Multiobjective Analysis of the Multi-Location Newsvendor and Transshipment Models

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ABSTRACT

Unlike the Newsvendor model, a system based on lateral transshipments allows the unsold inventories to be moved from locations with surplus inventory to fulfill more unmet demands at stocked out locations. Both models were thoroughly studied and researches were usually confined to cost minimization or profit maximization. In this paper, we proposed a more realistic multiobjective study of both multi-location Transshipment and Newsvendor inventory models. The aggregate cost, the fill rate, and the shared inventory quantity are formulated as conflicting objectives and solved using two reference multiobjective evolutionary algorithms (SPEA2 and NSGA-II). The proposed models take into account the presence of storage capacity constraints. The obtained Pareto fronts revealed interesting information. When transshipments are allowed, both low aggregate cost and high fill rate levels are ensured. The required shared inventory may have an important variability. The considered objective functions are conflicting and very sensitive to local storage capacities.

Keywords: Multiobjective optimization; Evolutionary algorithms; Inventory pooling; Transshipment and Newsvendor models.

INTRODUCTION

In the past, research in operations management focused on single-firm analysis. Its goal was to provide managers in practice with suitable tools to improve the performance of their firm by calculating optimal inventory quantities, among others. Recently, research in operations management has shifted its focus from single-firm analysis to multi-firm analysis, in particular to improving the efficiency and performance of supply chains under centralized control. New strategies and technologies are being implemented in response to the challenges and customer demands. Recently, two generic strategies for supply chain design emerged: efficiency and responsiveness. Efficiency aims to reduce operational costs; responsiveness, on the other hand, is designed to react quickly to satisfy customer demands and save costs.

The proactive use of transshipments is an illustrative example of internal coordination in multi-firm inventory systems. Referred to as pooling, transfer, and redistribution, transshipments are recognized as the monitored movement of material among locations at the same echelon (Herer et al., 2002). They afford a valuable mechanism for correcting the discrepancies between the locations’ observed demand and their on-hand inventory. Subsequently, transshipments may reduce costs and improve service without increasing the system-wide inventories. The background is drawn mostly from Operations Research, where work has been done to characterize the optimal solution to the inventory problem with correlated demands. With today's communication technologies, the implementation of the transshipment idea into an inventory management system is quite easy. Thanks to the developments in information and communication technologies in the last twenty years, it is possible to electronically view the inventory levels of other stocking locations to incorporate the transshipment option into the planning phase.

The newsvendor model (or newsboy or single-period (William et al., 2009)), is a mathematical model in operations management and applied economics used to determine optimal inventory levels in the presence of uncertain demand and simple cost structure. It is often considered as the basic model on which were based most of the proposed Transshipment models. In the Newsvendor problem, a decision maker orders inventory before the starting of a selling season characterized by a stochastic demand. If too much is ordered, stock is left over at the end of the period and the retailer incurs additional holding costs. The manager must dispose of the remaining stock at a loss. If the order quantity is lower than the realized demand, the manager forgoes some profit. Customers’ disappointment is implicitly considered as a paid penalty cost. Therefore, in choosing an order quantity, the manager must balance the costs of “ordering too much” against the costs of “ordering too little”. The Newsvendor problem applies in a broad array of settings. In the fast-moving computer business, International Business Machine produced $700 million of excess inventory of their Value-Point line, but in another year, they under produced their Aptiva® PC line, and lost potential revenues of more than $100 million (Ziegler 1995). Both oversized inventories and low fill rates are costly management strategies. Business may be lost through canceled orders, and the company's reputation may be severely damaged. It is therefore in a company's interest to balance inventory holding cost and the cost of imperfect customer satisfaction. The trade-off inventory vs. customer satisfaction is one of the classic issues of logistics and supply chain management. In traditional inventory systems, minimizing costs or maximizing profits as a single objective is often the focus.
However, very few inventory systems are single objective problems. Multiobjective formulation has to be considered whose solutions will be a set of Pareto alternatives representing the tradeoffs among different objectives.

Evolutionary Algorithms (EAs) are adaptive heuristic search algorithm inspired by evolutionary theory: natural selection and survival of the fittest. They are known to be efficient-solving and easy-adaptive, especially those where traditional methods failed to provide good solutions (Wang and Prabhu 2009)). The use of EA in multiobjective optimization has been widely studied, experimented and applied in many fields due to its particular suitableness for multiobjective optimization: 1) its ability to work simultaneously with a population of promising solutions which would evolve into a set of Pareto optimal solutions at termination; 2) its insusceptibility to the shape or continuity of the Pareto front. As an adaptive search algorithm, MOEA simulates the survival of the fittest among individuals over consecutive generations for solving a problem. Each generation consists of a population of chromosomes representing possible solutions in the solution space. As the population evolves from generation to generation, MOEA identifies non-dominated individuals in each population and have them evolve towards the final Pareto set. Recently, multiobjective evolutionary algorithms (EMOAs) have become prevailing since the pioneering work (Shaffer 1985). There are many efficient MOEAs that are possible to find Pareto optimal solutions as well as widely distributed sets of solutions; NSGA-II and SPEA2 (Zitzler and Thiele 1999) are among the most successful approaches.

This paper is organized as follows. In section 2, a literature review shows that little research has dealt with multiobjective optimization of Transshipment and Newsvendor models. In section 3, all parameters, decision variables and constraints are defined. The pooling policy is modeled as a linear programming problem. A mathematical formulation of the cost, fill rate and shared inventory objective functions is described. In section 4, we introduce the basic concepts of multiobjective optimization. We briefly described SPEA2 and NSGA-II algorithms and the capability of evolutionary approaches to tackle hard problems. In section 5, numerical examples are given. The optimization results are illustrated and explained. Useful information is revealed based on the obtained Pareto fronts. We showed that a multiobjective analysis of both Transshipment and Newsvendor problems is more accurate and comprehensive than single-objective one. Multiobjective performance measures are given to allow comparison with other solving approaches. Finally, discussion and outlooks are proposed in section 6.

LITERATURE REVIEW

There is a considerable amount of Supply Chain Management in the last past decades. Some papers provided interesting surveys. Pokharel (2008) indicated that various objectives could be considered for strategic decision making on Supply Chain Network: (1) increasing service level, (2) decreasing warehouse costs, (3) decreasing total fixed and variable costs, (4) decreasing lead time (order processing and supply lead times), (5) consolidating supplier base, (6) increasing supplier reliability, (7) increasing capacity utilization and, (8) increasing total quality of supply. In the same work, Pokharel (2008) developed a two-objective decision-making model for the choice of suppliers and warehouses for a supply chain network design. Arshinder et al. (2008) presented a systematic literature review on the importance of Supply Chain coordination. They reported various perspectives on Supply chains coordination issues and explained various mechanisms available for coordination. Multiple objectives were investigated and diverse optimization approaches were used. Liao and Rittscher (2007) simultaneously considered the optimization of the total cost, the quality rejection rate, the late delivery rate and the flexibility rate in their stochastic supplier selection problem while involving constraints of demand satisfaction and capacity. Zhou et al. (2003) studied the bi-criteria allocation problem involving multiple warehouses with different capacities using a genetic algorithm based solution procedure. Liberopoulos and Koukoumialos (2005), numerically investigated tradeoffs between near-optimal base stock levels, numbers of kanbans, and planned supply lead times in base stock policies and hybrid base stock/kanban policies. Special interest in the Newsvendor has increased since the nineties. Abdel-Malek et al. (2008) presented a series of articles addressing issues regarding the newsvendor models. They developed new models to extend the existing ones and designated them as the Gardener Problem. The models are based on the application of Lagrange multipliers, Leibniz’s rule and Newton’s method to obtain the optimum solution for the considered random yield and probabilistic demand situations. Other newsvendor extensions in situations of strategic interaction have previously been considered as well, but in different contexts. Parlar (1988) and Lippman and McCardle (1997) considered models with more than one newsvendor where the demand is transferable. In other words, in their models, if a newsvendor is stocked out, it is the customer who moves to a rival newsvendor. Khouja (1999) studied a variation of the Newsvendor Problem based on the analysis of Cost-Volume-Profit optimization. Other researchers have also examined the effectiveness of lateral transshipment and centralized coordination as modern inventory management strategies. Sharma and Jana (2009) presented a multiobjective transshipment planning model for the petroleum refinery industry. The considered objectives were to minimize the total transshipment cost, maximize production, satisfy storage requirements at depots and meet the demand for oil in these sales areas. The goals are defined in a fuzzy sense and a Fuzzy Goal Programming (FGP) model is developed. Olsson
(2008) optimized the ordering policies for normal replenishment in systems where lateral transshipments are used as emergency supply in case of stock out. The difference with other models was that the rule for lateral transshipments is predefined. Köchel and Nielländer (2004) proposed a successful simulation optimization approach where a simulator is combined with an appropriate optimization tool to define optimal policies in very general multi-echelon inventory systems. Xu et al. (2003) estimated customer service in a two-location continuous review inventory model with emergency transshipments. Evers (1997) used a simulation model to examine whether emergency transshipments outperform split orders. These recent researches were based on some early and basic inventory models such as the well-known study of Gross (1963) where it was determined both optimal redistribution and replenishments policies for a two-store inventory system. Later, Krishnan and Rao (1965) considered the analytical determination of optimal base stock levels that would minimize the one-period inventory and transportation costs for inventory systems with emergency transshipments. Tagaras and Cohen (1992) discussed the possible effects of replenishment lead times on pooling policies. They performed a simulation analysis on different pooling policies for a two-location inventory system.

**THE MODELS**

In this section, we establish a mathematical formulation for both Newsvendor and Transshipment based inventory models showing the analytical relation between them. A bi-objective model is outlined based on cost and fill rate objective functions. Finally, we discuss the simulation methodology in the multiobjective optimization process.

**The Multi-location Newsvendor Model**

In the newsvendor problem, the decision-maker, facing uncertain demand distribution, has to decide how many units to buy each day. The newsvendor problem focuses on the purchase of perishable products. The mathematical model maximizes the expected profit by determining the optimal order-size. Two modeling approaches have been used to solve the classical Newsvendor problem (Khouja 1999). In the first approach, the expected shortage and holding costs are minimized. In the second approach, the expected profit is maximized. Both approaches yield the same results. We use the first approach in this study. Define the following notation:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n )</td>
<td>Number of locations</td>
</tr>
<tr>
<td>( h_i )</td>
<td>Unit inventory holding cost at location ( i )</td>
</tr>
<tr>
<td>( p_j )</td>
<td>Unit penalty cost for shortage at location ( j )</td>
</tr>
<tr>
<td>( S_i )</td>
<td>Replenishment quantities for location ( i )</td>
</tr>
<tr>
<td>( S )</td>
<td>Vector of replenishment quantities, ( S = (S_1, S_2, \ldots, S_n) ) (Decision variables)</td>
</tr>
<tr>
<td>( S_{\text{max},i} )</td>
<td>Maximum storage capacity of location ( i )</td>
</tr>
<tr>
<td>( S_{\text{max}} )</td>
<td>Vector of storage capacities, ( S_{\text{max}} = (S_{\text{max},1}, S_{\text{max},2}, \ldots, S_{\text{max},n}) )</td>
</tr>
<tr>
<td>( D_i )</td>
<td>Demand realized at location ( i ) (Random variable)</td>
</tr>
<tr>
<td>( D )</td>
<td>Vector of demands, ( D = (D_1, D_2, \ldots, D_n) )</td>
</tr>
<tr>
<td>( f_i(D_i) )</td>
<td>Probability density function of demand distribution at location ( i )</td>
</tr>
<tr>
<td>( F_i(D_i) )</td>
<td>Cumulative distribution function of demand at location ( i )</td>
</tr>
<tr>
<td>( I^+ )</td>
<td>Set of locations with surplus inventory: ( I^+ = {i; S_i &gt; D_i \text{ for } i = 1..n} )</td>
</tr>
<tr>
<td>( I^- )</td>
<td>Set of locations with unmet demands: ( I^- = {j; D_j &gt; S_j \text{ for } j = 1..n} )</td>
</tr>
</tbody>
</table>

**Newsvendor Cost Function**

At the end of a given period, the cost \( C_{i}^{NV} \) at location \( i \) is either a holding cost or shortage cost (Arrow et al. (1951)):

\[
C_{i}^{NV} (S_i, D_i) = h_i \max(0, S_i - D_i) + p_i \max(0, D_i - S_i) \tag{1}
\]
Since the demands are stochastic with known joint distributions, the expected cost at location \( i \) is given by computing the following integral:

\[
C^{\text{NV}}_i (S_i) = h_i \int_0^S (S_i - D_i) f_i(D_i) dD_i + p_i \int_S^\infty (D_i - S_i) f_i(D_i) dD_i
\]

Using Leibniz’s rule to obtain the first and second derivatives shows that the local cost function \( (2) \) is convex. Let \( S^*_i \) be the optimal replenishment quantity that minimizes the cost expected function \( (2) \). The optimality condition is given by the following formulae:

\[
S^*_i = F_i^{-1} \left( \frac{p_i}{h_i + p_i} \right)
\]

The ratio \( p_i/(h_i+p_i) \) is called the critical ratio. Hence, to minimize the expected Newsvendor cost of location \( i \), we must choose \( S_i^* \) such that we do not have unsold units (i.e., \( D_i < S_i \)) with a probability that equals the critical ratio. Notice that in the presence of storage capacity constraints, the inventory level of each location should not exceed the predefined storage capacity \( (S_i < S_{\text{max},i}) \). In other words, if the optimal inventory level \( S_i^* \) is greater than the storage limit \( S_{\text{max},i} \), the decision maker should apply equation \( (4) \).

\[
F^{\text{NV}} (S_i,D_i) = \frac{\min(D_i,S_i)}{D_i}
\]

The overall fill rate of \( n \) Newsvendor locations denoted \( F^{\text{NV}} \) is given by dividing the demands satisfied with the on-hand stock by the overall observed demand.

\[
F^{\text{NV}} (S,D) = \frac{\sum_{i=1}^n \min(D_i,S_i)}{\sum_{i=1}^n D_i}
\]

If a location \( i \) has a surplus stock, then, the satisfied demand at \( i \) is equal to the observed demand \( D_i \). However, if a location \( j \) has insufficient stock quantity, the satisfied demand at \( j \) would be equal to the base stock level \( S_j \). Thus, by introducing the sets \( I^+ \) and \( I^- \), we obtain a different formulation of the aggregate fill rate which is useful for further comparison with the Transshipment model (see equation 17).
\[
F^{SN}(S,D) = \frac{\sum_{i=1}^{n} D_i + \sum_{j=1}^{x} S_j}{\sum_{k=1}^{n} D_k}
\]

### The Multi-location Transshipment Model

A transshipment based inventory model defines an additional feature which is the possibility of sharing stock with according to a defined pooling policy. In this study, we consider a real life Transshipment model where \( n \) locations are selling a single product. They may differ in their costs and demands parameters. The system inventory is reviewed periodically. At the beginning of the period and long before the demands realization (figure 1), replenishments take place in location \( i \) to increase the stock level up to \( S_i \). The storage capacity of each location is limited to \( S_{max,i} \). In other way, the replenishment quantities should not exceed \( S_{max,i} \) inventory units. Thus, the inventory level of location \( i \) will be always less or equal to \( \min(S_i, S_{max,j}) \). After the replenishment, the observed demands \( D \) which represent the only uncertain event in the period are totally or partially satisfied depending on the on-hand inventory of local locations. However, some locations may be run out of stock while others still have unsold goods. In such situation, it would be possible to transfer these goods from locations with surplus inventory to locations with still unmet demands. This is called \textit{Lateral Transshipment} within the same echelon level (Herer et al. 2006). It means that locations in some sense share the stocks. The set of locations holding inventory \( I^* \) can be considered as temporary suppliers since they may provide other locations at the same echelon level with stock units. Let \( \sigma_{ij} \) be the transshipment cost of each unit sent by location \( i \) to satisfy a one-unit unmet demand at location \( j \). After the end of the transshipment process, if location \( i \) still has a surplus inventory, it will be penalized by a per-unit holding cost of \( h_i \). If location \( j \) still has unmet demands, it will be penalized by a per-unit shortage cost of \( p_j \). Fixed cost transshipment costs are assumed to be negligible in our model. Herer et al. (2006) proved that, in the absence of fixed costs, if transshipments are made to compensate for an actual shortage and not to build up inventory at another location, there exists an optimal base stock policy \( S^* \) for all possible stationary policies. The following notation is used:

| \( \sigma_{ij} \) | Unit transshipment cost from location \( i \) to location \( j \) |
| \( T_{ij} \) | Amount of inventory transshipped from location \( i \) to location \( j \) |
| \( T \) | Matrix of transshipped inventory. \( T = (T_{ij}) \) |

\[ \begin{array}{c|c}
\sigma_{ij} & \text{Unit transshipment cost from location } i \text{ to location } j \\
T_{ij} & \text{Amount of inventory transshipped from location } i \text{ to location } j \\
T & \text{Matrix of transshipped inventory. } T = (T_{ij}) \\
\end{array} \]

**Modeling assumptions**

- Demands could not be canceled after the beginning of the transshipment process and can be partially fulfilled. In other words, if a customer ordered 100 units and finally gets only 40 units, he cannot cancel his demand although it is not entirely satisfied.
- Replenishment and transshipment lead times do not influence the system parameters. No exaggerated delays will occur. No impact on the replenishment or the transshipment policies. Assume there are enough available vehicles to ensure the transshipment process. More precisely, if location \( i \) is willing to send \( T_{ij} \) units to location \( j \), there should be a transport vehicle allocated for that. Although it is a restrictive assumption, it greatly simplifies the modeling of pooling policy sub-problem. An extension of this research would study the situation when the number of vehicles is limited.

### The Pooling Policy

In (9), problem \( K \) can be recognized as the maximum aggregate income due to the transshipment. \( T_{ij} \) denotes the optimal quantity that should be shipped from \( i \) to fill unmet demands at location \( j \) so that the resulting profit due to transshipments is maximal. Constraints (10) and (11) say that the shipped quantities cannot exceed the available quantities at location \( i \) and the unmet demand at location \( j \). Constraint (12) indicates that the transshipments quantities are positive real valued parameters, and that the transport vehicle are not capacitated.
\[ K(S,D) = \max_{T} \left\{ \sum_{i \in I^+} \sum_{j \in I^-} \left( h_i + p_j - \sigma_{ij} \right) T_{ij} \right\} \]  

Subject to:

\[ \sum_{j \in I^-} T_{ij} \leq S_i - D_i \quad \forall i \in I^+ \]  

\[ \sum_{i \in I^+} T_{ij} \leq D_j - S_j \quad \forall j \in I^- \]  

\[ T_{ij} \geq 0 \]  

**Transshipment Cost function**

Since inventory choices in each location are centrally coordinated, it would be a common interest among the locations to minimize the aggregate cost \( C^{TR} \). At the end of the period, the system cost is given by (13):

\[ C^{TR}(S,D) = \sum_{i \in I^+} h_i (S_i - D_i) + \sum_{j \in I^-} p_j (D_j - S_j) - K(S,D) \]  

The first and the second term on the right hand side of (13) can be respectively recognized as the total holding cost and shortage cost before the transshipment. The third term is recognized as the aggregate transshipment profit since every unit shipped from \( i \) to \( j \) decreases the holding cost at \( i \) by \( h_i \) and the shortage cost at \( j \) by \( p_j \). However, the total cost is increased by the per-unit transshipment cost \( \sigma_{ij} \).

Grouping the first and the second term on the right hand side of equation (13), we obtain a simpler expression (14) shows the important relationship between both the Newsvendor and the Transshipment problem. By setting high transshipment costs, i.e. \( \sigma_{ij} > h_i + p_j \), no transshipments will occur. Problem \( K \) (equation 9) will then return zero.

\[ C^{TR}(S,D) = C^{NV}(S,D) - K(S,D) \]  

**Transshipment Fill Rate Function**

Let \( F^{TR} \) be the aggregate fill rate measure after the transshipment realization. The satisfied demand depends not only on the inventory level \( S_i \) and the observed demand \( D_i \), but also on the stock transferred among locations.

\[ F^{TR}(S,D) = \frac{\sum_{j=1}^{n} \min(D_j, S_j + \sum_{i=1}^{n} T_{ij})}{\sum_{j=1}^{n} D_j} \]  

Using sets \( I^+ \) and \( I^- \), we get the following expression:

\[ F^{TR}(S,D) = \frac{\sum_{i \in I^+} D_i + \sum_{j \in I^-} S_j + \sum_{i \in I^+} \sum_{j \in I^-} T_{ij}}{\sum_{k=1}^{n} D_k} \]  

The difference between Newsvendor and Transshipment fill rate functions is underlined in equation (17). The second term on the right hand side of \( F^{TR} \) is recognized as the contribution of the transshipments in the fill rate. It shows that the share of stock is always beneficial and cannot deteriorate the fill rate.
\[ F^{TR}(S, D) = F^{NV}(S, D) + \frac{\sum_{i \in I} \sum_{j \in I'} T_{ij}}{\sum_{k=1}^{n} D_k} \] (17)

**Shared Inventory Quantity**

The main difference between Newsboy and Transshipment models is the possibility to fulfill unmet demands in locations \( I \) with unsold units brought from locations \( I' \). In situations where most of the system locations are far from each other in terms of geographical distances, it would be better to rely more on on-hand stock instead of waiting for shipped goods from distant locations. Special interest must then be given to the quantity of inventory shared in the system. If too much stock is exchanged, many problems can arise such as unexpected vehicles unavailability, long lead times, inventory deterioration due to transport accidents... However, if too few stock is shared while there are not used transportation vehicles, the transshipment strategy could be inappropriate for the system configuration (i.e. a surprising increase of transshipment costs, etc.). A more realistic inventory model which involves transshipments should give the decision maker the possibility to have an idea about the way inventory is managed in the system. Equation (18) states a formulation of the Shared Inventory Quantity (SIQ) where \( T_{ij} \) are the optimal solutions of problem \( K \) (see equation 9).

\[ SIQ(S, D) = \sum_{i \in I} \sum_{j \in I'} T_{ij} \]

s.t.

\[ T = \arg \max_{\tau} \left[ K(S, D, T) \right] \] (18)

**Objective functions approximation**

The considered objective functions are stochastic because of the demand randomness modeled by the continuous random variables \( D_i \) with known joint distributions \( f_i \). Thus, computing the expected value of each objective function is needed to start the optimization process. We mentioned previously that in our Newsvendor model, both expected aggregate cost and fill rate are computed via integration over all the possible demands realization (see equation 2). Hence, we do not need any simulation methods to compute the average functions values. It is not the case for the transshipment problem we studied (see figure 2). We modeled the pooling strategy as a linear programming problem we named \( K \) (see equation 9). An analytical tractable expression for problem \( K \) exists only in several special cases (a generalized two-location problem or an n-location problem with identical cost structures (Krishnan and Rao 1965). In the general case (many locations \( n>2 \) with different cost structures), we should have recourse to a linear programming technique to solve problem \( K \). In this study, we used the standard Simplex Method. The most common method to deal with noise or randomness is re-sampling or re-evaluation of objective values (Beyer 2000). It is a simple and widely used method that leads to an unbiased estimation of random variables with unknown mean or variances. With the re-sampling method, if we evaluate a solution \( S \) for \( m \) times, the estimated objective value is obtained as in equation (19) and the noise \( \sigma \) is reduced by a factor of \( m^{-2} \). For this purpose, draw \( m \) random scenarios \( D'_1, \ldots, D'_m \) independently from each other. In this paper, a scenario \( D'_k \) is equivalent to a demand vector \( D'_k=(D'_k1, \ldots, D'_kn) \). In this study, we ran simulations with at least \( m=30000 \) scenarios.

\[
\overline{f(S)} \approx \frac{1}{m} \sum_{i=1}^{m} f(S, D'_i) \Rightarrow \\
\sigma = \sqrt{\text{Var}[f(S)]} \approx \frac{\sigma}{\sqrt{m}}
\] (19)

The approximation quality is as good as the number of samples \( m \) is big:

\[ \overline{E(f(S, D))} = f(S) = \lim_{m \to \infty} \overline{f(S)} \] (20)
MULTIOBJECTIVE OPTIMIZATION

Most real world problems have several (usually conflicting) objectives to be satisfied. A general multiobjective optimization problem has the following form:

$$\min \left[ f_1(S), f_2(S), \ldots, f_k(S) \right]$$

Subject to the $m$ inequality constraints and the $p$ equality constraints:

$$g_i(S) \geq 0, \quad i = 1, 2, \ldots, m$$

$$h_i(S) = 0, \quad i = 1, 2, \ldots, p$$

The most popular approach to handle multiobjective problems is to find a set of the best alternatives that represent the optimal tradeoffs of the problem. After a set of such trade-off solutions are found, a decision maker can then make appropriate choices. In a simple optimization problem, the notion of optimality is simple. The best element is the one that realizes the minimum (or the maximum) of the objective function. In a multiobjective optimization problem, the notion of optimality is not so obvious. This set of solutions is known as the optimal solutions of the Pareto set or nondominated solutions. This is the most commonly adopted notion of optimality. A vector of decision variable $S^*$ is said to be Pareto optimal if there does not exist another $S$ such that:

$$f_i(S) \leq f_i(S^*), \quad \forall i = 1, 2, \ldots, k$$

$$\text{and}$$

$$f_j(S) < f_j(S^*), \quad \text{for at least one } j$$

In other words, this definition says that $S^*$ is Pareto optimal if there exists no feasible vector of decision variable $S$ that would decrease some criterion without causing a simultaneous increase in at least one criterion. The plot of the objective functions whose nondominated vectors are in the Pareto optimal set is called the Pareto front.

Evolutionary Multiobjective Optimization

The classical methods such as weighted sum approach or $\varepsilon$-constraint method (Yu 1985) can only find one Pareto optimum in a single run. To obtain the whole Pareto front, a series of separate runs must be performed while dynamically varying the weights at each run. In addition, most of these approaches fail if the Pareto front is not convex.

A number of stochastic optimization techniques such as simulated annealing, Tabu search, ant colony optimization etc., could be used to generate the Pareto set. Evolutionary algorithm is characterized by a population of solution candidates and the reproduction process enables the combination of existing solutions to generate new solutions. This enables finding several members of the Pareto-optimal set in a single run. Some of the other advantages of having evolutionary algorithms is that they require very little knowledge about the problem being solved, less susceptible to the shape or continuity of the Pareto front, easy to implement, robust, and could be implemented in a parallel environment.

The absence of knowledge about either the Pareto fronts of our multiobjective problems considered in this study motivated us to solve the problem by using two conventional Evolutionary Algorithms (EAs) described in the next two subsections.
SPEA2: a brief description
Many multiobjective evolutionary algorithms have been proposed in the last few years. According to Zitzler and Thiele (1999), SPEA2 and NSGA-II showed the best performance on almost of benchmark functions. At the beginning of the SPEA2 optimization process, an initial population is generated randomly respecting the different local storage constraints ($S_i$ is less than $S_{max,i}$). In our multi-location problem, an individual is a base stock decision vector $S = (S_1, S_2, ..., S_n)$ consisting of $n$ genes $S_i$. At each generation, all individuals are evaluated. A fine-grained fitness assignment strategy is used to perform individuals’ evaluation. Good individuals are conserved in an external set (archive). If the archive is full, a truncation operator is used to determine which individuals should be removed from the archive. The resulting optimal Pareto front is located in the archive. Refer to Zitzler et al. (2002) for more details about SPEA2.

NSGA-II: a brief description
A Fast and Elitist Multi-objective Genetic Algorithm, NSGA-II (Deb et al. 2002) is the update of the previous Non-dominated Sorting Genetic Algorithm (NSGA). The main feature of the NSGA and NSGA-II in solving multi-objective problems is that they lead the search toward the global Pareto front while maintaining diversity of the solution set along the Pareto front. A random population is created and then sorted based on the non-domination level. These solutions are assigned fitness values based on their non-domination, where the first front allotted the highest fitness value. A crowding distance is taken into consideration to maintain diversity during the search procedure.

OPTIMIZATION RESULTS
In this section, we report on our numerical study. For the remaining of the paper, we keep the following notation:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>USN(n)</td>
<td>n-Location Unconstrained storage Newsvendor instance</td>
</tr>
<tr>
<td>CSN(n)</td>
<td>n-Location Constrained storage Newsvendor instance</td>
</tr>
<tr>
<td>UST(n)</td>
<td>n-Location Unconstrained storage Transshipment instance</td>
</tr>
<tr>
<td>CST(n)</td>
<td>n-Location Constrained storage Transshipment instance</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameters</th>
<th>SPEA2</th>
<th>NSGA-II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population size</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>Archive size</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>Max evaluations</td>
<td>50 000</td>
<td>50 000</td>
</tr>
<tr>
<td>Crossover probability</td>
<td>0.90</td>
<td>0.90</td>
</tr>
<tr>
<td>Mutation probability</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Crossover distribution index</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Mutation distribution index</td>
<td>10</td>
<td>10</td>
</tr>
</tbody>
</table>

Table 1: Optimization algorithms parameters settings

Case study with identical costs and demands structure
In this section, we maintain the same costs and demand structure: shortage cost=$5, holding cost=$2, demands are random variables normally distributed $N(200, 50)$. Illustrative examples based on identical cost and/or...
demand structure were widely used in most of the literature analytical studies. We follow this trend to allow further researches to be compared with our study.

**Aggregate Cost vs. Fill Rate**

Figure 3, it is given the 3D representation of both Cost and Fill rate function of the CST(2) instance. Figure 4 illustrates the Pareto front of Cost/Fill Rate problem for both Newsvendor and Transshipment unconstrained instances. Non-dominated solutions are well spread over the entire Pareto front which is convex. We notice that USN(5) solutions are dominated by UST(5) ones. This is another way to prove that Transshipment based system performs better than Newsvendor systems in term of cost reduction and fill rate increase. Pareto front of UST(5) is also located in tight region (cost: [500, 600], and Fill rate: [99.6%; 100%]) while USN(5) front is well spread especially in term of cost which varies from $550 to $1700. We conclude that Cost and fill rate are more conflicting in Newsvendor systems than in transshipment allowed systems.

Figure 5 illustrates the case where storage constraints are present. Remark the considerable degradation of costs and fill rate in CSN(5) while in CST(5), the 100% fill rate value still can be reached with low costs. The Transshipment plays an important role in optimizing costs and fill rate by ensuring goods movement intra-locations.

**Cost vs. Shared Inventory Quantity**

In figure 6, it is represented graphically the SIQ function of both CST(2) and UST(2) instances. The main difference is that when storage is not constrained (instance UST(2), the SIQ can be negligible when all replenishment quantities are too large. (Here S1 and S2 are greater than 400 units). Whereas in the presence of storage constraints (figure 6 CST(2)), the SIQ is not negligible because the highest possible levels of on-hand inventory is bounded. In other words, the probability that the demand exceeds the inventory at one location is not null. Figure 7 illustrates UST(5) and CST(5) Pareto fronts where both costs and SIQ are minimized. Notice that for both problems, lowest cost levels are obtained ($500 and $1500) when the quantity of shared inventory is maximal (respectively 40 and 90 units). When transshipments do not take place (usually when replenishment quantities are very low or very high), costs increase considerably. We compared the highest nondominated SIQ to its corresponding replenishment quantity for UST(5) problem, we observed that sharing 4% of the initial inventory ensures the best cost. While for CST(5) problem, 15% of the initial inventory should be shared to reach
the lowest cost level.

**Fill rate vs. Shared Inventory Quantity**

When storage capacity is not limited, fill rate and SIQ are not conflicting objectives. A 100% fill rate and no shared units could be achieved by ordering very high replenishment quantities in all locations (see figure 6 for a two-location illustrative instance). Thus, we examine the situation where storage is limited. Figure 8 shows that the best fill rate value is obtained only when the shared inventory is relatively high (45% of the inventory).

![Figure 6. SIQ function 3d representation of both CST(2) (left) and UST(2) (right) problems.](image)

![Figure 7. “Cost vs. SIQ” Pareto fronts. URQ and CRQ refer to Unconstrained (resp. constrained) Replenishment Quantity (x10^-1).](image)

![Figure 8. “Fill rate vs. SIQ” Pareto fronts of CST(5) problem. CRQ refers to the resulting replenishment quantity.](image)

**Case study with non-identical costs and demands structure**

In this case study, each location has its own setting. It is a more general case study that can be tackled only with optimization solving algorithms. Table 2 defines a 5-location system setting.

<table>
<thead>
<tr>
<th>Location index (i=1..5)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holding cost (hi)</td>
<td>$5</td>
<td>$6</td>
<td>$7</td>
<td>$8</td>
<td>$9</td>
</tr>
<tr>
<td>Shortage cost (pi)</td>
<td>$4</td>
<td>$4</td>
<td>$4</td>
<td>$4</td>
<td>$4</td>
</tr>
<tr>
<td>Normal demand D_i→N(μ, σ)</td>
<td>(250,50)</td>
<td>(300,100)</td>
<td>(350,150)</td>
<td>(400,200)</td>
<td>(450,250)</td>
</tr>
<tr>
<td>Storage capacity (S_max,i)</td>
<td>125</td>
<td>150</td>
<td>450</td>
<td>600</td>
<td>750</td>
</tr>
</tbody>
</table>

Table 2. System settings when costs and demands are not identical

The unit transshipment costs are infinite for all Newsvendor instances. But in the Transshipment case, they pursue the following rule: \( T_{ij} = 1+i \) for all \( j \neq i \). (Recall that \( T_{ii}=0 \)). The transshipment cost depends only on the sending location index. It is a simple procedure to vary the parameters settings.

Figure 9 presents two superposed plots. The first plot (with dashed points) is the Pareto front of instance CST(5). When comparing it to the Pareto front of figure 5, we remark that Figure 9 has better fill rate levels than figure 5. It is due to the high demand variance of the former. In fact, when the demand at some locations has a high
variability, the situation of having too many unsold units and too many unmet demand occurs frequently. Hence, the transshipments take place frequently too so that demands and inventory are balanced. The continuous plot of Figure 9 represents the global inventory level required for each non-dominated solution of Cost/Fill rate Pareto front. We observe that higher fill rate levels (greater than 99%) result not only in higher costs but also in global inventory increase. The decision maker may conclude that it is not interesting to try to enhance the fill rate whenever it reached the 99% level.

Figure 9. Variation of the global inventory level for each non-dominated solution of "Cost vs. Fill Rate" Pareto front of CST(5).

In Figure 10, it is drawn the variation of the SIQ for each non-dominated solution of Cost/Fill rate bi-objective problem CST(5). We identify three noticeable regions:

- Region 1: defined by solutions where costs are in [$3000, $3400]. The SIQ is increasing regularly from 130 to 155 units. According to Figure 9, it is about 4% to 9% of the global inventory is devoted to transshipments.

Figure 10. Variation of the SIQ for each non-dominated solution of "Cost vs. Fill Rate" Pareto front of CST(5).

- Region 2: defined by solutions where costs are in [$3400, $5000]. The SIQ is decreasing regularly. Thus, no need to share too much stocks in order to achieve fill rate of 96% to 99%. In fact, such situation occurs when the order quantity $S_i$ at each location is set so that stock out probability is low. Consequently, few units are shared between the locations.

- Region 3: defined by solutions where costs are in [$5000, $8400]. This region is of big interest. It shows the high variability of the SIQ for solutions with near optimal fill rate levels (99% to 100%). In other words, when moving regularly from one non-dominated solution to its neighbor, both cost and fill rate levels are similar while the SIQ is fluctuating from higher to lower values. Such situation occurs when both local under stocking and overstocking probabilities are important in the system locations. Thus, we may frequently find the set $I^+$ with a large quantity of unsold stock while having the set $I^-$ with a large amount of unmet demands. The SIQ will be very high.

Performance measures
To assess the performance of SPEA2 and NSGA-II in solving instances of Transshipment and Newsvendor problems, we used two reference metrics: Hypervolume (HV) and Spread ($Δ$). The HV quality indicator computes the volume covered by a nondominated set of solutions (Zitzler and Thiele 1999). For each solution $i$, a hypercube is constructed. It designates the region of objective space dominated by $i$. Thereafter, a union of all hypercubes is found and its Hypervolume is calculated. Since this metric is not free from arbitrary scaling objectives, we have evaluated it by using normalized objective functions values. Algorithms with larger values of HV are desirable.

The Spread metric $Δ$ is a diversity indicator that measures the extent of spread achieved among the obtained solutions (Deb et al. 2002). This metric takes a zero value for an ideal distribution. Before applying it, the
objective function values were normalized. Let's focus on \textit{CST(4)} optimization results. We assess the performance of the used optimization algorithms for further comparison with other approaches that may be used to tackle \textit{CST(4)} instance. After 100 runs of both SPEA2 and NSGA-II, mean and variance values of Hypervolume and Spread metrics are calculated. Table 3 indicates that there is no significant difference between SPEA2 and NSGA-II. Both of them converge to a well spread Pareto fronts.

<table>
<thead>
<tr>
<th>Objective Function Values</th>
<th>Hypervolume ($HV$)</th>
<th>Spread ($\Delta$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Variance (10$^3$)</td>
</tr>
<tr>
<td>\textit{CST(4)}</td>
<td>0.9951</td>
<td>3.35506</td>
</tr>
<tr>
<td>SPEA2</td>
<td>0.9952</td>
<td>3.10447</td>
</tr>
</tbody>
</table>

Table 3. Best results given by NSGA-II and SPEA2 in \textit{CST(4) Cost/Fill rate and Cost/SIQ optimization (system with identical cost structure).}

\section*{DISCUSSION AND FUTURE RESEARCHES}

In this research paper, we have dealt with the analysis and the optimization of a wide range of multi-location inventory problems instances belonging to both Newsvendor and Transshipment inventory models. It adds to the existing literature in several ways. Unlike most of the relevant researches in this field, multiple objectives were involved. For each model, the bi-objective Cost vs. Fill Rate problem is optimized. It is showed that when transshipments are allowed, aggregate cost and fill rate measures are considerably improved compared to the Newsvendor case especially in the presence of storage constraints. However, the shared inventory quantity may increase in this case. This conflicting situation is represented by a large number of nondominated solutions that should be analyzed thoroughly by the managers to make good decisions. We showed that considering multiple objectives is of big interest. Two reference multiobjective optimization evolutionary algorithms referred to as SPEA2 and NSGA-II were successful in generating well spread nondominated solutions in relatively reasonable running time. Hypervolume and Spread multiobjective metrics were used for the algorithmic performance assessment.

Although the models' descriptions already exhibit a significant level of detail, they can be further extended. The pooling policy stated in this strategy is based on the instantaneous physical transfer of stock among the system locations. Transport vehicles should be available during all the working period. Then, efficient logistics should be fully implemented. It should ensure immediate access to local inventory information and instant vehicles workload, etc.

The computational experiments presented in this study underlined that costs and fill rate are conflicting. However, in supply chain management, a wide range of performance measures are serving as helpers to the decision makers. We suggest incorporating new objectives to the current inventory models. Minimizing the stock-out and/or the overstocking probability are some interesting candidate objectives. Besides, we believe that managing stochastic lead time is an important objective. We expect that these extensions will make the problem much harder to solve and hence urge intensive future research to integrate them adequately.

\section*{References}


