Robust Power Control for Cognitive Radio in Spectrum Underlay Networks

Nan Zhao\textsuperscript{1} and Hongjian Sun\textsuperscript{2}

\textsuperscript{1}School of Information and Communication Engineering, Dalian University of Technology
Dalian, Liaoning, 116024, China
[e-mail: zhaonan@dlut.edu.cn]

\textsuperscript{2}Centre for Telecommunications Research, King’s College
London, United Kingdom
[e-mail: hongjian.sun@kcl.ac.uk]

*Corresponding author: Nan Zhao

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Abstract

Power control is a key technique in spectrum underlay cognitive network to guarantee the interference temperature limit of the primary users (PUs) and the quality of service of the secondary users (SUs). In this paper, a robust power control scheme via link gain pricing with $H_\infty$ estimator is proposed. The scheme guarantees the interference temperature of the PUs through operating in the network-centric manner, and keeps the fairness between the SUs through link gain pricing. Furthermore, the $H_\infty$ filter is also used in the proposed scheme to estimate the channel variation, and thus the power control scheme is robust to the severe channel fading. Plenty of simulations are taken, and prove its superior robust performance against the channel fading, and its effectiveness in guaranteeing the interference temperature limit of the PUs.

**Keywords:** Cognitive radio, spectrum sharing, power control, spectrum underlay network, $H_\infty$ filter

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1. Introduction

Wireless communication networks have developed rapidly over the last decade. This has led to growing demand for radio spectrum, and the spectrum is becoming a scarce resource. On the other hand, recent studies by the Federal Communications Commission (FCC) show that the current utilization of some spectrum bands is as low as 15% [1]. Therefore, in these years, cognitive radio (CR) network-related research has progressed rapidly [2], due to conflicts between the scarcity and low utilization of the wireless spectrum resource. In CR, the secondary users (SUs) need to opportunistically sense the idle channels [3][4], and then share the spectrum resource with the primary users (PUs). There are two models of spectrum sharing, which are overlay spectrum sharing model and underlay spectrum sharing model [5][6], as shown in Fig. 1.

![Fig. 1. Overlay spectrum sharing and underlay spectrum sharing](image)

In the overlay spectrum sharing model shown in Fig. 1, the SUs identify the idle spectrum via spectrum sensing, and the SUs are allowed only to access the spectrum that are completely empty of any primary operation. For the underlay spectrum sharing scheme, the SUs coexists with the PUs by sharing the primary spectrum, such that they do not violate the interference temperature defined by the PUs. Since the priority of the PUs should be preserved, the FCC proposed a quantitative standard, named interference temperature, to quantify and manage the interference caused by the SUs [7]. Hence, power control is a key technique for the underlay spectrum sharing model, and the power of SU signal should be well adjusted so that the interference at PUs caused by all the SUs is below the interference temperature limit set by PUs. In this paper, only spectrum underlay in CR networks is considered, and power control for spectrum underlay cognitive networks is researched.

Since power control is important for the spectrum underlay cognitive networks, plenty of algorithms have been proposed to pursue better quality of service (QoS) of SUs and satisfy the interference temperature limit of the PUs [8][9][10]. In [8], two auction mechanisms for sharing spectrum through allocating the transmit power are proposed, subject to a constraint on the interference temperature at the measurement point. However, the optimal solution can be obtained only when the manage center has the knowledge of all the users’ utility functions, and it is difficult to realize for a distributed cognitive radio network. Two centralized removal algorithms named I-SMIRA and I-SMART(R) are designed in [9], and through them the
number of admitted SUs is maximized with its QoS constraints and PU’s interference temperature limit. However, these two algorithms require the knowledge of instantaneous channel gains between nodes to be known at the central controller, and it is unpractical. In [10], an optimal power control algorithm for SUs to satisfy outage probability constraint of the PU is introduced, which is only an improvement or another form of the interference temperature limit. In these power control schemes in [8][9][10], none of them guarantee the fairness between the SUs using the information of the channel state. This may cause the transmit power in bad channel lower and that in good channel higher, and it is unfair for the SUs in the network. Furthermore, these algorithms are all iterative schemes and converge slowly; hence, the robustness to the channel fading of these algorithms is rather weak.

Traditional power control schemes using iterative algorithms [8][9][10][11][12][13][14][15] are inappropriate to compensate severe channel fading, because they require long time to converge. Usually, these algorithms assume that the link gains and fading conditions remain almost unchanged during several iterations period. Hence, filtering techniques for estimating interference have been used in power control to combat the channel fading. In [16][17], the Kalman filter is applied to predict the received interference, assuming that it is corrupted only by additive white Gaussian noise (AWGN). These power control schemes using Kalman filter are effective only if the fluctuation of the channel variation and the measurement noise are Gaussian distributed. However, the fluctuation of the channel variation is not Gaussian distributed in general. Therefore, the \( H_\infty \) filter is a more appropriate choice to estimate the interference and channel variation in power control [18][19][20][21]. Compared with the Kalman filter, the \( H_\infty \) filter considers the system filtering regardless of the nature of the system and measurement disturbances [22][23].

In this paper, the cognitive spectrum underlay network is considered, and a robust power control scheme through link gain pricing using \( H_\infty \) estimator is proposed. The scheme considers the interference temperature of the PUs and guarantees the fairness between the SUs through link gain pricing. The \( H_\infty \) filter is also applied to estimate the channel variation, and thus the power control scheme is robust to the channel fading.

The rest of this paper is organized as follows. In Section 2, we describe the system model of the cognitive spectrum underlay network, and give the goal of power control in this situation. In Section 3, the robust power control scheme via link gain pricing with \( H_\infty \) estimator is proposed and the existence of its global minimum is proved. The \( H_\infty \) filter used in this power control scheme is also demonstrated in Section 3. In Section 4, the advantages of the proposed robust power control scheme are illustrated through plenty of simulations. Conclusions are drawn in Section 5.

### 2. System Model

Consider a cognitive radio network with \( N \) SUs, and correspondingly there are \( N \) transmitters and \( N \) receiver of SUs using code division multiple access (CDMA) communication model for its low spectrum power density and great multi-access capability. There are also \( M \) Measurement points (MPs) in the network to measure the power of received interference at the PUs. The architecture of the cognitive radio network is shown in Fig. 2.

As depicted in Fig. 2, \( P_i \) is the transmit power of SU\( i \), \( h_{ij} \) is the link gain from the transmitter of SU\( i \) to the receiver of SU\( j \), \( g_{ik} \) is the link gain from the transmitter of SU\( i \) to the MP\( k \). For each secondary user \( i \), the signal to interference plus noise ratio (SINR) of receiver at SU\( i \) is given by
\[
\gamma_i(k) = \frac{Lh_i P_i(k)}{\sum_{j=1, j \neq i} h_j P_j(k) + \sigma^2}, \quad i = 1, 2, ..., N
\]  

(1)

where \( L \) is the processing gain in spread spectrum wireless systems, \( k \) stands for the \( k \)th time instant, and \( \sigma^2 \) is the background noise power that is assumed to be the same for the receivers of all the SUs.

Transmitter of SU 1

\[ g_{11} \]

\[ g_{1M} \]

\[ \vdots \]

\[ g_{NM} \]

Receiver of SU 1

\[ h_{11} \]

\[ h_{1N} \]

\[ \vdots \]

\[ h_{NN} \]

Transmitter of SU \( N \)

Denote the denominator in (1) by \( I_i \), which represents the received interference plus background noise at SU\( i \). Thus, the SINR in (1) can be reformulated as

\[
\gamma_i(k) = \frac{Lh_i P_i(k)}{I_i(k)}, \quad i = 1, 2, ..., N
\]  

(2)

Define

\[
\delta_i(k) = \frac{Lh_i}{I_i(k)}, \quad i = 1, 2, ..., N
\]  

(3)

where \( \delta_i(k) \) represents the channel variation (introduced in [24]), and it can be predicted by the \( H_\infty \) filter in the proposed power control algorithm of this paper.

The goal of the power control scheme is that every SU should achieve the value of its own SINR above a certain target value, and make sure that the received interference power at every MP is below the interference temperature. That is,

\[
\gamma_i \geq \gamma_i^{\text{tar}}, \quad i = 1, 2, ..., N
\]  

(4)
\[ \sum_{i=1}^{N} s_i P_i \leq Z_k, \quad k = 1, 2, \ldots, M \]  

(5)

where \( \gamma_i^{\text{tar}} \) is the target value of SINR for SU\( i \), and \( Z_k \) is the interference temperature set by PU\( k \).

### 3. Robust Power Control with Estimator for Spectrum Underlay Network

#### 3.1 Proposed Power Control Scheme via Link Gain Pricing with Estimator

Define SINR error of SU\( i \) at time \( k \) as

\[ E_i(k) = \gamma_i^{\text{tar}} - \gamma_i(k) \]  

(6)

which represents the deviation between the target QoS \( \gamma_i^{\text{tar}} \) and the received QoS \( \gamma_i(k) \) of SU\( i \) at time \( k \). A SU may adapt its transmit power according to this measurement so as to minimize its own SINR deviation and guarantee its own QoS. Thus this operation only benefits SU\( i \) regardless of the other SUs in the network, and this may cause the transmit power of SU\( i \) rather high and introduce much more interference to the SU network and PUs. Therefore the transmit power should also be priced in our power control algorithm, and it will benefit the whole SU network and guarantee the interference temperature limit of PUs. Accordingly, a novel performance criterion of SU\( i \) is proposed as

\[ J_i(k) = E_i^2(k+1) + a_i h_i P_i^2(k+1) \]  

(7)

In (7), the performance criterion at time \( k \) \( J_i(k) \) depends on the SINR error and the transmit power at time \( k+1 \) (\( E_i(k+1) \) and \( P_i(k+1) \)). Therefore in the proposed power control algorithm, the current transmit power of SU\( i \) is determined by the future status of the system, and it can combat the channel fading by estimating the channel variation using \( H_\infty \) filter. The nonnegative \( a_i \) defines the importance between satisfying the QoS of SU\( i \) and mitigating the interference introduced to the network by reducing the transmit power of SU\( i \). The larger \( a_i \) is, the lower the transmit power of SU\( i \) will be. \( h_i \) is link gain from the transmitter of SU\( i \) to the receiver of SU\( i \), and it guarantees the fairness between the SUs. When \( h_i \) is relatively large and the channel state is good, the transmit power can be smaller, and the QoS (SINR) will still be satisfied. Otherwise, when \( h_i \) is relatively small and the channel state is bad, the transmit power can become larger, and the SINR will be improved.

Suppose that the transmit power of SU\( i \) can be adapted according to the following distributive control law

\[ P_i(k+1) = P_i(k) u_i(k), \quad i = 1, 2, \ldots, N \]  

(8)

where the control variable \( u_i(k) \) should be chosen as an optimal solution that can minimize the cost function described in (7).

To obtain the optimal solution, apply the necessary conditions for Nash equilibrium [25], and we can get
\[
\frac{\partial J_i(k)}{\partial u_i(k)} = \frac{\partial \left( a_i h_{i} \left( P_i(k) u_i(k) \right)^2 + \left( \gamma_i \right)^2 \right)}{\partial u_i(k)} \\
= 2 a_i h_{i} \left( P_i(k) \right)^2 + \frac{\partial \left( \gamma_i - \delta_i \left( k + 1 \right) P_i(k) \right)^2}{\partial u_i(k)} \\
= 2 a_i h_{i} \left( P_i(k) \right)^2 + \frac{\partial \left( \gamma_i\left( k + 1 \right) P_i(k) u_i(k) \right)^2}{\partial u_i(k)} \\
= 2 a_i h_{i} \left( P_i(k) \right)^2 - 2 \delta_i \left( k + 1 \right) P_i(k) \left( \gamma_i\left( k + 1 \right) P_i(k) u_i(k) \right) \\
= 0
\]

From (9), the optimal solution \( u_i^{opt}(k) \) can be obtained as

\[
\langle 10 \rangle
u_i^{opt}(k) = \frac{\gamma_i}{P_i(k) \left( \frac{a_i h_{i}}{\delta_i \left( k + 1 \right) + \delta_i \left( k + 1 \right)} \right)}
\]

In (10), \( u_i^{opt}(k) \) is the optimal solution of \( u_i(k) \), and it will minimize the value of the cost function described in (7).

Take the second derivative of the performance criterion \( J_i(k) \) in (7), and we can obtain

\[
\frac{\partial^2 J_i(k)}{\partial u_i(k)} = 2 a_i h_{i} \left( P_i(k) \right)^2 + 2 \delta_i \left( k + 1 \right) P_i(k) \left( \gamma_i\left( k + 1 \right) P_i(k) u_i(k) \right) > 0
\]

The results in (11) confirm that the global minimum exists. Therefore, the corresponding optimal power updating function is given by

\[
\langle 12 \rangle P_i(k + 1) = P_i(k) u_i^{opt}(k) \\
= \frac{\gamma_i}{P_i(k) \left( \frac{a_i h_{i}}{\delta_i \left( k + 1 \right) + \delta_i \left( k + 1 \right)} \right)}
\]

In (12), \( \delta_i(k+1) \) should be estimated at every time instant \( k \), so that the transmit power can be updated instantaneously.

Tranditional power iterative schemes [8-15] usually assume that the power can update quickly enough and the link gains can be seen as unchanged during several iterations. On the contrary, our proposed robust power control scheme with estimator is not an iterative algorithm, and the assumption that the link gains are fixed during the power evolution is not required. The proposed power control scheme is suitable for the situations of fast channel fading, because immediate decisions of the optimal power allocation at the next time instant \( k+1 \) can be made based on the predictions of the channel variation \( \delta(k+1) \). Using an advanced estimator, e.g. \( H_{\infty} \) filter, all the effects of the channel are taken into consideration, and the proposed power control scheme will achieve much better performance than the traditional
iterative schemes.

The block diagram of the proposed robust power control scheme using estimator is shown in Fig. 3.

![Block Diagram](image)

**Fig. 3.** Proposed robust power control scheme via link gain pricing with estimator

In **Fig. 3**, $f(\cdot)$ is defined as

$$f(x) = \frac{1}{a_i h_{ii} + x}$$

and $f_1(x) = 10\log_{10}(x)$ and $f_2(x) = 10^{\frac{x}{10}}$ are converting functions between W and dBW. $I_i(k)$ and $h_{ii}$ can be tracked and calculated at the receiver of SU$i$, and the technique has already been well developed. The control signal from MPs could contain only one bit, and it determines how to adapt $a_i$ according to whether the power of received interferences at PUs is above the interference temperature.

In the user-centric manner [21], the values of $a_i$ are the same. Denote the control signal from MPs as control, and control is a one-bit signal. When control=1, it means the power of received interferences at PUs is above the interference temperature, and we should adjust $a_i$ through $a_i = a_i + \Delta a$, where $\Delta a$ is a positive parameter. When control=0, it means the power of received interferences at PUs is below the interference temperature, and $a_i$ remains unchanged.

As shown in **Fig. 3**, the performance of the estimator is important to the performance of the power control scheme. Thus, $\hat{\delta}(k+1)$ represents the estimated value of $\delta_i(k+1)$, and the power control function of (12) can be written as

$$P_i(k+1) = \frac{\gamma_{ii}^{\text{tar}}}{a_i h_{ii} + \hat{\delta}(k+1) + \delta_i(k+1)}$$

### 3.2 H∞ Filter as an Estimator
The Kalman filter has been widely used to estimate the states of the system through past measurements in the noise signal processing. The Kalman filter can obtain the optimal solution, however, the system model parameters should be already known, and the system and measurement noise should only be white Gaussian processes with known statistics. The Kalman filter is also not robust to the parameter uncertainty of the signal models. Hence, the Kalman filter cannot be applied to the power control problem with severe channel fading.

The $H_\infty$ filter can obtain the optimal solution no matter which kind the system’s nature and measurement disturbances are, and the only requirement is that the power of the disturbance should be bounded $[18, 22-23]$. In the proposed robust power control scheme for cognitive radio network, the channel variation $\delta_i(k)$ should be estimated, and its fluctuations are not normal distributions generally. Hence, the $H_\infty$ filter is adopted as an appropriate estimator in the proposed power control scheme.

The dynamics of $\delta_i$ (in dB) to be predicted can be expressed as

$$\delta_i(k+1) = \delta_i(k) + \omega_i(k)$$

where $\omega_i(k)$ is the process noise. Denote the measurement of $\delta_i$ as $y_i(k)$, and it can be given by

$$y_i(k) = \delta_i(k) + v_i(k)$$

where $v_i(k)$ represents the noise of the measurement. In nature the problem is scalar, so the system controllability and observability conditions can be satisfied. Therefore, the above conditions ensure the existence of the solution of the algebraic Riccati equation.

Thus, the measurement of the $H_\infty$ filter can be expressed as

$$\sup_{\omega, W, V} \left\{ \sum_{k=0}^{N-1} \left\| \delta_i(k) - \hat{\delta}_i(k) \right\|_{W_i}^2 + \sum_{k=0}^{N-1} \left\| \omega_i(k) \right\|_{W_i}^2 + \left\| v_i(k) \right\|_{V_i}^2 \right\} < \frac{1}{\lambda}$$

where $Q_i$, $W_i$, and $V_i$ are all positive parameters, and the prescribed level of the noise attenuation is determined by $\lambda$. Therefore, the $H_\infty$ filter in discrete-time form can be given by

$$\hat{\delta}_i(k+1) = \hat{\delta}_i(k) + K_i(k)(y_i(k) - \hat{\delta}_i(k))$$

where $K_i(k)$ is the optimal gain, and it can be calculated as

$$K_i(k) = R_i(k) \left( I - \lambda Q_i R_i(k) + V_i^{-1}(k) R_i(k) \right)^{-1} V_i^{-1}(k)$$

Because $R_i(k)$, $Q_i$, and $V_i$ in the power control problem are all scalars, $K_i(k)$ can be written as

$$K_i(k) = \frac{R_i(k)}{1 - \lambda Q_i R_i(k) + \frac{R_i(k)}{V_i}} V_i$$
where $R(k)$ is positive, and can be calculated via the following scalar difference Riccati equation.

$$R(k+1) = R(k)(I - \lambda Q R(k) + V^{-1}(k) R(k))^{-1} + W_i$$  \hspace{1cm} (21)

For $R(k)$, $Q_i$, $W_i$, and $V_i$ in the power control problem are all scalars, (21) can be reformulated as

$$R(k+1) = \frac{R(k)}{1 - \lambda Q_i R(k) + V_i^{-1}} + W_i$$  \hspace{1cm} (22)

The initial condition of the system $\delta_i(0)$ and the initial condition of the $H_\infty$ filter $R_i(0)$ are arbitrary positive quantities. The steady-state value of $R_i(k)$ satisfies

$$R_{ss} = \frac{W_i \left( \frac{1}{V_i} - \lambda Q \right) \pm \sqrt{W_i \left( \frac{1}{V_i} - \lambda Q \right)^2 + 4W_i \left( \frac{1}{V_i} - \lambda Q \right)}}{2 \left( \frac{1}{V_i} - \lambda Q \right)}$$  \hspace{1cm} (23)

where the steady-state solution of $R(k)$ should be positive. The choice of the optimization parameters $\lambda$, $Q_i$, $W_i$, and $V_i$ is important to the performance of the $H_\infty$ filter, and they are set as $Q_i=20$, $W_i=1$, $V_i=0.1$, and $\lambda$ can be 0.2, 0.1, or 0.01 in the simulations of this paper.

### 4. Simulation Results and Discussion

#### 4.1 Simulation Model and Parameters

In this section, plenty of simulations are presented in a cognitive spectrum underlay network to demonstrate the effectiveness and efficiency of the proposed robust power control scheme, and there are 20 SUs and 2 MPs in the network. CDMA communication system is taken by the SUs for its low spectrum power density and great multi-access capability. The transmitters of the SUs and the MPs are randomly located in a hexagonal cell with a diameter of 2km, and the receivers of the SUs are all at the origin of the cell. In the simulations, the background noise power at the receivers of SUs $\sigma_i^2$ are all set to 0.1mW. Assume that the link gains from the transmitters to the receivers of the SUs follow

$$h_{in}(k) = Ad_{in}^{-1}(k) S_{in}(k)$$  \hspace{1cm} (24)

In (24), $d_{in}(k)$ is the distance from the transmitter of SU$n$ to the receiver of SU$i$ at time instant $k$ with unit kilometer, $A$ is equal to $10^{-4}$, and $S_{in}$ is a log-normal distributed stochastic process with its standard deviation of 8dB. The processing gain $L$ is set to 128, and the target SINR $\gamma_i^{tar}$ is 5dB for all the SUs. The transmit power should not exceed $P_{\text{max}}=500$ mW, and each SU will transmit with a power level of $P_i(0)=P_{\text{min}}=0$ mW initially.

Distributed constrained power control (DCPC) scheme [13] is a widely known and accepted
power control algorithm, and the power updating function can be given by

$$P_i(k+1) = \min \left\{ \frac{\gamma_{ir}}{\gamma_i(k)} P_i(k), P^{\text{max}} \right\}$$  \hspace{1cm} (25)$$

We use the DCPC in (25) as a traditional iterative power control scheme to compare with our proposed power control scheme in the simulations.

The famous Nash game power control scheme [14] is also considered for comparison in the simulations, and the iterative function of the scheme can be described as

$$P_i(k+1) = \begin{cases} \frac{\gamma_{ir}}{\gamma_i(k)} - \frac{b_i}{2c_i} \left( \frac{P_i(k)}{\gamma_i(k)} \right)^2, & \text{if positive} \\ 0, & \text{otherwise} \end{cases}$$  \hspace{1cm} (26)$$

$b_i/(2c_i)$ is set to 0.05 in the simulation.

### 4.2 Simulation of Robustness to Channel Fading

First, we investigate the robustness of the proposed power control scheme to channel fading. $a_i$ is set to 0.5 and $\bar{\lambda}$ in $H_{\infty}$ filter is 0.1. The channel fading is severe and the link gains of the SUs follow (24). The SINR and transmit power evolutional process of the 20 SUs using the proposed robust power control scheme and DCPC scheme is shown in Fig. 4 to Fig. 5, respectively. As shown in Fig. 4 and Fig. 5, when the proposed robust $H_{\infty}$ filtering power control scheme is applied, the SINR curves of the 20 SUs are much more stationary than those using DCPC scheme. After the initial several steps in the proposed robust power control scheme, the SINR does not fluctuate much and stay around the target SINR of 5dB. Therefore, the proposed scheme can combat the degradation of the channel with excellent performance due to the immediate decision of the optimal allocation based on the estimations of the channel variations from the $H_{\infty}$ filter. On the contrary, it can hardly converge and the fluctuation is extremely large when the DCPC scheme is applied, because it operates in an iterative manner and converges slowly. The largest value of SINR is over 9dB, and the smallest is lower than 3dB. Thus, it is not suitable for practical use due to the channel fading.
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There are too many curves in Fig. 4 and Fig. 5, and in order to analyze the robust performance of the power control schemes more explicitly, the SINR and transmit power evolutional process of only one SU using the proposed robust scheme, DCPC scheme, and Nash game power control scheme is shown in Fig. 6. From the simulation results in Fig. 6, we can see that the SINR converging curves of SU1 do not fluctuate much and stay around the target SINR of 5dB with severe fading when the proposed robust power control scheme is applied, and they are almost the same with \( \lambda \) is equal to 0.2, 0.1, and 0.01 respectively. Thus the proposed power control scheme is robust to the channel fading. On the other hand, when DCPC scheme and Nash game power control scheme are applied, it can hardly converge, and the curves fluctuate greatly. The robustness to the channel fading of these two iterative power control schemes is poor, and they are unpractical to use when the communication channel is under severe fading.

Fig. 6. Comparison of SINR and transmit power evolutional process of only one SU using DCPC scheme, Nash game scheme, and the proposed scheme with various \( \lambda \) values with severe fading
4.3 Simulation Considering Interference Temperature Limit

Next, the interference temperature limit of the PUs is considered, and the power control scheme proposed in our previous paper [21] (power control in user-centric and network-centric manners, we can call it PCUNM scheme) is also compared. The channel fading is ignored for clarity. Assume that the interference temperature at the two MPs in the network is the same, which is set to 1mW, and the link gains from the transmitters to the receivers of the SUs follow

\[ g_{n_k}(k) = A d_{n_k}^{-4}(k) \]  \hspace{1cm} (27)

In (24), \( d_{n_k}(k) \) is the distance from the transmitter of SU\( n \) to the MP\( k \) at time instant \( k \) with unit kilometer. \( A \) is equal to \( 10^{-4} \).

The SINR and transmit power of SUs, and interference power at MPs evolutional process using the proposed power control scheme in the first 80 iterations and using PCUNM scheme in the last 40 iterations is shown in Fig. 7.

![SINR evolutional process](image1)

![Transmit power evolutional process](image2)

![Interference power at MPs evolutional process](image3)

**Fig. 7.** SINR and transmit power of SUs, and interference power at MPs evolutional process using the proposed power control scheme in the first 80 iterations and PCUNM scheme in the last 40 iterations

In Fig. 7, during the first 40 iterations, the proposed power control scheme is applied. Assume that through spectrum sensing no PUs are detected to be communicating at this stage,
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and thus all the SUs pursue their own QoS performance through assigning $a_i=0$. All the SUs can attain their target SIR value upon convergence of the transmit power. The average transmit power of the SUs is 97.1927 mW at this stage. However, for $40<k \leq 80$, two PUs are found to be working at this stage, when spectrum sensing is performed. The interference power at MP1 is above the interference temperature, and it will affect the normal communication of the PUs. Therefore, for $40<k \leq 80$, the proposed power control continues operating, and $a_i$ is set to 6. Through this operation of increasing $a_i$, the average transmit power of the SUs is reduced to 53.1759 mW, and the interference power at MPs is below the interference temperature $Z_k$. To compare the proposed scheme with PCUNM scheme, PCUNM is applied during the interval $80<k \leq 120$. To guarantee the interference temperature limit and ensure the fairness with the same average transmit power, $a_i$ in PCUNM is set to 0.0005. In this stage, the average transmit power of SUs is 53.3711 mW, which is almost the same as that in the during $40<k \leq 80$. Compare the curves in the during $40<k \leq 80$ with those in the during $80<k \leq 120$, and we can find that in the proposed scheme the fairness between the SUs can be guaranteed. When $h_{ii}$ is relatively large and the channel state is good, the transmit power can be smaller, and the QoS (SINR) will still be satisfied. Otherwise, when $h_{ii}$ is relatively small and the channel state is bad, the transmit power can become larger, and the SINR will be improved. The lowest SINR in the during $40<k \leq 80$ is 3.145 dB, while the lowest SINR in the during $80<k \leq 120$ is 2.403 dB. Thus the lowest SINR is improved by 0.742 dB through applying the proposed power control scheme. The proposed power control scheme considers the fairness of all the SUs, and can achieve better SINR performance of the SU in the worst channel than the PCUNM scheme.

4.4 Simulation according to Different Manners

The PCUNM scheme can work under different manners, and the user can choose to benefit the user itself or to benefit the whole network. The proposed power control scheme can also operate in these two manners, and this is demonstrated in Fig. 8.
In Fig. 8, during the first 40 iterations, the proposed power control scheme is applied. Assume that the spectrum sensing is performed, and no PUs are detected to be in communication. Thus all the SUs pursue their own QoS performance in the user-centric manner through assigning $a_i=0$. All the SUs can converge to their target SIR value 5dB. The average transmit power of the SUs is 97.1927mW at the initial stage. However, in the during $40<k\leq80$, two PUs are found to be working from the results of the spectrum sensing. The interference power at MP1 is above the interference temperature, and it will affect the normal communication of the PU. Therefore, for $40<k\leq80$, the proposed power control continues operating, and $a_i$ is all set to 1, and it means that all the SUs operate in the network-centric manner with the same level. By this adjustment, the average transmit power of the SUs is reduced to 80.1161mW. As a result, the lowest SINR of the SUs is also decreased to 4.3452dB. The interference power at MP1 is lower, however, it is still above the interference temperature limit 1mW. Therefore, in the during $80<k\leq120$, the two SUs with largest transmit power (SU5 and SU10) choose to further address energy efficiency via assigning $a_5h_{55}=a_{10}h_{10-10}=1$. This results in SU5 and SU10 opting out via a converged transmit power of $P_5(k)=P_{10}(k)=P_{\text{min}}=0$ mW for $80<k\leq120$. In response to the increased network-centric performance of SU5 and SU10, the average transmit power of the other 18 SUs is reduced to 48.5113mW, and the interference power at MP1 is below the interference temperature. The lowest SINR of these 18 SUs is also increased to 4.6789dB. The SU5 and SU10 will restart transmission in response to an improved channel state or the results from the spectrum sensing that no PUs is working.

5. Conclusion

A robust power control scheme via link gain pricing with $H_\infty$ estimator for cognitive spectrum underlay network is proposed in this paper. The scheme can guarantee the interference temperature of the SUs through working in the network-centric manner, and it keeps the fairness between the SUs through link gain pricing, which can improve the QoS of the SU in the bad channel by increasing its transmit power. In addition, the $H_\infty$ filter is also applied, which can estimate the channel variation, and therefore the proposed power control scheme is robust to the severe channel fading. Several simulations have been carried out, and the results demonstrate the effectiveness and efficiency of the proposed scheme.
References


Nan Zhao was born in Dalian, China, in 1982. He received the B.S. degree in electronics and information engineering in 2005, the M.E. degree in signal and information processing in 2007, and the Ph. D. degree in Information and communication engineering in 2011 from Harbin Institute of Technology, Harbin, China. He is currently a lecturer and a postdoc at School of Information and Telecommunication Engineering, Dalian University of Technology. His research interests are multiuser detection and power control in CDMA, spectrum sensing and resource allocation in cognitive radio, chaotic theory, and ant colony optimization.

Hongjian Sun received the B.S. and M.S. degree from Harbin Institute of Technology, China, in 2005 and 2007, respectively, and obtained the Ph.D. Degree in Electronic Engineering from the University of Edinburgh, UK, in 2010. From December 2010, he is a Postdoctoral researcher in the Centre for Telecommunications Research within King’s College, London, UK. His recent research interests include cognitive radio, cooperative communication, power line communication, and smart grid.