Modelling Interference Margins in FFR enabled WiMAX Systems for Cell Dimensioning

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Abstract—The interference margins ($I_m$) for the downlink and uplink of a FFR enabled WiMAX network are modelled in this paper. This simple yet accurate model is suited for cell dimensioning applications and it draws many of the input parameters from cell dimensioning. The average collision probabilities are modelled first and then related to the interference margins. The results show the similarities in collision probability and $I_m$ for both the downlink and uplink and the impact of FFR in reducing interference.

Index Terms—Cell dimensioning, FFR, Interference margin, OFDMA, PUSC, WiMAX

I. INTRODUCTION

WiMAX technology has gained worldwide attention due to its potential for providing ubiquitous, broadband wireless access at a competitive cost and quality basis to the current DSL (Digital Subscriber Line) services. The IEEE 802.16e standard [1] is the basis for mobile WiMAX, which has been further enhanced by the work carried out by the Technical Working Group (TWG) of the WiMAX forum [2]. Major wireless operators around the world are gearing up to deploy WiMAX networks, on highly challenging time scales. Accurate cell dimensioning and network planning will be an essential pre-requisite for the smooth functionality of these WiMAX networks.

One of the critical parameters in WiMAX cell dimensioning is the Interference margin ($I_m$), which quantifies the impact of interference in the link budgets. The 802.16e standard allows adjacent cells to utilize the same frequency resource (i.e. frequency reuse=1), which would lead to very high interference levels for the cell edge users. The proposed solution to this problem is the use of Fractional Frequency Reuse (FFR) [3]. With FFR, the cell edge users operate with only a fraction of the available sub-channels (the basic resource allocation), while the in-cell users operate with all available sub-channels. In a cell dimensioning perspective, the inclusion of FFR reflects a basic trade-off between coverage (i.e. signal quality at cell edge) and the offered capacity through the available resources. Hence accurate representation of interference effects is essential to properly dimension a FFR enabled WiMAX network.

In this paper we present a novel, simplistic approach for quantifying the interference margins for WiMAX networks with FFR. WiMAX 802.16e being an OFDMA (Orthogonal Frequency Duplex Multiple Access) based system, the interference is mostly generated when inter-cell collisions occur with the usage of same time/ frequency resources [4]. In section II, we present a theoretical approach to quantify the average collision probability for the uplink and downlink and validate these results with simulations as per the 802.16e standards. In section III, we develop equations to model the worst case interference margins based on these average collision probabilities and other parameters extracted from cell dimensioning. The simulation results are presented and discussed in section IV. The conclusions and further work planned in this area are presented in section V.

II. THE AVERAGE COLLISION PROBABILITY

The collision is a WiMAX terminology for the usage of the same time/ frequency allocations in adjacent cells/ sectors. The standards have introduced random sub-channelization schemes in order to minimize these collisions at low resource utilizations. In this work we consider the PUSC (Partial Usage of Sub Carrier) allocation scheme [1] in conjunction with FFR. In the common FFR implementation, the users at the cell edges operate on a sub-frame zone with frequency reuse=3 (called the R3 zone) while the inner cell users operate on a zone with all sub-channels available, called the R1 zone.

The modes of resource allocations for the WiMAX downlink (DL) and uplink (UL) differ significantly. In the DL, the data bursts for each active user can start from any symbol position (in the time axis). However in the UL, the emphasis is to limit the number of sub-channels allocated to each user, so the Mobile Station (MS) transmit power is constrained. Hence the data bursts in the UL tend to limit to a minimum number of sub-carriers and span the totality of time symbols. A generalized format for the DL and UL sub-frames is shown in Fig. 1.

![Fig. 1. Transmission frame format for mobile WiMAX (from [3])](image)

With FFR, the sub-frames will be further split into R1 and R3 zones. The R3 zone will only carry 1/3 of the sub-channels...
available for the R1 zone. The resource allocations always start from the top-left corner of the sub-frames.

In this work, we will make several assumptions on DL and UL sub-frame loading, which are in tune with cell dimensioning requirements. For the DL, we consider a worst case collision scenario, where each user is allocated the minimum number of time symbols. A slot is the minimum resource allocation for DL, which contains 2 sub-channels [1] (or 2 time symbols with 24 sub-carriers each). Hence loading in the DL is considered to expand on a 2 symbol basis, with all the sub-channels in those 2 symbols filling up before moving onto the next 2 symbol segment. For the UL the power limitations in the MS ensure that resource allocations span the total number of symbols in the sub-frame. Hence the sub-channel loading (down the frequency axis) is equivalent to cell loading in the UL. For simplicity, we also assume that the R1 and R3 zones are experiencing equal loading. Although this assumption will not hold true for a practical system, it will be valid for cell dimensioning scenarios.

### A. Theoretical derivation

The PUSC and FUSC sub-channelisation schemes in 802.16e use pseudo-random permutation sequences to draw-up different sub-carriers which make up the sub-channels. There is a PermBase parameter unique to each Base Station (BS) [1] and this offsets the pattern of sub-carrier selection. Hence the physical sub-carrier sequence for each BS would differ from the logical sub-carrier (or sub-channel) sequence shown in Figure 1. However when all sub-carriers get used up (down the rows of Fig.1), the collision probability reaches 1. The pseudo-random allocations have an impact only when all the sub-channels are not fully utilized.

The collision probability when only a portion of the sub-channels have been utilized was studied in [5], and it has been shown that the average collision probability can be equated to averages drawn from a purely random sub-carrier allocation process. The probability of registering c sub-carrier collisions when \( N_c \) (out of a total of \( N_d \)) sub-carriers make up a sub-channel and L sub-channels are utilized, is given by [5]:

\[
P_{\text{random}}(c) = \frac{N_{\text{data}}-N_c}{N_{\text{data}}} \frac{N_{\text{data}}}{N_{\text{data}}}! \frac{N_c}{N_c}! \frac{C_L}{C_L}^{N_{\text{data}}-N_c} \frac{C_c}{C_c}^{N_{\text{data}}}
\]

Where \( ^nC_r = n!/(n-r)!r! \) gives the number of un-ordered permutations possible on selecting a subset with r elements out of a total of n elements. n! is the factorial of n. This random sub-channelisation process can also be represented by the binomial distribution and similar comparisons have been drawn on random sub-channelisation and the actual PUSC performance [6].

![Image](image_url)

Fig. 2. Collision PDF’s for random sub-carrier allocations

The average collision probability \( P_{\text{coll,av}} \) can be found by averaging the number of collided sub-carriers (\( N_{\text{coll}} \)) by the number of used sub-carriers, \( N_{\text{used}} \) [5].

\[
P_{\text{coll,av}} = \frac{N_{\text{coll}}}{N_{\text{used}}} = \frac{\sum c \times P_{\text{random}}(c)}{N_{\text{used}}}
\]

It can quickly be seen that \( P_{\text{coll,av}} \) increases linearly with sub-channel loading. We will analyse how the average collision probability varies in the DL sub-frame loading (or DL cell loading), when the sub-channels are repeatedly used in the 2 symbol columns (which we term as segments). Depending on the number of segments s in the sub-frame the average collision probability for the DL can be noted as;

\[
P_{\text{coll,DL}} = \frac{\sum c \times P_{\text{random}}(c)}{(s-1) \times N_{\text{data}} + \sum c \times P_{\text{random}}(c)}
\]

Equation (3) accounts for the fact that when a segment is fully utilized, all \( N_{\text{data}} \) sub-carriers are in collision. Considering the collision probability varies linearly with the loading in a single segment (denoted by \( x \)), the above expression can be simplified as;

\[
P_{\text{coll,DL}} = \frac{\sum c \times P_{\text{random}}(c)}{(s-1) + \frac{\sum c \times P_{\text{random}}(c)}{N_{\text{used}}}} \frac{N_{\text{used}}}{N_{\text{data}}}
\]

\[
P_{\text{coll,DL}} = \frac{s-1 + x^2}{s-1 + x}
\]

For incremental loading in a segment, both \( N_{\text{used}}/N_{\text{data}} = x \) and \( N_{\text{coll}}/N_{\text{data}} = x \). Equation (4) shows that \( P_{\text{coll,DL}} \) will reach the maxima of 1, s times (as given by the number of segments). This theoretical expression is plotted against the load factor in Fig.3 (as the black solid line) for a DL zone of 14 symbols, which make up s=7 segments in our ‘worst case’ scenario. The blue dots are a scatter plot obtained from collision simulations for the DL-PUSC, as explained below in
For the UL with PUSC sub-channelisation, the same theoretic approximations as given by equations (1) and (2) can be applied, although the specific values for \( N_{\text{data}} \), \( N_{\text{sc}} \) and \( L \) are different. The UL sub-frame loading is proportionate to the UL sub-channel loading, with each allocation for a user spanning the whole symbol length. Hence the average collision probability for the UL, \( P_{\text{coll,UL}} \) can be modelled to increase linearly with the UL sub-frame loading. This behaviour is plotted in Fig.4, as the black solid line. Again the blue dots form a scatter plot taken from UL-PUSC collision simulations explained below in section II(B).

**B. Validation through collision simulations**

The DL–PUSC mapping process consists of mapping sub-carriers to clusters and then mapping these clusters to sub-channels. This process is governed by sub-carrier mapping equations and cluster renumbering sequences given in section 8.4.6.1.2.1 and section 8.4.6.1.2.2.2 (this section is common to the FUSC as well) of the 802.16e standards [1]. For the UL-PUSC, the process contains mapping sub-carriers to tiles and mapping is tiles to sub-channels. The procedure is explained in section 8.4.6.2.2 of the standards [1].

The PUSC sub-carrier allocations for a 2 tier (7cell) configuration were simulated as per the above specifications. The collision probabilities were calculated with 1000 iterations for the DL and UL with incrementing loading. The randomness in the iterations comes from the random selection of the \( \text{PermBase} \) parameter [1]. Additionally for the DL, a random number of sub-channels (moving down the 2-symbol segment) is assigned per user, which changes the DL cell loading in irregular steps. 7 segments were considered for the DL, emulating a zone with 14 symbols. The results are shown as scatter plots in Figures 2 and 3 in section II(A). The mean trends of these scatter plots agree well with the average collision probabilities calculated in section II(A).

**III. THE MODEL FOR INTERFERENCE MARGIN CALCULATIONS**

The main source of interference for mobile WiMAX is considered to be the inter-cell interference, generated through collisions as evaluated in section II. The interference margin is regarded as a ‘noise rise’ above the noise level, as analysed for 3G Wideband CDMA networks in [7]. Hence the interference margin \( I_m \) is defined with noise power \( N \) and interference power \( I \) as:

\[
I_m = 10 \times \log \left( \frac{N + I}{N} \right)
\]

The noise power \( N \) is considered to be the noise seen by the receiver terminal, either in the DL or UL. This noise power contains the thermal noise (dependent on the system bandwidth \( BW \)), the noise figure (\( NF \)) and any implementation losses (\( L_{\text{imp}} \)) at the receiver. The values for these parameters are drawn from the cell dimensioning process. The noise power is given by:

\[
N_{dBm} = -174 + 10 \times \log(BW_{Hz}) + NF + L_{\text{imp}}
\]

The interference powers are modelled as the worst case scenarios for the DL and UL, with a 2 tier network. For the UL, the interference effects the serving BS and it originates from multiple MS at cell edge, connected to neighbouring BS's. For the DL, a cell edge MS experiences interference from the neighbouring BS's. The relevant distances for these interactions are calculated and converted to path loss (\( PL \)) as per the appropriate path loss model used in cell dimensioning. Also the shadow margin and fast fade margins are added onto \( PL \) to reflect these effects on the received interference signal. The received interference power at a particular cell loading point is thus the sum of these interference components weighted by the average collision probability for DL or UL (denoted by \( P_{\text{coll}} \)).

\[
I = P_T \times G_T \times P_{\text{coll}} \left[ \sum_{i=1}^{N} \frac{1}{PL_i} \times \cos^2 \theta_i \right]
\]

Here \( P_T \) denotes transmit power and \( G_T \) any relevant antenna gains. These parameters are drawn from cell dimensioning. The \( \cos^2 \theta_i \) is a term that models azimuth signal power with a directional antenna pattern, where \( \theta \) is the angle from antenna bore-sight.
With the inclusion of FFR, the interference for R1 and R3 zones are calculated separately. A geographical zone boundary is required to simplify these calculations. Although such fixed boundaries may not exist in the practice, such a simplistic approach is required for both coverage and capacity dimensioning processes. In this work we assume that the users are uniformly distributed in the cell and the cell loading in both R1 and R3 zones are equal. Thus the geographic zone areas reflect the resources available in each transmission zone, given by the zone symbol split in DL and UL sub-frames. For the example of 3 sectored cell deployments shown in Figure 5, a circular R1 zone is assumed, with its area given by the proportion of resources allocated by the R1:R3 resource split. We give priority to the DL and assume that the UL follows (roughly) the same resource split.

The cell radius for the 3-sectored sites is given by the initial link budget from cell dimensioning. Directional antennas are considered for the BS and interference is only received (or emitted) from a ±60º angular span from the BS antenna bore-sight. The 1st tier interference sources for the R1 zone in DL are shown in solid red lines, while the R3 zone interference (with frequency re-use=3) are further apart shown in dashed red lines. The interference sources for the UL are shown in blue lines (solid for R1 and dashed for R3).

The sub-channelisation gain is resulting from limiting the transmit power of the MS to a minimum of 1 sub-channel. The urban macro-cell environment is considered in the simulations.

**IV. Simulation Parameters Results**

We conduct simulations with our interference model and parameters in Table 1 are utilized for the DL and UL respectively. All these parameters are drawn from an example cell dimensioning process. A SISO (Single Input Single Output) antenna configuration is considered, with no gains from antenna diversity or beam-forming.

<table>
<thead>
<tr>
<th>Link Parameters</th>
<th>Downlink</th>
<th>Uplink</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symbol split (data symbols)</td>
<td>22</td>
<td>18</td>
</tr>
<tr>
<td>Transmit power (dBm)</td>
<td>40</td>
<td>23</td>
</tr>
</tbody>
</table>

The sub-channelisation gain is resulting from limiting the transmit power of the MS to a minimum of 1 sub-channel. The urban macro-cell environment is considered in the simulations.

**A. Results for the Downlink**

For the downlink, a total of 22 data symbols is considered which is split at 4 different ratios of R1:R3. The \( I_m \) values of each scenario of the R1 zone and R3 zone are weighted with their actual resource splits (i.e. total available sub-channels) to derive an average \( I_m \). For the DL R3 zone, the interfering BS power is boosted by a 4.7dB sub-channelization gain (as only 1/3 of available sub-channels are utilized in R3). This is accounted for in the \( I_m \) calculations for R3. However as the sub-carriers are PUSC distributed, the receiver needs to scan the total 10MHz bandwidth and there is no reduction in the noise power of (6). The reuse=1 only and reuse=3 only scenarios are also simulated and the results are shown below in Fig.6.
The traditional method of DL interference management by relaxing the cell load [6] will not be effective in the WiMAX DL, even with FFR. We have only considered 2 zones (R1 and R3) in the DL, but WiMAX standards [1] allow the usage of many optional zones (like AMC zone and adaptive MIMO switching zones [8]), which split the frame into smaller zones. Then one would expect the loading point where the \( I_m \) plateau is reached to further increase.

**B. Results for the Uplink**

The uplink simulations consider the availability of 18 data symbols, split (roughly on 2 symbol granularity) at the same resource splits as for the DL. The simulation process is slightly different such that the successive interfering MS at the cell edge gets activated as the cell load increases. The simulations are iterated to represent different sequences of cell edge users contributing interference with increasing load. The averaged results are shown in Fig.7. In the UL, the sub-channelisation gain is already considered at 1 sub-channel usage per MS level, hence there is no further gain in shifting from R1 to R3.

The average collision probability (Fig.4) varies smoothly with the cell loading in the UL, and this is reflected in the \( I_m \) results. Hence the interference can be more effectively controlled by relaxing the loading in the UL. The impact of FFR in reducing the interference is also evident in the UL, but it is not as pronounced in the DL. In the DL, the \( I_m \) in R3 had more weight due to the additional sub-channelisation gain of 4.7dB described earlier. This is not available in the UL and it can be the reason for the FFR to show less impact in reducing \( I_m \) with larger R3 zones.

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**REFERENCES**


