A Monte Carlo Optimization and Dynamic Programming Approach for Managing MRI Examinations of Stroke Patients

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Abstract—Quick diagnosis is critical to stroke patients, but it relies on expensive and heavily used imaging equipment. This results in long waiting times with potential threats to the patient’s life. It is important for neurovascular departments treating stroke patients to reduce waiting times for diagnosis. This paper proposes a reservation process of magnetic resonance imaging (MRI) examinations for stroke patients. The neurovascular department reserves a certain number of appropriately distributed contracted time slots (CTS) to ensure quick diagnosis of stroke patients. Additional MRI time slots can also be reserved by regular reservations (RTS). The problem consists in determining the contract and the control policy to assign patients to either CTS or RTS in order to reach the best compromise between the waiting times and unused CTS. Structural properties of the optimal control policy are proved by an average-cost Markov decision process (MDP) approach. The contract is determined by combining a Monte Carlo approximation approach and local search. Extensive numerical experiments are performed to show the efficiency of the proposed approach and to investigate the impact of different parameters.

Index Terms—Magnetic resonance imaging (MRI), Markov decision process (MDP), Monte Carlo optimization, stochastic programming model, stroke patients.

I. INTRODUCTION

T HIS paper is motivated by our collaborations with a large French university teaching hospital in order to reduce the length of stay (LoS) of stroke patients treated in the neurovascular department. A stroke is a sudden loss of the brain function caused by lack of blood supply to the brain or rupture of blood vessels in the brain. The brain cannot tolerate long periods without blood flow, and stroke patients need the appropriate treatment as soon as possible.

Before starting the treatment, a number of examinations are needed for diagnostic purpose. The LoS of a patient greatly depends on the delay of having all necessary examinations. In the related hospital, requests for examinations of non-emergency patients are handled by secretaries by making appointments to patients by fax. If patients are available, then examinations are scheduled. The same-day examinations are reserved for emergency patients by neurologists by phone. This process seems simple, but is time-consuming as it needs the collaboration of neurovascular department, medical imaging department, and the patients.

Significant delays are observed as many key examinations rely on expensive and heavily used imaging facilities facing demand from all medical units of the hospital. For example, the delay for a magnetic resonance imaging (MRI) examination is over 30 days. Examination delays can be improved either from the service provider side, i.e., better schedule of examination demands, or from the neurovascular department side. The improvement of the operations of the imaging department involves the whole hospital and concerns all medical units. For this reason, we start from the neurovascular department perspective and develop implementable solutions.

A detailed analysis of data from a six-month field observation reveals that the neurovascular department has rather stable demand for medical examinations, and it is one of the largest customers of the imaging department.

Based on these observations, we proposed a new contract-based reservation process: The imaging department reserves each day some so-called contracted time slots (CTS) for the neurovascular department. Time slots by regular reservation (RTS) are still possible in case of arrival surges of stroke patients in order to avoid long waiting times. The efficiency of the new reservation process greatly depends on two closely related decisions: 1) the contract planning decision, i.e., the number of CTS and its distribution over time; and 2) the real-time control policy for assigning incoming patients to either CTS or RTS.

The use of a contract gives a long-term view of diagnostic capacity available, and the neurovascular department can better manage stroke patients of different priority and reduce the waiting times for examination. From the perspective of the imaging department, although the use of a contract potentially leads to unused time slots, it gives the imaging department stable demands that can be used to improve the scheduling of its staff and diagnostic facilities. Another advantage of contract-based approach is the possibility for the neurovascular department to better match different diagnostic examinations
of the same patient and available contracted CTS for different facilities.

This paper provides a theoretical analysis of the contract-based approach under some restrictions. First, we restrict ourselves to MRI examinations for two reasons. Delays for MRI examinations are by far the longest ones. Joint optimization of all medical examinations is fairly complex and will be a subject of our future research. Insights gained from this study will be exploited in the joint optimization of multiple medical examinations. Another major assumption is the focus on the neurovascular department, and the impact of the MRI examination reservations from this department on other departments sharing the same MRI facilities is neglected. Other assumptions include patients of the same priority and stable weekly demands that are justified by the field observations.

Under these assumptions, we propose in this paper a stochastic Markov decision process (MDP) approach to establish structural properties of the optimal control policy and to prove the optimality of threshold policies. The contract optimization problem is solved in two steps by a combination of MDP and Monte Carlo optimization. The long-term average cost is first approximated by the average cost defined on a finite sample path of patient arrivals. This Monte Carlo contract optimization problem is then simplified by relaxing the nonanticipativeness of the control policy and reformulated as a linear program (LP). Resulting contract is further improved by local search. Numerical results show that our approach is able to provide solutions close to real optimum. Except for one instance, the best contract is reached if exact criterion values are used in local search. The relaxed Monte Carlo approximation always leads to a contract which is at most two local moves away from the best contract identified by exhaustive search for small-size problems and by multiple runs of our approach for large-size problems. Sensitivity analysis is performed to show the impact of problem parameters on the contract and the control policy.

With regards to the implementation of the proposed approach, we do not recommend the contract-based approach for all departments, but only for critical diagnostic facilities and for major consumers with stable demands. Results of this paper can be directly used to design separately the contract for each department and for each critical facility. If this leads to the overusage of a diagnostic facility, the contract of each department can be refined by limiting the number of time slots to contract each day, and all results of this paper still hold. The joint design of contract-based solutions of multiple departments is an interesting research direction to be discussed in Section VI.

Our MRI examination reservation problem belongs to the general class of capacity planning and allocation problems. Allocating service capacity among competing customer classes has been studied in diverse applications including airlines seat management [3], hotels rooms [5], car rental [7], and call center management [6]. It is not our intention to provide a comprehensive survey of capacity planning and allocation. Instead, we limit ourselves to planning and allocation of medical service capacity most relevant to our problem.

Allocating medical service capacity between different demand streams has received limited coverage in the research literature. Effective allocation of expensive imaging diagnostic capacity among competing groups of patients was formulated in [10] as a finite-horizon dynamic program. Properties of optimal policies were identified in order to design the outpatient appointment schedule and establish dynamic priority rules for admitting patients into services. A simple approach for dividing the available diagnostic capacity between emergency, inpatients, and outpatients was proposed in [12]. A dynamical scheduling problem of multipriority patients to a diagnostic facility was formulated in [13] as an MDP, and an approximate dynamic programming approach was proposed to overcome the state-space explosion problem. An MDP method was also proposed in [16] to allocate and reallocate patients to different floors of a hospital during demand surges. Decisions such as patient assignment and reactive or proactive patient transfers were considered. Stochastic programming was used in [8] to characterize the optimal policy that determines at the start of each day how many additional elective surgeries to assign for that day under uncertain demand for emergency surgery. A stochastic model for operating room planning with two types of surgery demands: Elective and emergency was proposed in [11] and solved by a Monte Carlo optimization method. Organ allocation is an area of intense research, and sophisticated MDP models have been proposed for optimization of organ allocation from different perspectives [15], [1].

Our paper differs from the above studies by investigating the problem from a totally different perspective and explores solutions from the client side, i.e., the neurovascular department. Most existing studies investigate medical service capacity allocation problems from the perspective of service provider side. For diagnostic facility scheduling, [10] considered the real-time patient scheduling of a diagnostic facility within a day. Reference [12] addressed the daily capacity management of a computed tomography (CT) scanning department and looked at the benefit of reserving space for carrying over a percentage of non-emergency inpatient demand to the next day. Reference [13] addressed the admission of multipriority patients on a waiting queue to a diagnostic resource.

From a methodological point of view, our approach seems related to capacity allocation such as staffing in call center management [6]. Capacity allocation in this context also has to take into account random demands. The major difference with our problem is the acceptable waiting time. In the call center case, the acceptable waiting time is fairly short and, as a result, customers overflowing from one capacity planning time slot to another one can be neglected. In our case, the waiting of several days for MRI examination is common, and the capacity planning has to take into account patients untreated overflowing from one day to the next. This makes the capacity planning closely linked to CTS waiting queue control.

This paper is an extension of our preliminary work [2], which analyzed the contract design problem of MRI examinations of stroke patients by using discrete event simulation and experimental design. This paper provides an in-depth mathematical analysis of the optimal control and proposes an efficient contract optimization approach.

The rest of the paper is organized as follows. The problem formulation is given in Section II. Section III proposes an average-cost MDP model for exploring the structural properties...
of the optimal control policy for a given contract. Contract optimization is addressed in Section IV. Section V presents computational results to assess the efficiency of the proposed approach and the impact of problem parameters. Conclusions and perspectives are given in Section VI.

II. PROBLEM FORMULATION

Contract design for MRI examinations of stroke patients consists in determining: 1) a contract decision, i.e., planning the number of CTS for each day that we call contract; and 2) a patient assignment control policy, i.e., the determination of the optimal real-time control policy to assign incoming stroke patients to either CTS or RTS. These two decisions are mutually dependent. The optimal control policy depends on the contract decisions, while the control policy has an impact in the planning of CTS to be used.

The following assumptions are made throughout the paper.

Assumption A1: Only MRI examination is considered, and each patient requires one MRI time slot. Each patient can be assigned to either one CTS or one RTS. Our field observation shows that MRI examination of stroke patients takes nearly the same time, i.e., one time slot of about 30 min.

Assumption A2: Emergency stroke patients are not considered, and all other patients have the same priority. In practice, emergency stroke patients are given immediate access to MRI with no waiting time, and all other patients have the same priority. In practice, the same time, i.e., one time slot of about 30 min.

Assumption A3: Patient arrival varies during a week, but is stationary from one week to another. Furthermore, the number of arrivals in one day is independent of the arrivals of other days.

The MRI examination reservation problem is defined by the following notation:

- \( t \) index of days, \( t = 1, \ldots, T \);
- \( i \) index of days in one week, \( i = 1, \ldots, 7 \);
- \( d(t) \) corresponding weekday of \( t \) with \( d(t) \in \{1, \ldots, 7\} \);
- \( T^R \) average number of days for a patient to have his/her MRI examination through regular reservation with \( T^R > 1 \);
- \( c \) penalty factor of an unused CTS. It serves as a weighting factor in order to balance the waiting times and unused MRI time slots;
- \( a_t \) number of patients arrived in day \( t \). By Assumption A3, daily arrivals \( a_t \) for \( t \in \mathbb{N} \) are mutually independent random variables, and weekly arrivals \( a_1, a_2, \ldots, a_7 \) are identically distributed for all \( j = 0, 1, \ldots \). As a result, the arrival process is characterized by probability matrix \( P = [P_{ij}] \) for \( i = 1, \ldots, 7 \) and for all \( j \geq 0 \), with \( P_{ij} \) denoting the probability of \( j \) arrivals in day \( i \).

Decision variables are the following:

- \( n_t \) number of CTS of day \( t \);
- \( x_t \) number of patients waiting for CTS at the end of day \( t \) with \( x_0 \) being given;
- \( y_t \) number of patients directed to RTS in day \( t \) with \( y_t = f_t(x_{t-1} + a_t) \).

Remark 1: The average waiting time of the stroke patients served by RTS is approximated by a constant \( T^R \). In practice, the use of contract at neurovascular department is expected to modify \( T^R \). The use of a constant \( T^R \) is still reasonable for the following reasons. First, in practice the proportion of MRI usage by neurovascular department in large hospitals is usually not too high. For example, in our partner hospital, its five MRI machines are shared among many diagnostic, research, and teaching activities from 61 medical services of the hospital, external services, and the imaging department itself. The change of reservation process of the neurovascular department alone is expected to have limited impact on the delay of regular reservation. Furthermore, the sensitivity analysis performed in Section V shows that the optimal contract is quite insensitive to the change of \( T^R \) in a relative large range with respect to the current delay. Of course, as also shown, the optimal contract becomes sensitive to \( T^R \) when it becomes small enough. How to design a contract solution by taking into account its impact on \( T^R \) is an interesting subject of future research.

Remark 2: No constraint on the number of CTS is made in this paper. The contract determined this way best reflects the requirement of the neurovascular department. Nevertheless, all results of this paper still hold under constraints about the maximum number of CTS for each week and each day.

The sequence of events during a day \( t \) is as follows. First, the CTS queue length \( x_{t-1} \) at the beginning of the day is known. The number \( n_t \) of new incoming patients arrived during the day becomes known. \( \min\{n_t, x_{t-1} + a_t\} \) patients are served by the \( n_t \) CTS of the day, and \( \max\{0, n_t - x_{t-1} - a_t\} \) CTS cannot be filled. \( y_t = f_t(x_{t-1} + a_t) \) patients are directed to RTS and will have the MRI examination after an average of \( T^R \) days. The remaining patients will wait for CTS of the subsequent days.

Assumption A4: The same contract is used for different weeks, i.e., \( n_t = n_7 + 7 \) for all \( t \). As a result, the contract can be represented by a seven-entry integer-valued vector \( \mathbf{n} = \{n_1, \ldots, n_7\} \).

Remark 3: Due to Assumption A3 on the stationary weekly patient arrival, Assumption A4 is reasonable, and numerical results in Section V-B (see Remark 9) show that optimal biweekly contracts are the same as optimal weekly contracts. Under Assumption A4, the control policy will be proved to be stationary over weeks, i.e., \( f_{t+7}(\cdot) = f_t(\cdot) \) and is a threshold control policy. For nonstationary weekly arrival, Assumption A4 no longer holds, but results on the optimality of threshold control policies still hold.

The MRI examination reservation problem can be formally stated by the following stochastic programming model:

\[
\min_{n_t} \quad E \left[ \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} (T^R y_t + x_t + c(n_t - x_{t-1} - a_t)^+) \right]
\]

subject to:

\[
y_t = f_t(x_{t-1} + a_t) \leq x_{t-1} + a_t \tag{2}
\]

\[
x_t = (x_{t-1} + a_t - y_t - n_t)^+ \tag{3}
\]

\[
(n_1, n_2, \ldots, n_7) \in \mathbb{N}^7, f_t : \mathbb{N} \to \mathbb{N}. \tag{4}
\]

In this formulation, criterion (1) contains three terms: waiting days \( T^R y_t \) of patients served by RTS, additional waiting days \( x_t \)
of patients in the CTS queue, and the penalty cost of unused CTS. Constraint \((2)\) defines the control policy for the use of RTS. Constraint \((3)\) updates the CTS queue length.

Note that only the average waiting time of stroke patients is considered. Patient scheduling is not considered. How to schedule patients in order to reduce waiting time variance is an issue of future research.

In the following, we first investigate the structural properties of the optimal control policy and then propose an optimization method for determining the contract.

III. STRUCTURAL PROPERTIES OF THE OPTIMAL CONTROL

This section considers the optimal control policy for the average-cost MDP under a given contract \(\pi\). Structural properties of the average-cost MDP \((1)-(4)\) are established via discounted cost MDP by value iteration and by using relations between these two MDP models.

Let \(z_t = x_{t-1} + a_t\) denote the state variable after patient arrivals. The control policy \(\pi = \{\pi_1, \pi_2, \ldots\}\) is defined as \(x_t = \pi_t(z_t)\) with \(0 \leq t \leq (z_t - n_t)^+\). Note that the new definition of control policy is equivalent to that of relation \((2)\) as a result of relations \((2)\) and \((3)\).

The objective is to minimize over all policies \(\pi = \{\pi_1, \pi_2, \ldots\}\) the average cost

\[
J_\pi(i, z) = \lim_{T \to \infty} \frac{1}{T} \mathbb{E} \left[ \sum_{t=0}^{T+i} g_t(z_t, x_t) \right] z_i = z
\]

or the \(\alpha\)-discounted total cost with \(0 < \alpha < 1\)

\[
J_{\alpha, \pi}(i, z) = \lim_{T \to \infty} \alpha^T \mathbb{E} \left[ \sum_{t=0}^{T} \alpha^{t-i} g_t(z_t, x_t) \right] z_i = z
\]

for any given initial state \(z_i = z\) with \(i = 1, \ldots, 7\) where \(g_t(z_t, x_t)\) is the cost incurred at day \(t\),

\[
g_t(z_t, x_t) = c(n_t - z_t)^+ + x_t + T^R (z_t - n_t)^+ - x_t
\]

In the following, \(g_t(z_t, x_t)\) and \(n_t(z_t)\) are written as \(g_t(i)\) and \(n_t\) for convenience.

A. Discounted Cost Problem

Consider the following optimal cost function:

\[
V_\alpha(i, z) = \min_{\pi} J_{\alpha, \pi}(i, z)
\]

The index \(\alpha\) is omitted for simplicity in this subsection. From [4], since all stage costs \(g_t(z_t, x_t) \geq 0\) and the control constraint set is finite for each \(z_t\) as \(x_t \leq z_t\), the optimal cost function is a solution of the following optimality equations:

\[
V(i, z) = \min_{x} \left\{ c(n_i - z)^+ + x + T^R (z - n_i - x)^+ + \alpha \sum_a P_{t+1,a} V(i+1, x+a) \right\}
\]

\[
V(7, z) = \min_{x} \left\{ c(n_7 - z)^+ + x + T^R (z - n_7 - x)^+ + \alpha \sum_a P_{t,a} V(1, x+a) \right\}
\]

for all \(i = 1, \ldots, 6\), and the optimal control policy is given by the argument \(x\) that reaches the minimum in \((8)\) and \((9)\), and the optimal cost function is the limiting function of the following value iteration:

\[
V'_t(z_t) = \min_{x_t} \left\{ c(n_t - z_t)^+ + x_t + T^R (z_t - n_t - x_t)^+ + \alpha \sum_a P_{t+1,a} V^{t+1}(x_t+a) \right\}
\]

\[
V^0(z) = 0
\]

\[
V_t(z_t) = c(n_t - z_t)^+ + T^R (z_t - n_t)^+ + \min_{0 \leq i \leq (z_t - n_t)^+} \left\{ U^t+1(x_t) - (T^R - 1)x_t \right\}
\]

\[
U^t+1(x_t) = \alpha \sum_a P_{t+1,a} V^{t+1}(x_t + a)
\]

Similarly, optimality \((8)\) and \((9)\) can be rewritten as follows:

\[
V(i, z) = c(n_i - z)^+ + T^R (z - n_i)^+ + \min_{0 \leq x \leq (z_t - n_t)^+} \left\{ U(i+1, x) - (T^R - 1)x \right\}
\]

\[
V(7, z) = c(n_7 - z)^+ + T^R (z - n_7)^+ + \min_{0 \leq x \leq (z_t - n_t)^+} \left\{ U(1, x) - (T^R - 1)x \right\}
\]

\[
U(i, x) = \alpha \sum_a P_{t,a} V(i, x+a)
\]

Remark 4: From [14, Theorem 6.10.4], the optimality \((8)\) and \((9)\) have a unique solution as [14, Assumption 6.10.1 and Condition 6.10.11] hold with \(g_t(z_t, x_t) \leq w(z_t) \equiv cn^* + TRz_t\) and

\[
\sum_a P_{t+1,a} w(x_t + a) \leq cn^* + TRx_t + TR E[a_t+1]
\]

\[
\leq cn^* + TRz_t + T R a^*
\]

where \(n^* = \max\{n_1, \ldots, n_7\}\). \(a^* = \max\{E[a_1], \ldots, E[a_7]\}\).

Property 1: In the value iteration by \((10)\) or equivalently by \((13)\), the optimal \(x_t\) is nondecreasing in \(z_t\).
Proofs of all the properties are given in the Appendix.

Property 2: \(-c \leq V^t(z_t + 1) - V^t(z_t) \leq T^R\), for any \(z_t\) and \(t\).

Definition: A function \(\phi(x) : \mathbb{Z} \to \mathbb{R}\) is said convex if \(\phi(x + 1) - \phi(x) \geq \phi(x) - \phi(x - 1)\), for all \(x\).

Property 3: \(V^t(z_t)\) is convex in \(z_t\) and \(U^{t+1}(x_t)\) is convex in \(x_t\).

From Property 3, the optimal control function can be written as follows:

\[
V^t(z_t) = \begin{cases} 
0, & \text{if } z_t = L_t \\
2z_t - n_t + U^{t+1}(z_t - n_t), & \text{if } n_t \leq z_t \leq L_t + n_t \\
L_t + T^R(z_t - n_t - L_t), & \text{if } z_t \geq L_t + n_t 
\end{cases}
\]

where

\[
L_t = \arg\min_{x_t} \left( U^{t+1}(x_t) - (T^R - 1)x_t \right).
\]

Theorem 1: \(V(i, z)\) and \(U(i, x)\) are convex functions respectively in \(z\) and \(x\) for all \(i = 1, \ldots, 7\). Furthermore, the optimal control policy is of the following form:

\[
x^*_i = \begin{cases} 
0, & \text{if } z - n_i \leq 0 \\
L_i, & \text{if } z - n_i \geq L_i 
\end{cases}
\]

Proof: The theorem is a direct consequence of \((12), (15)-(17),\) and Property 3.

B. Average Cost Problem

In this section, the optimality equations and the form of the optimal control policy for the average cost problem will be established via the \(q\)-discounted problem.

Property 4: For any optimal average cost policy, \(x_t \leq T^R n^*\) for all \(t > 0\).

As our final goal is the average cost minimization, without loss of generality, the following assumption is made.

Assumption A5: \(x_t \leq T^R n^*\) for all \(t > 0\).

It can be checked that all results of the discounted cost case still hold under Assumption A5. Even through \(x_t\) is bounded, \(z_t = x_t - n_t\) can be unbounded. For the sake of readability, we first consider the case of bounded \(z_t\) and then establish properties of optimal control for the unbounded case.

Assumption A6: The number of patients arrived in a day is bounded from above by \(M\), i.e., \(a_t \leq M\) for some given positive integer \(M\).

Assumptions A5–A6 imply that \(z_t\) is bounded from above by \(z_{\text{max}} = M + T^R n^*\). Therefore, \(g_t(z_t, x_t)\) is bounded from above and \(0 \leq g_t(z_t, x_t) \leq c_n + T^R z_{\text{max}} - n_t\). Hence,

\[
\alpha_t \leq M + T^R n^* + T^R z_{\text{max}}.
\]

Property 5: Under Assumptions A4–A6, there exists \(\Gamma > 0\) such that \(V_{\alpha}(i, z) - V_{\alpha}(7, 0) \leq \Gamma\), for all \(i = 1, \ldots, 7\) and for all \(z\).

Theorem 2: Under Assumptions A4–A6, there exists an optimal stationary control policy of the form of \((19)\) for the average cost model \((5)\). Furthermore, the optimal average cost is independent of the initial state \((i, z)\).

Proof: From [4, Proposition 4.2.6] and Property 5, the optimal average cost per day exists and has the same value \(\lambda\) for all initial states and \(\lambda\) satisfies

\[
\lambda = \lim_{\alpha \to 1} (1 - \alpha)V_{\alpha}(i, z).
\]

The differential cost functions

\[
h(i, z) = \lim_{\alpha \to 1} (V_{\alpha}(i, z) - V_{\alpha}(0, 0))
\]

satisfy the following optimality equations:

\[
\lambda + h(i, z) = c(n - z)^+ + T^R(z - n_i)^+ + \min_{0 \leq z \leq (z - n_i)^+} \{H(i + 1, x) - (T^R - 1)x\}
\]

\[
\lambda + h(7, z) = c(n - z)^+ + T^R(z - n_i)^+ + \min_{0 \leq z \leq (z - n_i)^+} \{H(1, x) - (T^R - 1)x\}
\]

for \(i = 1, \ldots, 6\) with

\[
H(i, x) = \alpha \sum_{a} P_{i,a} h(i, x + a).
\]

Furthermore, the optimal control policy is defined by the argument \(x\) that reaches the minimum in \((22)\) and \((23)\). From Property 3, \((12)\), and \((21)\), \(h(i, z)\) is convex in \(z\) and \(H(i, x)\) is convex in \(x\) for all \(i\). This implies that the optimal control policy for the average cost problem is of the form \((19)\).

We consider now the unbounded arrival case and relax Assumption A6.

Theorem 3: Under Assumptions A4–A5: (a) there exists a constant \(\lambda\) satisfying relation \((20)\) for all \((i, z)\), a matrix \(h(i, z)\) satisfying relations \((22)\)–\((24)\); (b) the optimal control policy is given by the argument \(x\) that reaches the minimum in relations \((22)\)–\((24)\); (c) there exists an optimal stationary control policy of the form of \((19)\) for the average cost problem.

Proof: The proof is based on [14, Theorem 8.10.7], and the conditions to check are the following ones:

C1) For each state \((i, z)\), \(\infty < R < g(i, z) < \infty\).

C2) For each \((i, z)\) and \(\alpha < 1\), \(V_{\alpha}(i, z) < \infty\).

C3) There exists \(G > \infty\) such that, for each \((i, z)\)

\[
h_{\alpha}(i, z) \equiv V_{\alpha}(i, z) - V_{\alpha}(7, 0) \geq G \quad \forall \alpha < 1.
\]

C4) There exists a nonnegative function \(W(i, z)\) such that:

a) \(W(i, z) < \infty\);

b) for each \((i, z)\), \(h_{\alpha}(i, z) \leq W(i, z)\), \(\forall \alpha < 1\);

c) for each \((i, z)\) and \(x, \sum_{a} P_{i,a} W(i + 1, x + a) < \infty\).

According to [14, Theorem 8.10.7], Assumption A5 implies that the control constraint set for each state \((i, z)\) is finite and (a) and (b) of the theorem hold. Furthermore, \(h_{\alpha}(i, z)\) is the limit of a sequence \(h_{\alpha_{\lim}}(i, z)\) such that \(h_{\alpha_{\lim}}(i, z)\) converges to \(1\) and \(h_{\alpha_{\lim}}(i, z)\) converges for all \((i, z)\). From Property 3, \((12)\), and \((21)\), \(h_{\alpha}(i, z)\) is convex in \(z\) and \((c)\) of the theorem can be proved as for Theorem 2.

Let us now prove conditions C1–C4. Condition C1 clearly holds as \(g(i, z) \geq 0\). Condition C2 holds as well as the following apply.

(i) By Assumption A5, \(x_t \leq T^R n^* + \alpha t\),
(ii) $g_k(z_t, x_t) \leq cn^* + TRz_t$ and

$$E[g_k(z_t, x_t)] \leq E[cn^* + TRz_t] \leq cn^* + TR(T^Rn^* + \alpha^*) .$$

(iii) $V_0(i, z) \leq cn^* + TRz + (\alpha/1 - \alpha)(cn^* + TR(T^Rn^* + \alpha^*)) .

Condition C3 holds as the following apply.
(i) By Property 2 and relation (12)

$$-\epsilon \leq V_\alpha(i, z + 1) - V_\alpha(i, z) \leq TR .$$

(ii) By Assumption A5 and Theorem 1, the control threshold $L_i \leq TRn^* .

(iii) By relations (18) and (12), $V_\alpha(i, z)$ is increasing for $z \geq L_i + n_i .

(iv) By (i)-(iii), $-C(T^Rn^* + n^*) \leq V_\alpha(i, z) - V_\alpha(i, z')$ with $C = \max(c_i, T^R) .

(v) $V_\alpha(7, 0) \leq icn^* + TR \sum_{s=1}^{i-1} E[\alpha_s] + \sum_{s=1}^{i-1} \sum_{s=1}^s E[\zeta_t] + \sum_{s=1}^s Q_{(r, z', (r+1, z')} \alpha_\pi(i, z') .

where $Q_{(r, z', (r+1, z')}$ is the probability of reaching state $z'$ at the beginning of day $t + 1$ by starting from state $z$ at day $t$ under policy $\pi .

(vi) Combining with (iv)

$$V_\alpha(7, 0) \leq icn^* + TRi\alpha^* + i^2\alpha^* + (V_\alpha(i, z) + C(T^Rn^* + n^*)) .$$

which proves C3.

Condition C4 holds as the following apply.
(i) By starting from state $(i, z)$, the average stage cost of any period $t$ is bounded from above by

$$cn^* + TR\left(z + \sum_{t=1}^{t} E[\alpha_{t+1}]\right) \leq cn^* + TR(z + ta^*) .

(ii) From (i)

$$V_\alpha(i, z) \leq \sum_{t=0}^{t-1} (cn^* + TR(z + ta^*)) + \sum_{s=1}^{s} Q_{(r, z', (r+1, z')} \alpha_\pi(i, z') .

(iii) Combining (ii) with $i + t = 7$ and Property 2

$$V_\alpha(i, z) \leq 6cn^* + 6TRz + 36TR\alpha^* + V_\alpha(7, 0) + TR E[z_\gamma] .

(iv) Combining (iii) with $E[z_\gamma] \leq z + \sum_{t=1}^{7} E[\alpha_t] \leq z + 6\alpha^*$

leads to $h_\alpha(i, z) \leq W(i, z) + W(i, z) \leq 7TRz + 6cn^* + 4TR^2\alpha^*$.

Condition C4.a-C4.c clearly hold. This proves C4.

Remark 5: From Theorems 2 and 3, there exists a stationary optimal control policy of form (19) with a threshold $L_i$ for each day $i$ of the week. The optimal control consists in keeping patients for CTS if the CTS queue length at the end of a day is below $L_i$, otherwise sending some patients to RTS to ensure that the CTS queue ends at $L_i$. The existence of optimal threshold control makes the implementation easy. The optimal control policy for a given contract $n$ can be determined by solving optimality equations (22)-(24). This can be done by either value iteration or the linear program (see [14] and the Appendix). In the following, we will restrict us to threshold policies and denote each policy by its threshold vector $L .

Assumption A7: For each day $i = 1, \ldots, 7$, the probability of no patient arrival is nonnull, i.e., $P_{i0} \geq \delta$, for some $\delta > 0 .

Remark 6: Assumption A7 is not restrictive in this paper as an equivalent formulation of the problem that fulfills Assumption A7 can be easily obtained as follows. If the number of patient arrivals of a day $i$ is at least one, i.e., $\alpha_i \geq 1$, there will be at least one CTS for day $i$ in the optimal contract $n$, i.e., $n_i \geq 1$. Furthermore, the contract $n'$ with $n'_i = n_i - 1$ and $n'_{ij} = n_{ij}$, for all $j \neq i$ is the optimal contract for patient arrivals $a'$ with $a'_i = a_i - 1$ and $a'_{ij} = a_{ij}$. The reverse is also true. As a result, the optimal contract and control policies for $a$ can be derived for the optimal solution for $a'$.

Property 6: Under Assumptions A4, A5, and A7, for any stationary control policy $L$, i.e., $I(t) = L(t + T)$, the underlying stochastic process $(\alpha(t), z(t))$ and $(\alpha(t), x(t))$ are Markov chains with a unique positive recurrent class including all states $(i, 0)$ for $i = 1, \ldots, 7 .

IV. CONTRACT OPTIMIZATION

This section proposes a two-step approach for optimization of the contract $n$ combining a Monte Carlo approximation to identify an initial contract and a local search for contract improvement.

A. Monte Carlo Approximation

The contract optimization problem (1)-(4) is difficult to solve as it involves integer variables, optimal control policies, and the random demand. Structural properties of the optimal control policies identified in Section III lead to the following equivalent reformulation of model (1)-(4):}

\[
F^* = \min_{n \in L} \{F[n, L] \} \\
= \mathbb{E} \left[ \lim_{T \to \infty} \frac{1}{T} \sum_{t=1}^{T} (TRy_t + x_t + c(n_d(t) - x_{t-1} - \alpha_t)^+) \right] \\
\text{subject to:} \\
y_t = (x_{t-1} + \alpha_t - n_d(t) - L_d(t))^+ \\
x_t = (x_{t-1} + \alpha_t - y_t - n_d(t))^+ \\
n, L \in \mathcal{N}^7 .
\]

Theorem 4: Any $n$ such that $\sum_{i=1}^{7} n_i > \Pi$ with $\Pi = (TR + c/c) \sum_{i=1}^{7} E[\alpha_i]$ is not an optimal contract.
Proof: For the contract \( n \) with \( n_t = 0 \), all patients are sent to RTS, and hence
\[
F_{0,0} = T R \sum_{i=1}^{7} E[a_i] / 7.
\]
For any other contract \( n \)
\[
F_{n,L} \geq C + \frac{\sum_{i=1}^{7} n_t - \sum_{i=1}^{7} E[a_i]}{7}.
\]
The combination of the two relations completes the proof. \( \square \)

To simplify the problem, model (25)–(28) is further approximated by a deterministic optimization problem by using a single given but long enough sample path \( \bar{a} = (a_1, a_2, \ldots, a_T) \) of patient arrivals. This together with Theorem 4 and Property 4 leads to the following Monte Carlo approximation:
\[
F_T(a) = \text{MIN}_{n,L} F_{T,n,L}(a)
\approx \left( \sum_{t=1}^{T} \left( T R y_t + x_t + c \left( n_{d(t)} - x_{t-1} - a_t \right) \right) + K(x_T; n) \right) / T
\]
subject to (26)–(28), \( n_t \leq \Pi \), \( L_t \leq T R \Pi \) where \( K(\cdot) \) is the total waiting times of patients remaining in the CTS queue at the end of the time horizon \( T \).

Theorem 5: With probability 1, \( F_T(a) \) converges to \( F^* \) when \( T \) goes to infinity.

Proof: From Property 4 and Theorem 4, conditions \( n_t \leq \Pi \) and \( L_t \leq T R \Pi \) do not exclude any optimal contract and control policy. For the contract \( n = 0, F_{T,0,0}(a) \geq F_{T,0,0}(a), F_{0,0} \geq F_{0,0} \) and
\[
\lim_{T \to \infty} F_{T,0,0}(a) = \lim_{T \to \infty} \left( \sum_{t=1}^{T} (T R a_t) \right) / T
= \sum_{i=1}^{7} T R E[a_i] / 7 = F_{0,0}.
\]

As a result, we only need to show the convergence of \( F_{T,n,L}(a) \) for each \((n,L)\) with \( n \neq 0 \). From Property 6, \((d(t),x_t)\) forms a finite state Markov chain with unique positive recurrent class including \((7,0)\) which implies that \((d(t),x_t)\) is a regenerative process with \((7,0)\) as a regenerative point. Hence
\[
\lim_{T \to \infty} \sum_{t=1}^{T} \left( T R y_t + x_t + c \left( n_{d(t)} - x_{t-1} - a_t \right) \right) / T = F_{n,L}
\]
with probability 1.

Since \( L_t \leq T R \Pi, x_T \leq T R \Pi \). Since \( n \neq 0 \)
\[
K(x_T; n) \leq \sum_{j=1}^{7} T_j = 7 T R (x_T + 1) / 2
\]
\[
\leq 7 T R (T R + 1) / 2
\]
and \( F_{T,n,L}(a) \) converges to \( F_{n,L} \) as \( T \) goes to infinity. \( \square \)

The above Monte Carlo optimization problem is still difficult to solve due to the nonlinear constraint (26) related to the control policy. We further omit this constraint and consider the following relaxed Monte Carlo optimization problem:
\[
\text{LB}(a) = \min \left( \sum_{t=1}^{T} T R y_t + \sum_{t=1}^{T+D} x_t + c \sum_{t=1}^{T} u_t \right) / T
\]
subject to
\[
u_t = \max \left( n_t - (a_t + x_{t-1}), 0 \right) \quad \forall t=1, \ldots, T \quad (31)
\]
\[
x_t = x_{t-1} + a_t - y_t - n_t + u_t \quad \forall t=1, \ldots, T \quad (32)
\]
\[
u_t = \max \left( n_t - x_{t-1}, 0 \right) \quad \forall t=1, \ldots, T+D \quad (33)
\]
\[
x_t = x_{t-1} - n_t + u_t \quad \forall t=T+1, \ldots, T+D \quad (34)
\]
\[
x_t, y_t, u_t, n_t \in \mathbb{N} \quad \forall t=1, \ldots, T+D \quad (35)
\]
where \( u_t \) denotes the number of unused CTS of day \( t \). The extra days introduced to determine the waiting times of patients remaining at the end of time horizon \( T \). As \( x_T \leq T R \Pi \) for any optimal control policy, we can set \( D = T R \Pi \).

This formulation provides a lower bound of the Monte Carlo optimization problem (29) as any feasible solution of problem (29) corresponds to a feasible solution of the above problem. Furthermore, the decision variables \( y_t \) are determined with the full knowledge of the demand, both past demand and future demand. This contradicts the requirement of the so-called nonanticipativeness of any feasible control policy. However, the contract determined by this relaxed Monte Carlo approximation is expected to be a good contract. This statement will be confirmed by the following numerical experiments. From Theorem 5, \( \text{LB}(a) \) becomes a lower bound of the optimal average cost when the sample path \( a \) is longer enough.

The two nonlinear constraints (31) and (33) can be made linear. Since reduce \( u_t \) leads to the reduction of both \( x_t \) and \( y_t \) and hence the reduction of the criterion value, constraints (31) can be equivalently replaced by \( u_t \geq n_t - (a_t + x_{t-1}) \) and \( u_t \geq 0 \) and constraints (33) by \( u_t \geq n_t - x_{t-1} \) and \( u_t \geq 0 \). As a result, model (30)–(35) can be equivalently defined as follows:
\[
\text{LB}(a) = \min \left( \sum_{t=1}^{T} T R y_t + \sum_{t=1}^{T+D} x_t + c \sum_{t=1}^{T} u_t \right) / T
\]
subject to
\[
x_{t-1} + u_t \geq n_t - a_t \quad \forall t=1, \ldots, T \quad (37)
\]
\[
x_t - x_{t-1} + y_t - u_t = a_t - n_t \quad \forall t=1, \ldots, T \quad (38)
\]
\[
x_{t-1} + u_t \geq n_t \quad \forall t=T+1, \ldots, T+D \quad (39)
\]
\[
x_t - x_{t-1} = u_t \quad \forall t=T+1, \ldots, T+D \quad (40)
\]
\[
x_t, y_t, u_t, n_t \in \mathbb{N} \quad \forall t=1, \ldots, T+D \quad (41)
\]

We further show that integrity constraints of variables \( x_t, y_t, u_t \) can be relaxed. This greatly reduces the computation
effort for solving $LB(a)$ which contains only seven integer variables for contract $n$.

Property 7 [9]: An $m \times n$ matrix $A$ is total unimodular if and only if for every $C \subseteq \{1, \ldots, n\}$ there exists a partition $(C^1, C^2)$ of $C$ such that

$$\sum_{j \in C^1} a_{ij} - \sum_{j \in C^2} a_{ij} \leq 1 \quad \forall = 1, \ldots, m.$$ 

Theorem 6: The constraint matrix of the left-hand side terms of (37)–(40) is total unimodular. Hence, the integrality constraints of variables $x_t, y_t, u_t$ of model (36)–(41) can be relaxed.

Proof: From Property 7, it is enough to prove that, for any subset $C$ of variables $x_t, y_t, u_t$, the condition of Property 7 holds. This is established by the following partition. All variables $x_t \in C$ belong to set $C^1$. For every $t$, variables $y_t, u_t \in C$ are partitioned as follows: $y_t \in C^1$, if $x_{t-1} \in C$ and $u_t \notin C$; $y_t, u_t \in C^2$, if $x_{t-1} \in C$ and $u_t \in C$; $y_t \in C^2$, if $x_{t-1} \notin C$ and $u_t \notin C$; $y_t, u_t \in C^3$, if $x_{t-1} \notin C$ and $u_t \in C$. 

B. Improving the Contract by Local Search

Starting from the contract of the Monte Carlo approximation, this subsection presents a local search to further improve the contract.

The following notation is needed:

$F_n$ optimal average cost under contract $n$, i.e., $F_n = \min L F_n L$. $F_n$ and the related optimal control policy $L(n)$ can be determined by using standard linear programming approach to solve the optimality equations (22)–(24) (see Appendix);

$F_{nL}(a)$ average cost of policy $L$ under contract $n$ and sample path $a$ estimated by relation (29). Note that compared with relation (29), index $T$ is omitted for simplicity. By definition, $LB(a) \leq F_{nL}(a)$;

$e_i$ a seven-dimension vector with $i$th entry equal to 1 and all other entries equal to 0.

The local search starts from the contract $n$ determined by the Monte Carlo approximation. At each iteration of the local search, it determines the best neighbor solution among the set of contracts: $n + e_i$ (add one time slot in day $i$), $n - e_i$ (remove one time slot in day $i$), $n + e_i - e_j$ (move one time slot from day $i$ to day $j$). This process repeats until no improvement can be found.

The overall algorithm for the contract optimization is summarized as follows.

Algorithm 1: (Contract optimization)

Step 1. Generate a long enough sample path $a$ of patient arrivals;

Step 2. Solve the Monte Carlo approximation problem to determine $LB(a)$ and an initial contract $n_0$;

Step 3. Determine the optimal control policy $L(n_0)$ and the optimal average cost $F_{n_0}$ under contract $n_0$

by linear programming for optimality equations (22)–(24);

Step 4. Let $n^* = n_0$: $F_{n^*} = F_{n_0}$;

Step 5. Determine the neighbor solution $n'$ with the smallest average cost as follows:

$$n' = \arg \min_{n \in \{n_0, n_0 + e_i, n_0 - e_i, n_0 + e_i - e_j\} \cap \mathcal{N}} F_n.$$ 

Step 6. Determine the optimal control policy $L(n')$ and the optimal average cost $F_{n'}$ as in Step 3 if necessary;

Step 7. If $F_{n'} < F_{n^*}$, set $n^* = n'$ and go to step 5;

Step 8. The final contract is $n^*$ and the final control policy is $L(n^*)$.

For a high-demand case with high patient arrival rate, the state space is large, and it is time-consuming for solving the optimality equations (22)–(24) for determining $F_n$. In order to reduce the computational burden, $F_{nL}(a)$, where $a$ is the sample path of Step 1, can be used to replace $F_n$ in step 5. This leads to Algorithm 2. Note that the contract $n'$ selected in each iteration is still exactly evaluated in Algorithm 2.

To evaluate the performance of this solution strategy, the global optimal contract and control policies can also be determined by exhaustive search by comparing $F_n$ for all $n$ in order to determine the search is possible when the demand is low and becomes too time consuming when the demand is high and the state space becomes large.

Remark 7: Although the Monte Carlo approximation and relaxations are used for determining the contract, all our solution algorithms use MDP to find the exact criterion value of the resulting contract. Furthermore, both Algorithms 1 and 2 start with the same contract provided by the single sample path Monte Carlo approximation.

V. COMPUTATIONAL RESULTS

This section presents numerical experiments to evaluate the performance of the solution strategies. The solution strategies are compared to the exact optimum $F^*$ obtained by exhaustive search for small-size problems, and for large-size problems, with the best solution $F_{\text{best}}$ of all independent runs of Algorithms 1 and 2 for the same problem instance. Sensitivity analysis is also performed to show how the optimal contract depends on different factors such as $T^R$ and $c$, patient arrival patterns and patient arrival rates. All numerical results are performed on a PC with Core 2 Quad Q9500 CPU running at 2.83 GHz with 4.0 GB RAM. LP models are solved by the CPLEX 12 solver.

A. Numerical Experiments

We first describe the base case corresponding to our real case study. The average numbers of patient arrivals during the week are as follows: {1, 0.89, 0.95, 1.16, 1.53, 0.16, 0.05}. The number of patients arrived each day is assumed to follow a Poisson distribution. The average waiting time for RTS is in the range of 30–40 days with an average of $T^R = 35$ days. The weight, $c$, is set to 15.
The base case is then modified to investigate the impact of parameters $T^R$ and $c$, the patient arrival patterns, and patient arrival rates. More precisely, the following numerical experiments are considered:

Case 1) base case, but with weighing factor $c \in \{1, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 60, 70, 80, 90, 100\}$.

Case 2) base case, but with delay for RTS $T^R \in \{2, 4, 6, 8, 10, 15, 20, 25, 30, 35, 40, 45, 50, 60, 70, 80, 90, 100\}$.

Case 3) base case, but with different patient arrival pattern. The peak demand on Friday of the base case is interchanged with the demand of another weekday. We also consider the case of stationary demand for all weekdays.

Case 4) base case, but with different patient arrival rate. Two other cases are derived from the base case by multiplying the patient arrival rate by 5 and 10.

The sample path $\alpha$ needed for the Monte Carlo optimization is generated on a time horizon of 1000 weeks, i.e., $T = 7000$ days. The number of patients arrival $a_t$ for $t = 1, \ldots, 7000$ is generated according to a Poisson distribution truncated at 20, 40, and 60 for respectively low-, medium-, and high-demand instances. $D = 100$ extra days are used in the Monte Carlo approximation to determine the waiting times of patients in the CTS queue at the end of the planning horizon. Optimality equations (22)–(24) are solved by restricting the queue length $x_t$ below a given parameter $LL$ set by trial-and-error to $LL = 30, 40,$ and 40 for low-, medium-, and high-demand instances. We also use very long simulation for evaluation of other performance measures such as average delays.

For each problem instance, 10 sample paths $\alpha$ of patient arrivals are generated, Algorithms 1 and 2 are run using each sample path to investigate their performances.

### B. Performances of Algorithms and Cost of Unused CTS

This subsection considers Case 1 to investigate the performances of Algorithms 1 and 2 and to show the impact of the weighting factor $c$. As patient arrival rate is rather low and the state space is small enough, exhaustive search of optimal contract can be done in reasonable time.

The exhaustive search is performed for all contract $n$ with $n_t \in \{0, 1, 2, 3\}$. The range was selected based on the observation of different contracts provided by Algorithms 1 and 2. The optimal control policy and the average cost $F_n$ of each contract are determined by solving optimality equations (22)–(24) by linear programming. The cost of the resulting optimal contract is denoted as $F^\ast$.

Table I summarizes optimal solutions for different weighting factors $c$ of unused CTS. The exhaustive search takes more than 1 h for each instance with CPU time varying from 4699 to 4824 s. The total number of CTS decreases and the criterion value increases when $c$ increases. The contract and control policy are sensitive to the change of $c$. With the same contract, for example for $c = 25$ to $c = 80$, the threshold for assigning patients to RTS increases with the increase of $c$.

Table II contains simulation results of optimal contract and optimal control policy to show the impact of $c$ on average delays (Delay), ratio of unused CTS (Unused CTS) and percentages of patients directed to RTS (RTS). When $c$ increases, the corresponding waiting time increases from less than one day to about 10 days, while the unused CTS ratio decreases from 42.6% to 2.47%. Therefore, by varying $c$, the decision makers are able to obtain different contracts with different waiting times.
and different ratios of unused CTS. This allows them to set the right value of the weighting factor $c$ and to select the contract reaching the best compromise between the waiting times and unused CTS. This is reasonable in practice as the contract is not subject to frequent changes.

The average delay of the contract approach is very small compared to the delay of 35 days for regular reservation. This is basically due to the rather stable demand of the neurovascular department and is the key advantage of addressing the MRI examination reservation problem from the neurovascular department point of view.

Given the small average waiting time of Table II, it raises a question on the appropriateness of directing patients to RTS that requires $T_R$ days waiting time. The explanation of the threshold policy proved earlier is as follows. Due to random arrival process, the CTS queue can become large. When this happens, it is better to direct some incoming patients to RTS. Otherwise, the waiting times of these patients will be long and, furthermore, their insertion in the CTS queue will lead to longer waiting times of subsequent patients.

Table III summarizes results of Algorithms 1 and 2 for different weighting factors. For each problem instance, each algorithm is applied 10 times with 10 different sample paths. Let $n_1$ and $n_2$ be the contract provided respectively by Algorithms 1 and 2. Let $F_{n_k}$ be the exact criterion value of contract $n_k$ of Algorithm $k$, where $k = 1, 2$. Columns “Gap$^1_k$” show the minimum, average, and maximum of the average deviations of $F_{n_k}$ from $F^*$, i.e., $(F_{n_k} - F^*)/F_{n_k}$. Column “Move$^k$” gives the minimum, average, and maximum of local search moves in Algorithms $k$. $RT^1$ and $RT^2$ are the average CPU time. The optimal contract is always found by Algorithm 1 except for the instance $c = 1$ for which the 10 criterion values of Algorithm 1 are all within 1% of the optimum. Results in column “Move$^1$” show that the contract of the Monte Carlo approximation is close to the optimum as the optimal contract can be obtained with less than two local moves. Recall that, from Remark 7, Algorithms 1 and 2 actually start from the same Monte Carlo solution for each of the 10 sample paths. Results of Algorithm 2 are good with average deviation from the optimum of less than 2.2% and with a deviation of about 13.60% for one run of $c = 50$. However, the CPU time of Algorithm 2 is much smaller than that of Algorithm 1.

We now analyze the quality of the sample path lower bound $LB(a)$. Fig. 1 shows the deviation gap between $LB(a)$ and $F_{n^*_L}^{-L}(n^*)$ where $n^*$ is the optimal contract obtained by exhaustive search, i.e., $(F_{n^*_L}^{-L}(n^*) - LB(a))/F_{n^*_L}^{-L}(n^*)$. As it can be seen, the quality of the lower bound becomes worse when the weighting factor $c$ increases and the lower bound is in general very loose.

Remark 8: Even though the sample path lower bound $LB(a)$ is loose, the contract of the Monte Carlo approximation is actually close to the optimal contract and is at most two local moves away from the optimal contract in all our numerical experiments.

**C. Performances of Algorithms and Delay for RTS**

This subsection considers Case 2 to investigate the performances of Algorithms 1 and 2 and to show the impact of regular reservation delay $T_R$.

Table IV summarizes optimal solutions given by exhaustive search with similar CPU times as in Case 1. The total number of CTS is nondecreasing and the criterion value increases when delay $T_R$ for regular reservation increases. The contract of seven CTS remains unchanged in the range $T_R \in [25, 100]$ covering the actual delay of 35 days. A contract of six CTS is used for $T_R \in [10, 20]$. For $T_R < 10$, the contract uses at most five CTS per week, which does not cover the weekly demand of 5.695. The control policy is sensitive to the change of $T_R$. With the increase of $T_R$, the control threshold $L$ increases.

Table V shows the impact of $T_R$ on delays, unused CTS and ratio of patients directed to RTS. The average delay varies from 1.79 to 3.64 and is not sensitive to the change of $T_R$. The ratio of patients assigned to RTS decreases from 83.77% to 0.00% when $T_R$ increases. The unused CTS ratio increases when $T_R$ increases from 2 to 25, and then decreases slightly. This is reasonable as the same contract is used when $T_R$ increases from 25 to 100.

Table VI summarizes results of Algorithms 1 and 2 for Case 2. The optimal contract is always found by Algorithm 1. Results of Algorithm 2 deviate from the optimum by less than 12.84%
with the worst performance reached at $T^R = 2$. The CPU time of Algorithm 2 is much smaller than that of Algorithm 1.

Fig. 2 shows the deviation gap between $L(x)$ for different $T^R$. As it can be seen, the lower bound is in general poor, but becomes tighter when $T^R$ increases.

**Remark 9:** In order to check whether Assumption A4 requiring stable weekly contract is strong, we apply Algorithm 1 to optimize biweek contract and the related optimal control policy, i.e., for contract $n$ and control $L$ defined over 14 days. Numerical experiments are performed for Cases 1 and 2. The resulting biweek contracts and the optimal criterion values remain the same as weekly contracts of Table I. This implies that Assumption A4 is not really strong in these cases.

### D. Impact of Patient Arrival Pattern

This subsection considers Case 3 to show the impact of patient arrival pattern. Table VII summarizes the optimal solutions obtained by exhaustive search for different patient arrival patterns. The row “Mon.” corresponds to patient arrival pattern derived from the base case by interchanging the patient arrival rate of Monday with that of Friday (actual peak arrival). The next four rows are defined similarly. The row “Ave.” corresponds to the case of equal arrival rate for all workday with the same weekly patient arrival rate as the base case. From this table, the total number of CTS of the optimal contract does not change with respect to the patient arrival pattern. However, one CTS moves from Friday to the day of peak arrival, and there are still two CTS for Friday in order to serve patient arrival during the week. In the case of stationary weekday arrival, one CTS moves from Friday to Wednesday. The control policy seems to be insensitive to the patient arrival patterns.

Table VIII summarizes average delays, unused CTS ratios, and RTS percentages for different arrival patterns. These performance measures seem to be insensitive to the change of patient arrival pattern. Table IX presents results of Algorithms 1 and 2 for the same case. The same conclusions can be made as in Section V-A, and optimal contract is always reached by Algorithm 1.

### E. Impact of Patient Arrival Rate

As the patient arrival rate increases, the solution space becomes larger and the CPU time for exhaustive search is beyond acceptable limit, and we limit ourselves to Algorithms 1 and 2 in this experiment.

Tables X and XI summarize best contracts and corresponding control policies and the performances of the two algorithms for different patient arrival rates where three scenarios are considered: “Low” (base case), “Medium” (patient arrival rates five times larger), and “High” (patient arrival rates 10 times larger). “Gap1” and “Gap2” in this case are the deviation gap of $F_{nk}$ from $F_{best}$, i.e., $Gap_k = (F_{nk} - F_{best})/F_{nk}$, where $F_{best}$ is the best solution of 20 solutions of the 10 runs of Algorithms 1 and 2.

From these tables, Algorithm 1 is always able to find the best solutions given in Table X whatever the sample path used, while...
TABLE VI
PERFORMANCES OF ALGORITHMS 1 AND 2 FOR CASE 2

<table>
<thead>
<tr>
<th>T&lt;sup&gt;R&lt;/sup&gt;</th>
<th>Gap2 (%)</th>
<th>Move1</th>
<th>Move2</th>
<th>RT1 (s)</th>
<th>RT2 (s)</th>
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</tr>
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<td>10</td>
<td>[0.1,9,4,0]</td>
<td>[1.1,1]</td>
<td>[0.0,9,2]</td>
<td>99</td>
<td>6</td>
</tr>
<tr>
<td>15</td>
<td>[0.0,0]</td>
<td>[0.0,0]</td>
<td>[0.0,0]</td>
<td>58</td>
<td>5</td>
</tr>
<tr>
<td>20</td>
<td>[0.0,0]</td>
<td>[0.0,0]</td>
<td>[0.0,3,1]</td>
<td>56</td>
<td>5</td>
</tr>
<tr>
<td>25</td>
<td>[0.1,5,3,0]</td>
<td>[1.1,1]</td>
<td>[0.1,2]</td>
<td>110</td>
<td>6</td>
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<tr>
<td>30</td>
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<td>[0.1,5,3]</td>
<td>113</td>
<td>11</td>
</tr>
<tr>
<td>35</td>
<td>[0.1,1,1,11,0]</td>
<td>[0.0,3,1]</td>
<td>[0.0,1,1]</td>
<td>102</td>
<td>15</td>
</tr>
<tr>
<td>40</td>
<td>[0.0,0,2,0,1]</td>
<td>[0.0,6,1]</td>
<td>[0.1,1,3]</td>
<td>97</td>
<td>14</td>
</tr>
<tr>
<td>45</td>
<td>[0.0,0,4,0,1]</td>
<td>[0.0,5,1]</td>
<td>[0.0,6,2]</td>
<td>93</td>
<td>15</td>
</tr>
<tr>
<td>50</td>
<td>[0.0,0,4,0,1]</td>
<td>[0.0,5,1]</td>
<td>[0.0,6,2]</td>
<td>95</td>
<td>17</td>
</tr>
<tr>
<td>60</td>
<td>[0.0,0,6,0,34]</td>
<td>[0.0,4,1]</td>
<td>[0.0,2,2]</td>
<td>96</td>
<td>20</td>
</tr>
<tr>
<td>70</td>
<td>[0.0,0,6,0,4]</td>
<td>[0.0,3,1]</td>
<td>[0.0,1,1]</td>
<td>85</td>
<td>14</td>
</tr>
<tr>
<td>80</td>
<td>[0.0,0,6,0,4]</td>
<td>[0.0,3,1]</td>
<td>[0.0,0]</td>
<td>82</td>
<td>12</td>
</tr>
<tr>
<td>90</td>
<td>[0.0,1,0,4]</td>
<td>[0.0,4,1]</td>
<td>[0.0,0]</td>
<td>86</td>
<td>10</td>
</tr>
<tr>
<td>100</td>
<td>[0.0,1,0,4]</td>
<td>[0.0,4,1]</td>
<td>[0.0,0]</td>
<td>84</td>
<td>11</td>
</tr>
</tbody>
</table>

TABLE VII
OPTIMAL SOLUTIONS OF DIFFERENT PATIENT ARRIVAL PATTERNS BY EXHAUSTIVE SEARCH

<table>
<thead>
<tr>
<th>Peak arrival</th>
<th>f&lt;sup&gt;n&lt;/sup&gt;</th>
<th>CPU Time (s)</th>
<th>n</th>
<th>L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mon.</td>
<td>4.506</td>
<td>5033</td>
<td>{1,1,1,1,2,0,0}</td>
<td>{1,10,1,1,1,1,10,10,10,10,11}</td>
</tr>
<tr>
<td>Tues.</td>
<td>4.496</td>
<td>4834</td>
<td>{1,2,1,1,2,0,0}</td>
<td>{1,10,1,1,1,10,10,10,10,10,11}</td>
</tr>
<tr>
<td>Wed.</td>
<td>4.487</td>
<td>4866</td>
<td>{1,1,1,2,1,2,0,0}</td>
<td>{1,11,1,1,10,11,10,10,11}</td>
</tr>
<tr>
<td>Thurs.</td>
<td>4.476</td>
<td>4838</td>
<td>{1,1,1,2,2,0,0}</td>
<td>{1,11,1,1,10,9,10,10}</td>
</tr>
<tr>
<td>Fri.</td>
<td>4.501</td>
<td>4808</td>
<td>{1,1,1,1,1,3,0,0}</td>
<td>{1,11,1,1,11,11,9,10,10}</td>
</tr>
<tr>
<td>Ave.</td>
<td>4.517</td>
<td>4834</td>
<td>{1,1,2,1,2,0,0}</td>
<td>{1,11,1,1,10,10,10,10,11}</td>
</tr>
</tbody>
</table>

The quality of the contract obtained by Algorithm 2 is more sensitive to the sample path used. Move1 shows that the contract obtained by Monte Carlo approximation is very close to the best contract and is at most two local moves from the best contract. This highlights the quality of contract given by the Monte Carlo Approximation as 61 CTS (resp. 31 and 7 CTS) are needed for high- (resp. medium- and low-) demand instance. Finally, Algorithm 1 becomes too slow for high-demand instance, while Algorithm 2 is much faster and is able to provide a good solution with a reasonable CPU time.
TABLE XII

<table>
<thead>
<tr>
<th>Arrival rate</th>
<th>Algorithm 1</th>
<th>Algorithm 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Delay (days)</td>
<td>Unused RTS (%)</td>
</tr>
<tr>
<td>Low</td>
<td>2.13</td>
<td>18.01</td>
</tr>
<tr>
<td>Med</td>
<td>1.15</td>
<td>7.65</td>
</tr>
<tr>
<td>High</td>
<td>0.75</td>
<td>6.00</td>
</tr>
</tbody>
</table>

Table XII summarizes average delay, unused CTS ratios, and RTS percentages for different patient arrival rates. All the performance measures decrease when the arrival rate increases.

VI. CONCLUSION AND PERSPECTIVES

This paper proposed a new reservation process of MRI time slots for stroke patients in order to reduce the waiting times of patients without degrading the utilization ratio of MRI facilities. The new method requires the determination of the number of contracted time slots and a control policy for the assignment of patients to either CTS or RTS. This results a stochastic combinatorial problem that combines combinatorial planning decision variables and dynamic control policies. This paper first explored the structural properties of the optimal control policies under a given contract and then proposed a two-step approach to obtain the efficient contract. A single sample-path Monte Carlo approximation is used to determine an initial contract, which is further improved by local search. Numerical results show that the deviation gap from the optimal solution is rather small, which means that the contract and the corresponding control policy are very close to the optimal ones.

Future research can be pursued in several directions. First, the form of the optimal contract is still an open issue even for purely stationary demand. It is unclear how to determine the optimal contract if Assumption A4 is relaxed. Another immediate extension is the development of real-time control strategies for advance cancellation of CTS in case of short CTS queue. Management of multiple classes of patients and multiple imaging examinations is a natural but challenging research direction. Extension to nonstationary patient arrival case is another interesting research avenue.

The relation between different medical departments is not addressed in this paper, but is crucial for implementing the contract-based approach in a hospital. The joint design of contract-based solutions of several departments raises some fundamental questions such as: 1) how many time slots of a diagnostic facility to contract; and 2) how to share these time slots among different departments. Results about the optimal control policies of this paper can be extended to evaluate a contract solution. However, new approaches are needed to coordinate the contracts for different departments.

From the service provider perspective, how to optimize the operational efficiency of the imaging department by taking into account different quality requirements of medical units is a rich research area.

APPENDIX

Optimal average cost and control policy for a given contract \( n \)

\[
F_n = \text{Maximize } \lambda
\]

subject to

\[
\lambda + h(d(t), z) \leq c(n_d(t) + z) + TR\left(z - n(t) - x\right) + x + \sum_a P_{a}h\left(d(t+1), x_a + a\right)
\]

\[
\forall z, \forall x \leq (z - n(t))^{+}, \forall t = 1, \ldots, 7.
\]

Proof of Property 1.: Denote \( x_1^t \) and \( x_2^t \) as the optimal \( x_t \) for \( V^t(z_t + 1) \) and \( V^t(z_t) \). From (13)

\[
0 \leq x_1^t \leq (z_t + 1 - n_t)^+ \leq (z_t - n_t)^+ + 1
\]

\[
0 \leq x_2^t \leq (z_t - n_t)^+.
\]

Therefore, \( x_1^t \) is equal to either \( x_2^t \) or \( (z_t - n_t)^+ + 1 \), and hence \( x_1^t \geq x_2^t \). This completes the proof.

Proof of Property 2.: The proof is made by induction. First, the property is clearly true for \( t = 0 \). Assume that the property holds for \( t + 1 \), and we prove that it also holds for \( t \). Let \( x_1^t \) and \( x_2^t \) as the optimal \( x_t \) for \( V^t(z_t + 1) \) and \( V^t(z_t) \). From Property 1, \( x_1^t \geq x_2^t \). As a result, by (13) for \( z_t + 1 \) with \( x_1^t = x_2^t \)

\[
V^t(z_t + 1) \leq c(n_t - z_t - 1)^+ + x_2^t + TR\left((z_t + 1 - n_t)^+ - x_2^t\right) + U^{t+1}(x_2^t).
\]

By definition

\[
V^t(z_t) = c(n_t - z_t)^+ + x_2^t + TR\left((z_t - n_t)^+ - x_2^t\right) + U^{t+1}(x_2^t).
\]

Combining the two relations

\[
V^t(z_t + 1) - V^t(z_t) \leq c(n_t - z_t - 1)^+ + c(n_t - z_t)^+ + TR\left((z_t + 1 - n_t)^+ - TR(z_t - n_t)^+
\]

\[
\leq TR.
\]

Three cases are considered for the proof of the first inequality of Property 2. Case: \( z_t + 1 - n_t \leq 0 \). First, \( x_1^t = x_2^t = 0 \)

\[
V^t(z_t + 1) = c(n_t - z_t - 1) + U^{t+1}(0)
\]

\[
V^t(z_t) = c(n_t - z_t) + U^{t+1}(0).
\]

\[
V^t(z_t + 1) - V^t(z_t) = - c.
\]

Case: \( z_t + 1 - n_t > 0 \) and \( 0 < x_1^t < z_t + 1 - n_t \). From (13), there is no unused CTS for both \( V^t(z_t + 1) \) and \( V^t(z_t) \) and \( x_2^t = x_1^t \). Hence

\[
V^t(z_t + 1) = x_1^t + TR\left((z_t + 1 - n_t) - x_1^t\right) + U^{t+1}(x_1^t)
\]

\[
V^t(z_t) = x_1^t + TR\left((z_t - n_t) - x_1^t\right) + U^{t+1}(x_1^t)
\]

\[
V^t(z_t + 1) - V^t(z_t) = TR \geq -c.
\]

Case: \( z_t + 1 - n_t > 0 \) and \( x_1^t = z_t + 1 - n_t \). From (13), there is no unused CTS for both \( V^t(z_t + 1) \) and \( V^t(z_t) \). Hence

\[
V^t(z_t + 1) = x_1^t + U^{t+1}(x_1^t).
\]

By (13) for \( z_t \) with \( x_1^t = x_1^t - 1 \)

\[
V^t(z_t) \leq x_1^t - 1 + U^{t+1}(x_1^t - 1).
\]
Combining the two relations with (14)

\[ V^t(z_t + 1) - V^t(z_t) \geq 1 + \alpha \sum_a P_{t+1,a} 
\times (V^{t+1}(x_{t+1}^a) - V^{t+1}(x_{t}^a + a - 1)) \, . \]

By induction assumption

\[ V^{t+1}(x_{t}^1 + a) - V^{t+1}(x_{t}^1 + a - 1) \geq -c \]

and hence

\[ V^t(z_t + 1) - V^t(z_t) \geq 1 - \alpha c \geq -c. \]

**Proof of Property 3.** First, the property holds for \( t = 0 \) as \( V^t(z_t) = 0 \). Assume that \( V^{t+1}(z_{t+1}) \) is convex, and we prove the property for \( t \). From (14), the convexity of \( V^{t+1}(z_{t+1}) \) implies the convexity of \( U^{t+1}(x_t) \). Hence, \( U^{t+1}(x_t) - (T^R - 1)x_t \) is also convex. Let

\[ L_t = \arg \min_{x_t} (U^{t+1}(x_t) - (T^R - 1)x_t) \, . \]

Equation (13) can be written as

\[ V^t(z_t) = \begin{cases} 
0 & \text{if } z_t = n_t \\
 z_t - n_t + U^{t+1}(z_t - n_t) & \text{if } n_t \leq z_t \leq L_t + n_t \\
 L_t + T^R(z_t - n_t - L_t) & \text{if } z_t \geq L_t + n_t. 
\end{cases} \]

By convexity of \( U^{t+1}(x_t) \), \( V^t(z_t) \) is convex in \( z_t \) in the following internal \( [0, n_t] \), \( (n_t, L_t + n_t) \), and \( (L_t + n_t, \infty) \). We still need to prove the convexity of \( V^t(z_t) \) for \( z_t = n_t \) and \( z_t = L_t + n_t \).

The convexity of \( V^t(z_t) \) at \( z_t = n_t \) holds as, by Property 2

\[ V^t(z_t + 1) - V^t(z_t) \geq -c = V^t(z_t) - V^t(z_t - 1). \]

The convexity of \( V^t(z_t) \) at \( z_t = L_t + n_t \) holds as, by Property 2

\[ V^t(z_t) - V^t(z_t - 1) \leq T^R = V^t(z_t + 1) - V^t(z_t). \]

By induction, this completes the proof.

**Proof of Property 4.** For any day \( t \) ending with a queue length \( x_t \), the waiting time of the last patient in this queue is at least \( x_t/n^* \) days. If \( 3/2/n^* > T^R \), then direct this patient to RTS would result in a total cost reduction. This contradicts the optimality of the control policy and concludes the proof.

**Proof of Property 5.** From Property 2

\[ -c \leq V^t_t(z + 1) - V^t_0(z) \leq T^R \]

which, together with \( 0 \leq z \leq z_{\max} \) implies, for all \( z \) and \( z' \) such that \( z \leq z' \)

\[ -cz_{\max} \leq V^t_0(z) - V^t_0(z') \leq T^R z_{\max}. \]

Combining with (12)

\[ -cz_{\max} \leq V^t_0(i, z) - V^t_0(i, z') \leq T^R z_{\max}. \] (42)

This establishes the property for \( i = 7 \). Consider now the case \( i = 1, \ldots, 6 \). From the optimality (8) and (9) and letting \( \pi \) be the optimal control policy

\[ V^0_0(i, z) = g_i(i, z, \pi(z)) + \alpha \sum_a P_{t+1,a} V^0_0(i + 1, \pi(z) + a) \]

\[ V^0_0(7, z) = g_7(i, z, \pi(z)) + \alpha \sum_a P_{t+1,a} V^0_0(1, \pi(z) + a) \]

where \( g_i(i, z, x) = (c(n_i - z)^c + x + T^R(z - n_i - x)^c \). From Assumption A6, \( g_i(i, z, x) \leq B \) with \( B = \alpha e^* + T^R z_{\max} \). As a result

\[ V^0_0(i, z) \leq B + \sum_a P_{t+1,a} V^0_0(i + 1, \pi(z) + a) \]

\[ V^0_0(7, z) \leq B + \sum_a P_{t+1,a} V^0_0(1, \pi(z) + a) \]

for \( i = 1, \ldots, 6 \). Repeat the above relations for \( t \) subsequent days leads to

\[ V^0_0(i, z) \leq tB + \sum_{i'} Q_{i+1}^T(i, t+i', z, z') V^0_0(t+i', z') \] (43)

where \( Q_{i+1}^T(i, t+i', z, z') \) is the probability of reaching state \( z' \) at the beginning of day \( t+i \) by starting from state \( z \) at day \( i \) under policy \( \pi \). Combining (42) and (43) with \( t+i = 7 \)

\[ V^0_0(i, z) \leq 6B + \sum_{i'} Q_{7}^T(i, t+i', z', z') V^0_0(t+i', z') \]

\[ \leq 6B + V^0_0(7, 0) + T^R z_{\max}. \]

Similarly

\[ V^0_0(7, 0) \leq 6B + \sum_{z'} Q_{7}^T(7, t+i', z'), z') V^0_0(i, z') \]

\[ \leq 6B + V^0_0(7, z) + \max(c, T^R) z_{\max}. \]

The above two properties concludes the proof.

**Proof of Property 6.** Under the assumptions of the property, it is clear that \( (d(t), x_t) \) is a finite-state Markov chain and \( (d(t), z_t) \) is a Markov chain. This property obviously holds for the case \( n = 0 \), and we assume \( n \neq 0 \), i.e., \( n^* > 0 \) in the following. Starting from any initial state \( (i, z) \), for any control policy \( L \), any state \( (d(t), x_t) = (i, 0) \) can be reached in at most \( T^R + 6 \) days with probability \( \delta^T \leq T^R n^* \). As a result, \( (d(t), x_t) \) is a finite state Markov chain with a unique positive recurrent class including all states \( (i, 0) \). Since \( z_t = x_t + a_t \), Assumption A7 and the property of \( a_t \) imply that \( (d(t), z_t) \) is a Markov chain with a unique positive recurrent class including all states \( (i, 0) \).

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REFERENCES


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