Collision-Free Formations with Reactively-Controlled Virtual Head Robot Tracking

A.D. Nguyen, N.M. Kwok, V.T. Ngo and Q.P. Ha
ARC Centre of Excellence for Autonomous Systems
University of Technology, Sydney
PO Box 123 Broadway NSW 2007 AUSTRALIA

Abstract – This paper presents an effective methodology for the control of a group of mobile robots moving in desired formations.

Virtual head robot tracking and three-point \( l-l \) controllers incorporated with reactive control schemes are proposed here to overcome the cases when the control law is undefined and subject to potential inter-robot collision. In order for the whole group to enter a desired formation, a general procedure is suggested in the initialisation stage. Advantages of our approach include the establishment and maintenance of any formation from arbitrary initial conditions with collision avoidance for a large group of mobile robots. The approach is validated through simulation cases to illustrate the capability of handling singularities and avoiding collision in a group of \( N \) robots.

Index Terms – formation, collision avoidance, virtual head robot, reactive control.

I. INTRODUCTION

Formations of multiple robots, emerging as a key technology in mobile robotics, have attracted intense research effort over recent years. The control of a group of robots to operate in a formation has several advantages over the control of single robots, including overall system enhanced performance (increased instrument resolution, reduced cost), and the capability of executing tasks that a single agent cannot accomplish. Potential applications can range from industrial coordination in agriculture, construction, mining and to diverse missions such as surveillance, wide-area search and rescue, environmental mapping, defense, and health care. Examples that have been described in the literature include box pushing [1], load transportation [2] and capturing/enclosing an invader [3].

The pattern formation problem in multi-robot systems is defined as the coordination of a group of robots to get into and maintain a formation with a desired shape such as a column, a line, a wedge, a ring or a chain [4]. Solutions for this problem are currently applied in search and rescue operations, landmine removal, remote terrain and space exploration, and also the control of satellites and unmanned aerial vehicles. The issue of control and coordination for multiple mobile robots has revolved around two major tasks. Firstly, the robot platoon must maintain desired shapes. Secondly, the robots have to simultaneously avoid collisions between themselves and with obstacles in the environment.

While there are different approaches to the robotic formation control problem, the common theme remains the global coordination of multi agents to accomplish intelligently and/or autonomously some task objectives. In the control context, approaches to robotic formation can be classified into three broad groups: behaviour-based, leader-following, and virtual-structure [5]. In behavior-based approaches [6-8], after the assignment of some simple and intuitive behaviors or motion primitives for each individual agent, more complex motion patterns are generated through the interaction of several agents by using a weighted sum of these simple primitives. In general, it would be difficult to analyze the characteristics of this approach in a rigorous and formal way. However, some proofs of the stability and convergence do exist in simple schemes [9]. In leader-following approaches [10, 11], one or more agents are designated as leaders and responsible for guiding the formation. The other robots are required to follow the leader with predefined clearances. In virtual structure approaches [1, 5], the entire formation is treated globally as a single structure or so-called virtual structure. If the desired dynamics of the virtual structure can be translated into the desired motion of each agent then one can design local controllers to achieve global performance.

For leader-following in a group of multiple robots formation stability has been analyzed in the framework of interconnected systems [12]. Practically, two main problems are concerned in a leader-follower approach: leader tracking and collision avoidance. For tracking, Desai et al. [13] proposed two types of feedback controllers for maintaining formations of multiple mobile robots: \( l-l \) controller and \( l-l \) controller. Alternatively, three types of controllers (Basic Leader-Following, Leader-Obstacle, and Three-Robot Shape) are used to maintain a formation under appropriate assumptions on the motion of the lead robot [14]. For a single mobile robot tracking a moving object, “Vertical” tracking (path following) and “Horizontal” tracking are proposed in [15]. In these approaches the problem of initialization of the formation from arbitrary conditions has not been explicitly addressed. Virtual robot tracking control, proposed in [16], could be used to initialize a formation of three mobile robots but remains invalid to form a group of mobile robots into a “line” and still subject to a singularity case when three robots are aligned.

In this paper, we propose a new framework that features virtual-head robot tracking and three-point \( l-l \) control along with reactive control schemes to initialize and maintain a desired formation for a large group of mobile robots with the capability of inter-robot collision avoidance and singularity.
alleviation. The paper is organized as follows. After the introduction, section II presents the model and problem formulation. The control design is detailed in section III. The formation initialisation procedure is given in Section IV. Section V presents the simulation results and a conclusion is drawn in section VI.

II. MODEL AND PROBLEM FORMULATION

A mobile robot can be described by a common kinematic model as:

$$\dot{x} = v \cos \theta, \quad \dot{y} = v \sin \theta, \quad \dot{\theta} = \omega,$$

where \((x, y)\) is the centre point on the wheel axis, \(\theta \in \mathbb{R}\) is the orientation, inputs \(v\) and \(\omega\) are the translational and angular velocities respectively. From the non-holonomic velocity constraints, it is required the robots satisfy strictly pure rolling and non-slipping conditions, ie. \(\dot{x} \cos \theta + \dot{y} \sin \theta = v\), and \(\dot{x} \sin \theta - \dot{y} \cos \theta = 0\). The following assumptions are made:

**Assumption 1:** Each robot is indexed by a unique number indicating the priority level according to its role in the group: the lower the index, the higher the priority.

**Assumption 2:** Each robot can get any necessary information about its position and orientation and information of its leader in a global coordinates from its communication channel.

**Assumption 3:** The leader follows a smooth trajectory and the workspace is flat and obstacle-free.

Possible collision between any two robots may be detected by associating each robot with a circle having its center at the control point on the robot’s wheel axis, and radius \(r\) determined by the robot’s dimensions with an additional distance as a safety margin. Denoting \(\rho\) the distance between control points of two robots \(i\) and \(j\), a measure describing the possibility of collision between robots can be described as

$$f_y = \rho - 2r,$$

where \(f_y > 0 \rightarrow \text{safe}, \quad f_y \leq 0 \rightarrow \text{unsafe}.

Our objective here is to design controllers for a platoon of robots to achieve (i) any desired formation, (ii) no inter-robot collision, and (iii) the desired group motion that satisfies the limitation of the communication range.

III. CONTROL DEVELOPMENT

Our control framework is based on the so-called virtual-head robot tracking (VHRT) and three-point 1-1 control (3PLL) coupled with reactive schemes to establish any desired formation while ensuring collision-free motion of the robots.

A. Virtual-Head Robot Tracking and Three-Point 1-1 Control

Virtual robot (VR) is a hypothetical robot whose orientation is identical to that of its host robot, but its position is displaced apart from it with the predefined \(R\) and \(L\) clearances. The symbols \(L\) and \(R\) denote respectively the longitudinal clearance and clearance along rear wheel axis.

VR tracking control, proposed in [16], has proved to be effective in formation control for a pair of leader-follower robots by ensuring the convergence of the VR position to the reference position. As the VR tracking control law requires the longitudinal clearance to be non-zero, some desired shape (e.g. line formation) may however, not be formed directly. In addition, despite stability of the zero dynamics in terms of orientation, for some initial conditions the orientation of the VR (and its host) may not converge to the reference orientation. In these situations, formation may not be established or collisions may take place. To overcome this problem, we have proposed the Virtual-Head Robot Tracking approach [17], motivated by the ideas of virtual robot [16] and virtual reference point \((x_L, y_L)\) [15].

A head robot (HR) is defined also as a hypothetical robot whose orientation is identical to that of its host, but positioned at distance \(d > 0\) ahead from its host. The relation between HR and its host can be written as:

$$\begin{align*}
    x_{hj} &= x_j + d \cos \theta_j \\
y_{hj} &= y_j + d \sin \theta_j \\
\theta_{hj} &= \theta_j,
\end{align*}$$

where \((x_j, y_j, \theta_j)\) and \((x_{hj}, y_{hj}, \theta_{hj})\) denote respectively coordinates of the host (robot \(j\)) and its head robot.

Fig. 1 describes the model and parameters of VR-HR. Here, HR is designated to serve as a virtual robot of the follower with \(d\) to be chosen adequately small as a tracking margin. HR is identical with its host when \(d = 0\). Position errors between VR of leader \(i\) and HR of follower \(j\) are:

$$\begin{align*}
    e_{xji} &= x_{hj} - x_{wi} = (x_j + d \cos \theta_j) - (x_i + R \sin \theta_i - L \cos \theta_j) \\
    e_{yji} &= y_{hj} - y_{wi} = (y_j + d \sin \theta_j) - (y_i - R \cos \theta_i - L \sin \theta_j).
\end{align*}$$

The objective here is to design a control law applied to the follower robot \(j\) so that its head robot can track the VR of the leader \(i\) and as \(t \to \infty\), a desired \(R-L\) configuration of leader \(i\) and follower \(j\) is obtained with position errors decreasing monotonically to the chosen margin \(d\).
From the error model (4), we have
\[ \dot{e}_{ji} = B_j u_j - b_i u_i, \]
where
\[ e_{ji} = \begin{bmatrix} e_{xji} \\ e_{yji} \end{bmatrix}, \quad u_j = \begin{bmatrix} v_j \\ \omega_j \end{bmatrix}, \quad u_i = \begin{bmatrix} v_i \\ \omega_i \end{bmatrix}, \]
\[ B_j = \begin{bmatrix} \cos \theta_j & -d \sin \theta_j \\ \sin \theta_j & d \cos \theta_j \end{bmatrix}, \quad b_i = \begin{bmatrix} \cos \theta_i & R \cos \theta_i + L \sin \theta_i \\ \sin \theta_i & R \sin \theta_i - L \cos \theta_i \end{bmatrix}, \]
and the following control law can be obtained:
\[ u_j = B_j^{-1} \left[ b_i u_i - \Lambda_j E_{ji} \right], \]
where \( \Lambda_j = \begin{bmatrix} \lambda_{ji} & 0 \\ 0 & \lambda_{j2} \end{bmatrix} \) is a positive-definite diagonal matrix, \( B_j^{-1} = \frac{1}{d} \begin{bmatrix} d \cos \theta_j & d \sin \theta_j \\ -\sin \theta_j & \cos \theta_j \end{bmatrix} \), and the zero dynamics proved to be stable [17].

In the VR tracking approach [16], to establish formation for multiple robots, \( l-l \) control, originated by Desai et al. [13], has been modified to allow a finite time, \( T_r \), for the formation establishment, by using a terminal attractor. The \( l-l \) control law, aimed to maintain the desired lengths, \( l_{13}^d \) and \( l_{23}^d \) of a robot (indexed 3) from its two leader robots (indexed 1 and 2), is however undefined when the three robots are aligned on a straight line. To deal with this problem, three-point \( l-l \) controllers have been proposed in [17], by suitably switching between \( l-l \) controllers with respect to three virtual points, located around the centre of robot 3 and formed into a certain triangle. By introducing virtual velocities for the VR of robot \( i \), with predefined clearances \( R = r_K, L = 0 \) for an arbitrary virtual head point \( K \) on robot 3, shown in Fig. 2, as
\[ v_{i3} = v_3 + r_K \omega_3, \quad \omega_{i3} = \omega_3, \]
where \( v_i \) and \( \omega_i \) are velocities of robot \( i \), and using the same terminal attractor as in [16], one can obtain the following control law for virtual robot \( R_3 \):

![Fig 2. l-l control with respect to virtual point K](image)

\[ F_{dcos0}^{ctrl} \]

\[ u_{3} = \begin{bmatrix} v_3 \\ \omega_3 \end{bmatrix} = B_{3K}^{-1} (v_{aK} + \alpha_{K} l_{ek}), \quad 0 \leq r \leq T_r; \]
where \( \gamma_{iK} = \theta_i + \psi_{i3K} - \theta_3, (i = 1,2) \),
\[ l_{ek} = \begin{bmatrix} \frac{(l_{13K}^d - l_{13K})(0)}{3} \\ \frac{(l_{23K}^d - l_{23K})(0)}{3} \end{bmatrix}, \quad \alpha_K = \begin{bmatrix} \alpha_{1K} & 0 \\ 0 & \alpha_{2K} \end{bmatrix}, \]
\[ \alpha_{1K} = \text{sign}(l_{13K}^d - l_{13K}(0)) (l_{13K}^d - l_{13K}(0))^{\frac{1}{3}} \left( \frac{T_r}{3} \right)^{-1}, \]
\[ \alpha_{2K} = \text{sign}(l_{23K}^d - l_{23K}(0)) (l_{23K}^d - l_{23K}(0))^{\frac{1}{3}} \left( \frac{T_r}{3} \right)^{-1}. \]

From (3), the control law for robot 3 can be obtained as:
\[ u_3 = \begin{bmatrix} v_3 \\ \omega_3 \end{bmatrix} = \begin{bmatrix} v_{i3} - r_K \omega_{i3} \\ \omega_{i3} \end{bmatrix}. \]

As stated, in 3PLL control, a suitable controller is to be chosen correspondingly, for example, to three virtual head points of robot 3, namely A, B, and C (Fig. 3), selected to form the shape of a triangle. By that way, the singularity problem associated with \( l-l \) control, i.e., when the head point of robot 3 is aligned with the line connecting two other robots, can be completely overcome. Triggering the switching in practice depends on the level of sensitivity of the robot actuator control voltage with respect to singularities.

B. Reactive Control

The proposed control laws ensure there is no collision between two robots \( i \) and \( j \) at the beginning the centre of leader \( i \) is not found in the “critical area”, determined by initial positions of the two robots as illustrated in Fig. 4, where the safe distance referred to the centre of any robot in the group for avoidance of collision with others is defined as
\[ D_{\text{safe}} = d + d_{\text{safe}}, \]
with \( d_{\text{safe}} = 2r \).

It is noted that in the tracking phase if the safe distance given (10) is not preserved, the proposed controllers may not ensure the collision avoidance between two concerning robots. Reactive schemes are proposed as a treatment for these cases.

Fig. 3 Switching among three l-l controllers

![Fig 3 Switching among three l-l controllers](image)
The control parameters \( l_{13}^d \) and \( l_{23}^d \) can be designed as follow

\[
\begin{align*}
l_{13}^d &= 2r + D_{\text{max}} + \delta_1 \\
l_{23}^d &= \delta_2.
\end{align*}
\]

Using 3PLL control with the above parameters will drive robot 3 closer to the VR (robot 2) while going around the safe boundary of robot 1, which is a circle with centre of robot 1 and radius \( 2r \). This will obviously reduce the possibility of robot 3 to collide with the other robot. The reason that \( l_{13}^d \) is \( 2r + D_{\text{max}} \) rather than \( 2r \) is that in 3PLL control, the distances from one of three head points of robot 3 to the centres of its leaders is referred instead of the distances among their centres. Thus in order to ensure collision avoidance, \( l_{13}^d \) has to be increased by the largest distance from one of three head points of robot 3 to its centre. Margins \( \delta_1 \) and \( \delta_2 \) are deliberately augmented to \( l_{13}^d \) and \( l_{23}^d \) to ensure the distance between centres of robot 1 and robot 3 to be strictly greater than \( 2r \).

**Scheme 2: Potential collision between three robots**

This scheme is used when there is a possibility of collision between a robot and two other robots. The lowest priority robot will then apply 3PLL control with respect to two leaders, which are thee two other robots concerned. The control parameters \( l_{13}^d \) and \( l_{23}^d \) are proposed as,

\[
\begin{align*}
l_{13}^d &= 2r + D_{\text{max}} + \delta_1 \\
l_{23}^d &= 2r + D_{\text{max}} + \delta_2,
\end{align*}
\]

where again, \( D_{\text{max}}, \delta_1 \) and \( \delta_2 \) are augmented to \( l_{13}^d \) and \( l_{23}^d \) for the same reason as explained above.

**IV. FORMATION INITIALISATION PROCEDURE**

A step-by-step procedure is proposed in this section for initialization of a group of \( N \) mobile robots to enter a desired formation. At each step, robots in the group are classified as active or inactive. Active robots will participate to establish the formation while inactive robots stay at its initial position. The process of formation initialization will then run until all robots in the group become active and a desired formation is obtained. In addition, in the proposed framework, each robot can play the role of a follower (tracking another robot) or of a leader (guiding another one). The leader of the whole group is called the lead robot and is indexed by 1. Under the proposed reactively controlled VHRT and 3PLL control, inter-robot collision avoidance can be achieved with some margin in distance between robots, consequently passive robots are designated not to obstruct any active robot. Note that in any desired formation, distance between any two robots shall preserve a certain lower limit determined in a practical application. In this paper, this distance is taken as double of the safe distance, i.e. \( 4r \).

The proposed process of formation initialisation can now be performed with the following algorithm.
1. Make all robots in the group \textit{inactive}. Choose the \textit{lead} robot, indexed by \( i=1 \), to guide the whole group. Let it become \textit{active}.

2. Index (or reindex) all \textit{inactive} robots from \( j=(i+1) \) to \( N \), based on their initial position with respect to the motion of the \textit{lead} robot.

3. Let one or two \textit{inactive} robots with smallest indices become \textit{active}. Use reactively controlled VHRT-3PLL to get them into desired positions while avoiding collision with other robots until all \( i \) \textit{active} robots have reached their positions in the group. Go to step 2 if \( j<N \).

   If there is no \textit{inactive} robot left (or \( i=N \), the desired formation has established.

4. Exit.

By applying the proposed approach, a large group of mobile robots can be controlled to form and maintain a desired formation without inter-robot collision.

V. SIMULATION RESULTS

A. Case of singularities

To illustrate the capability of avoiding singularities and possibilities of collision among robots in a formation let us first consider the case of three mobile robots moving to form a wedge formation. Parameters and conditions used in this simulation were set as:

- Initial conditions of robots:
  Robot 1: \( x_1(0) = 30, y_1(0) = 0, \theta_1(0) = 0(\text{rad}), v_1 = 5, \omega_1 = 0 \),
  Robot 2: \( x_2(0) = 0, y_2(0) = -50, \theta_2(0) = 0(\text{rad}) \),
  Robot 3: \( x_3(0) = 20, y_3(0) = -90, \theta_3(0) = 1(\text{rad}) \);

- Parameters for the desired formation:
  Robot 2: \( R_2 = 40, L_2 = 30 \), Robot 3: \( R_3 = -40, L_3 = 30 \);

- Tracking margin for head robots: \( d = 1 \);

- Safe distance between any two robots: \( d_{\text{safe}} = 26 \);

- Parameters for Virtual-Head tracking control:
  Robot 2: \( \lambda_{21} = 1, \lambda_{22} = 2 \), Robot 3: \( \lambda_{31} = 1, \lambda_{32} = 2 \);

- Parameters for \( l_l \) control with point A:
  \( r_A = 0, l_A = 12, T_r = 3s, l_{13d} = 100, l_{13b} = 50 \);

- Parameters for \( l_l \) control with point B:
  \( r_B = 0, l_B = 6, T_r = 3s, l_{23d} = 100, l_{23b} = 50 \).

Figures 6 and 7 show respectively the global trajectories and time responses of orientation \( \theta \) for the three robots. With these initial conditions, potential collision could be observed at time points \( t = 0.11s; 3.21s; 6.33s; 9.46s; \) and 12.6s when using the \( l_l \) controller with head point A of robot 3 (Fig. 3). It is observed that at \( t = 9.46s \), due to singularity, the system switched to the \( l_l \) controller with respect to head point B. The results obtained indicate that the three robots could successfully get into and maintain a wedge formation without inter-robot collision even in the presence of singularities.

B. Formation initialisation

To illustrate the procedure proposed for a group of robots to enter a formation, we choose typically the case of five mobile robots moving to form a diamond-like formation to simulate. Parameters and conditions used were set as:

- Initial conditions of robots:
  Robot 1: \( x_1(0) = 30, y_1(0) = 0, \theta_1(0) = 0(\text{rad}), v_1 = 5, \omega_1 = 0 \),
  Robot 2: \( x_2(0) = 0, y_2(0) = 0, \theta_2(0) = 0(\text{rad}) \),
  Robot 3: \( x_3(0) = 10, y_3(0) = 50, \theta_3(0) = 0(\text{rad}) \),
  Robot 4: \( x_4(0) = 150, y_4(0) = 180, \theta_4(0) = 2(\text{rad}) \),
  Robot 5: \( x_5(0) = 150, y_5(0) = -150, \theta_5(0) = \pi / 2(\text{rad}) \);

- Parameters for the desired formation:
  Robot 2: \( R_2 = 85, L_2 = 40 \), Robot 3: \( R_3 = -85, L_3 = 40 \),
  Robot 4: \( R_4 = 0, L_4 = 40 \), Robot 5: \( R_5 = 0, L_5 = 80 \);

- Tracking margin for choosing head robots: \( d = 1 \);

- Safe distance between any two robots: \( d_{\text{safe}} = 22 \).

Following the proposed procedure, in the first step, three robots 1, 2, 3 formed a part of the desired formation (a
This step took 30 seconds. In second step, robot 4 and robot 5 tracked robot 1 to form the desired diamond shape. Figures 8 and 9 show respectively the global trajectories and the orientation \( \theta \)-time responses of the robots. It is observed robot 5 could possibly collide with robot 2 at \( t = 30.22 \text{s} \), and robot 4 with robot 3 at \( t = 30.23 \text{s} \). After avoiding collision by using the proposed reactive control schemes, robot 4 and robot 5 could switch back to tracking control to eventually establish the desired formation.

Our simulation results have shown the validity of the proposed reactively-controlled VHRT and 3PLL control approach in the initialisation and establishment of any desired formation for a large group of mobile robots while ensuring inter-robot collision avoidance.

V. CONCLUSION

We have presented a new approach for controlling multiple mobile robots in formation using the leader-following strategy while ensuring collision-free group motion. In the framework, we propose virtual head robot tracking (VHRT) for a follower to form with its leader any desired shape with a given tracking margin, three-point 1-1 (3PLL) control for avoiding singularities in establishing a formation of a follower with respect to two leaders, and reactive control schemes to ensure a safe distance between robots. For generally a group of \( N \) robots, a step-by-step procedure is then suggested for formation initialisation. The approach has been tested through extensive simulations to demonstrate its validity. Work is planned to tackle the problems of obstacle avoidance and formation changes.

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