu_A P_{2N1} + \lambda_B P_{1,N-1,1},

0 = -\mu_A P_{MN1} + \lambda_A P_{M-1,N,1} + \lambda_B P_{M,N-1,1},

0 = -\left( \lambda_A + \lambda_B + \mu_{BS} \right) P_{mn2} + \lambda_A p_{m-1,n,2} + \lambda_B p_{m,n-1,2}

with \( 1 \leq m < M \) and \( 2 \leq n < N \),

0 = -\left( \lambda_B + \mu_{BS} \right) P_{Mn2} + \lambda_A P_{M-1,n,2} + \lambda_B P_{M,n-1,1}

with \( 2 \leq n < N \),

0 = -\left( \lambda_A + \lambda_B + \mu_{BS} \right) P_{m12} + \lambda_A p_{m-1,1,2}

with \( 1 \leq m < M \),

0 = -\left( \lambda_A + \mu_{BS} \right) P_{M12} + \lambda_A P_{M-1,1,2},

0 = -\left( \lambda_A + \mu_{BS} \right) P_{mN2} + \lambda_A P_{m-1,N,2} + \lambda_B P_{M,N-1,2},

\text{The steady-state balance equations for the DSSPL model are defined as follows:}

0 = \left( \lambda_A + \lambda_B \right) P_{00} + \mu_A P_{001} + \mu_B P_{002} \text{ if } tl = 1\}

0 = -\left( \lambda_A + \lambda_B + \mu_A \right) P_{001} + \lambda_A P_{0} + \mu_{BS} P_{002} \text{ if } tl = 1\}

0 = -\left( \lambda_A + \lambda_B + \mu_A \right) P_{0m01} + \mu_A P_{0,m+1,0,1} + \mu_{BS} P_{m01} + \lambda_A P_{0,m-1,0,1}

with \( 2 \leq m < M \),

0 = -\left( \lambda_A + \lambda_B + \mu_A \right) P_{0M01} + \mu_A P_{1M01} + \mu_{BS} P_{0M12} + \lambda_A P_{0,M-1,0,1}

0 = -\left( \lambda_A + \lambda_B + \mu_A \right) P_{bM01} + \mu_A P_{b+1,M,0,1} + \mu_{BS} P_{b+1,M,0,1} + \lambda_A P_{b-1,M,0,1}

with \( 1 \leq b < B \),

0 = -\left( \lambda_A + \lambda_B + \mu_A \right) P_{01n1} + \mu_A P_{0221} + \lambda_B P_{0,1,n-1,1} + \mu_{BS} P_{0,1,n+1,2} \text{ if } tl = 1\}

with \( 1 \leq n < N \),
0 = - (λ_A + λ_B + μ_A)p_0mn1 + μ_A p_0, m + 1, n, 1 +
  λ_A p_0, m - 1, n, 1 + λ_B p_0, m, n - 1, 1 +
  \{ μ_Bp_0, m, n + 1, 2 if thl ≤ m \}

with (1 ≤ n < N) and (2 ≤ m < M),

0 = - (λ_A + λ_B + μ_A)p_0Mn1 + μ_A p_0, 0, M - 1, n, 1 +
  λ_B p_0, 0, M, n - 1, 1

with (1 ≤ n < N),

0 = - (λ_A + λ_B + μ_A)p_bMn1 + μ_A p_b + 1, M, n, 1 +
  λ_B p_b, M, n - 1, 1

with (1 ≤ b < B) and (1 ≤ n < N),

0 = - (λ_B + μ_A)p_bMN1 + μ_B p_B, M, n + 1, 2 +
  λ_B p_B - 1, M, n, 1 + λ_B p_B, M, n - 1, 1

with (1 ≤ n < N),

0 = - (λ_A + μ_A)p_0MN1 + μ_A p_1MN1 +
  λ_A p_0, M - 1, N, 1 + λ_B p_0, M, N - 1, 1,

0 = - (λ_A + μ_A)p_01N1 + μ_A p_02N1 +
  λ_B p_0, 1, N - 1, 1,

(3)

0 = - (λ_A + μ_A)p_0mn1 + μ_A p_0, m + 1, N, 1 +
  λ_A p_0, m - 1, N, 1 + λ_B p_0, m, N - 1, 2

with (2 ≤ m < M),

0 = - (λ_A + μ_A)p_bMN1 + μ_A p_b + 1, M, N, 1 +
  λ_A p_b - 1, M, N, 1 + λ_B p_b, M, N - 1, 1

with (1 ≤ b < B),

0 = - μ_A p_BMN1 + λ_A p_B - 1, M, N, 1 +
  λ_B p_B, M, N - 1, 1,

0 = - (λ_A + λ_B)p_0mn2 - \{ μ_Bp_0mn2 if thl > 1 \} +
  λ_A p_0, m - 1, n, 2 + λ_B p_0, m, n - 1, 2 +
  \{ μ_Bp_0mn2 if thl = 1 \} +
  \{ μ_Bp_0, m, n + 1, 2 if thl ≥ 1 \},

with (1 ≤ m < M) and (2 ≤ n < N),

0 = - (λ_A + λ_B + μ_B)p_0mn2 + λ_A p_0, M - 1, n, 2 +
  \{ μ_Bp_0mn2 if thl > 1 \}

with (1 ≤ m < M),

0 = - (λ_A + μ_B)p_0MN2 + λ_A p_0, M - 1, N, 2 +
  λ_A p_0, M, N - 1, 2,

0 = - (λ_A + μ_B)p_bMN2 + λ_A p_b, M - 1, N, 2 +
  λ_B p_b, M, N - 1, 2
with $1 \leq b < B$,

$$0 = -\mu_{BMN} + \lambda_{APB,M-1,N,2} + \lambda_{BPB,M,N-1,2}.$$ 

References


Figure 1  Concepts of the MRP-, MRPprior- and DSSPL-models
Figure 2 Comparison of logistic performance of MRP vs. MRPprior, DSSPL.1.1, DSSPL.1.05, DSSPL.2.1 and DSSPL.2.05 for setup = 0 and 0.5 ( _ symbol for stp = 0)

![Graph Comparison of logistic performance of MRP vs. MRPprior, DSSPL.1.1, DSSPL.1.05, DSSPL.2.1 and DSSPL.2.05 for setup = 0 and 0.5 ( _ symbol for stp = 0) with varying parameters RatioAB = 4; CostRatio = 1; lt = 15 and RatioAB = 4; CostRatio = 5; lt = 15]
Figure 3  Performance of MRP, MRPprior and DSSPL for increasing values of $lt$ (’_’ symbol for $lt = 10$)