Performance Analysis of Multiple Carrier $M$-ary FSK System with Diversity Combining Over Generalized Fading Channels

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Abstract—We evaluate the performance of the multiple carrier $M$-ary frequency shift keying (MC-FSK) system over generalized fading channels. MC-FSK system represents each transmitted signal by a series of orthogonal components, which are activated based on the balanced incomplete block (BIB) design from the combinatorial theory. The performance is quantified in terms of the system power efficiency, by deriving an upper bound expression on the average bit error probability (BEP), and calculating the system bandwidth efficiency. The average BEP expression considers the case where the fading envelopes are not identically distributed and/or statistically described by the same statistical distribution family. Furthermore, we compare between the performance of the MC-FSK and that of the conventional $M$-ary FSK system. The results show that both systems achieve the same power efficiency when their combined diversity orders are equal, but at the same time, MC-FSK system does provide better bandwidth efficiency, which is proportional to the number of orthogonal carriers per transmitted signal. We show that MC-FSK system can simultaneously provide the same power efficiency at different bandwidth efficiencies depending on the number of orthogonal components per transmitted signal.

I. INTRODUCTION

In wireless communications, the severity of the channel and the limited available bandwidth may incur considerable amount of performance degradation that can not be tolerated using conventional modulation schemes. Therefore, it is desirable to obtain modulation/coding schemes that enhance the system power and/or bandwidth efficiency, and relatively retains simple transmitter and receiver structures. Based on this, considerable amount of work has been done in scheming a suitable method that maintains the high power efficiency property of the $M$-ary frequency shift keying ($M$-ary FSK) system, and at the same time, improves its bandwidth efficiency. Some of this work has used algebraic tools in order to design appropriate coding methods that can be applied in conjunction with $M$-ary FSK system to satisfy this objective. One of such algebraic tools is the use of the balanced incomplete block design (BIB-design) from the combinatorial theory [1]–[3].

In this paper, we analyze the performance of the MC-FSK signals over generalized fading channels, which consists of $L$ diversity branches. The analysis is quantified in terms of the system power efficiency, by deriving an upper bound on the average bit error probability (BEP), and calculating the bandwidth efficiency. The average BEP is expressed in terms of the combined received SNR moment generating function (MGF), and allow for the case where the fading envelopes are not identically distributed nor following the same statistical distributions. The rest of the paper is organized as follows: section II presents an introduction to the BIB design and the generation process of the MC-FSK signals. Section III mathematically characterizes the communication channel and formulation of the decision variables. The average BEP is derived in section IV, and the system bandwidth efficiency is discussed in section V. Section VI presents comparisons between the MC-FSK and $M$-ary FSK systems. Finally, conclusions follow in section VII.

II. MC-FSK MODULATOR

A. BIB Design: An Introduction

Assume that the set $O = \{f_1, f_2, \ldots, f_v\}$ contains $v$ orthogonal carrier frequencies. The MC-FSK modulation is an arrangement of the set $O$ elements into $H$ signal waveforms $\psi_1, \psi_2, \ldots, \psi_H$, such that each signal waveform $\psi_m = \{f_{m_1}, f_{m_2}, \ldots, f_{m_w}\}$, for $m = 1, 2, \ldots, H$, simultaneously contains $w$ orthogonal frequencies. The selection process of orthogonal frequencies to represent each signal waveform $\psi_m$ is based on the rule that each carrier from the set $O$ occurs in exactly $\delta$ different waveforms, and each pair of the set $O$ elements occurs in exactly $\sigma$ signal waveforms. This arrangement rule of signal waveforms is referred to as a BIB-designed MC-FSK modulation. For a particular values of $v$, $w$, and $\sigma$, the total number of signal waveforms that can be generated in this design is $H = \sigma v (v - 1) / (w (w - 1)$ and the number of repetition of a particular frequency tone from set $O$ is $\delta = \sigma (v - 1) / (w - 1)$. In this paper, we consider the case when $\sigma = 1$, in which the BIB design system is known as the Steiner code system design, and denoted by $S(v, H, \delta, w, k_a)$.

The BIB (Steiner system) design enables representing the total number of orthogonal signal waveforms $M$ by $v \leq M$ orthogonal carriers, such that the $v$ carriers can be used to formulate $H \geq M$ signal waveforms, where each of which simultaneously contains $w$ orthogonal frequencies. This feature does greatly reduce the complexity of the system receiver.
since only \(v\) matched filters are needed for the demodulation process.

In Steiner system-based MC-FSK modulation, a given pair of orthogonal frequencies occurs in only one of the \(H\) signal waveforms. Therefore, for any transmitted signal waveform \(\psi_m\), for \(m = 1, 2, \ldots, H\), there will be \(H_1\) nonorthogonal waveforms having one single carrier frequency in common with \(\psi_m\), and \(H_2\) orthogonal waveforms having no common carriers with \(\psi_m\). The sum of the nonorthogonal and orthogonal signals must equal \(H - 1\), and they are respectively given by

\[
H_1 = w \frac{(v - w)}{w - 1}, \quad H_2 = H - \left(1 + \frac{w(v - w)}{w - 1}\right). \tag{1}
\]

Table I gives the number of orthogonal frequencies \(v\) required by Steiner system-based MC-FSK system to represent all possible combinations of \(k_s\) binary bits per symbol, the total number of signals \(H\) that can be generated using the \(v\) carriers for the cases when each signal contains three, four, or five orthogonal components, the number of nonorthogonal signals \(H_1\) and orthogonal signals \(H_2\) for each case. For the purpose of comparison, the required number of orthogonal carriers by the \(M\)-ary FSK system to represent every \(k_s\) binary bits are also provided in the table.

**B. MC-FSK Signals**

The modulation process of the MC-FSK signals based on the Steiner block design is shown in Fig. 1. In each symbol interval \(T_s\), \(k_s T_b\), where \(T_b\) is the bit duration, \(k_s\) binary bits are used by the Steiner system encoder to select one of \(H\) possible blocks that contains \(w\) distinct elements \((u_1, u_2, \ldots, u_w)\). After that, a \(v\)-ary frequency modulator simultaneously generates \(w\) orthogonal frequencies out of \(v\) possible ones corresponding to the activated block \((u_1, u_2, \ldots, u_w)\). The symbol energy \(E_s\) is equally divided among the \(w\) selected components, such that each of which contains \(E_{sw} = E_s/w\) amount of energy. The generated components are added with equal weights to formulate the MC-FSK modulated signal. Using the complex notation, the bandpass MC-FSK modulated signal can be written in the form

\[
u_m(t) = \sum_{\lambda=1}^{w} \sqrt{\frac{2E_w}{T_s}} e^{j2\pi(f_{sm} + f_c)t} u_{Ts} (t), \tag{2}
\]

where \(f_c\) is the center frequency, \(u_{Ts}(t)\) is the basic pulse waveform with unit amplitude and time duration of \(T_s\), \(w\) is the number of orthogonal components selected from the \(v\) possible ones based on the \(m\)th Steiner system block, and \(f_{sm}\) is one of the \(v\) orthogonal frequencies, and takes its values from the set \(\{2\lambda_m - 1 - v\} \Delta f/2\) \(\lambda_m=1\), where \(\Delta f\) is the frequency separation between any pair of adjacent orthogonal frequencies. The orthogonality between any pair of signal components is achieved, employing non-coherent detection, by choosing \(\Delta f = n/T_s\), where \(n\) is an integer.

**III. RECEIVER STRUCTURE**

**A. Modeling of Generalized Fading Channels**

The generalized fading channel consists of \(L\) independent diversity branches each of which is modeled as a frequency nonselective and slowly varying fading channel. Each transmitted signal is received over \(L\) independent branches. The energy of each received signal replica is one \(L\)th the symbol energy, and each received orthogonal component has energy of \(E_c = E_w/L = E_s/wL\). Assuming that the MC-FSK signal given in (2) is transmitted over such generalized fading channel, the received faded and noisy signal over the \(l\)th diversity branch becomes

\[
r_l(t) = \sum_{\lambda=1}^{w} \alpha_{l\lambda} \sqrt{\frac{2E_s}{T_s}} e^{j2\pi(f_{sm} + f_c)t + \phi_{l\lambda}} + n_{l\lambda}(t), \tag{3}
\]

where \(\alpha_{l\lambda}\) and \(\phi_{l\lambda}\) are the random channel envelope and phase shift that affect the \(\lambda\)th orthogonal component of the \(m\)th transmitted signal transmitted over the \(l\)th diversity branch and \(n_{l\lambda}(t)\) is a complex-valued additive white gaussian noise (AWGN) random process with zero mean and power spectral density (PSD) of \(N_0/(W/Hz)\). The AWGN random processes are assumed to be statistically independent from channel to channel and independent of the channel fading envelopes and phases.

**B. Formulation of Decision Variables**

The receiver of the MC-FSK signals is shown in Fig. 2. The received faded and noisy signals are first passed through \(v\) bandpass matched filters each of which is matched to one of the \(v\) carrier frequencies. The matched filters outputs are sampled every \(T_s\) and passed through square-law devices. Then the resultant random variables over all diversity branches are equally gain combined. The random variables, corresponding

**TABLE I**

<table>
<thead>
<tr>
<th>(k_s)</th>
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<th>(H_1)</th>
<th>(H_2)</th>
<th>(M)</th>
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**Data Source**

- \(k_s\) Bits
- Steiner Design
- Block Encoder
- \(v\)-FSK
- Modulator
- \(v\), \(f_c\)
- \(u_{Ts}\)
- \(f_{sm}\)
- \(n_{l\lambda}(t)\)

**Fig. 1.** Steiner design-based MC-FSK modulator.
to the $m^{th}$ transmitted signal, at the output of the first EGC process becomes

$$
\zeta_m = \left\{ \begin{array}{ll}
\sum_{l=1}^{L} \left| \alpha_{ik} \sqrt{2E_c} + n_{il} \right|^2, & \lambda = m, \\
\sum_{l=1}^{L} \left| n_{il} \right|^2, & \lambda = 1, 2, \ldots, w, \\
\sum_{l=1}^{L} \left| n_{il} \right|^2, & \lambda = w + 1, \ldots, v,
\end{array} \right. $$  
(4)

where the sets $\{n_{il}\}_{l=1}^{L}$ consist of independent identically distributed (i.i.d) complex-valued Gaussian random variables each with zero mean and $N_0$ (W/Hz) PSD, and the sets for $\lambda = 1, 2, \ldots, w, \ldots, v$ are mutually statistically independent of each other. Since each transmitted signal contains $w$ orthogonal components, another EGC process must be performed to attain the actual decision variable for each transmitted signal before performing the decision process. This combining process is referred to as the Steiner system EGC process. Therefore, the decision variables for all signals, assuming that the $m^{th}$ one is transmitted, are given by

$$
\chi_i = \sum_{\lambda=1}^{w} \zeta_{\lambda}, \quad \text{for } i = 1, 2, \ldots, H.
$$  
(5)

Based on (4) and (5), and assuming that the nonorthogonal signals share the frequency number $w$ with the $m^{th}$ transmitted signal, the decision variables for all possible transmitted signals can be written as

$$
\chi_i = \left\{ \begin{array}{ll}
\sum_{l=1}^{L} \sum_{\lambda=1}^{w} \left| \alpha_{ik} \sqrt{2E_c} + n_{il} \right|^2, & i = m, \\
\sum_{l=1}^{L} \left[ \sum_{\lambda=1}^{w-1} \left| n_{il} \right|^2 + \left| \alpha_{ik} \sqrt{2E_c} + n_{im} \right|^2 \right], & i \in \Gamma_1, \\
\sum_{l=1}^{L} \sum_{\lambda=1}^{w} \left| n_{il} \right|^2, & i \in \Gamma_2,
\end{array} \right. $$  
(6)

where $\Gamma_1$ and $\Gamma_2$ are the sets of signals that contain the nonorthogonal and orthogonal signals, respectively. The sets $\{\alpha_{ik}\}_{k=1}^{L}$ and $\{n_{il}\}_{l=1}^{L}$, for $\lambda = 1, 2, \ldots, w$, consist of mutually statistically independent random variables, and the two set are independent of each other. Therefore, (6) can be rewritten as the following

$$
\chi_i = \left\{ \begin{array}{ll}
\sum_{l=1}^{wL} \left| \alpha_{ik} \sqrt{2E_c} + n_{il} \right|^2, & i = m, \\
\sum_{l=1}^{(w-1)L} \left| n_{il} \right|^2 + \sum_{l=1}^{(w-1)L} \left| \alpha_{ik} \sqrt{2E_c} + n_{il} \right|^2, & i \in \Gamma_1, \\
\sum_{l=1}^{wL} \left| n_{il} \right|^2, & i \in \Gamma_2
\end{array} \right. $$  
(7)

where the sets $\{n_{il}\}_{l=1}^{wL}$, $\{n_{il}^{'}\}_{l=1}^{wL}$ and $\{n_{il}^{''}\}_{l=1}^{(w-1)L}$ are independent of each other, and each of which consists of i.i.d Gaussian random variables with zero means and $N_0$ (W/Hz) PSDs. From (7) we can conclude that Steiner design-based MC-FSK signalling in multichannel fading system provides a combined diversity that is directly proportional to the number of unshared orthogonal frequencies per transmitted signal.

### IV. Calculation of the Average Bit Error Probability

The detection process is accomplished based on the Maximum-Likelihood (ML) decision criterion, which selects, assuming that all transmitted waveforms are equally likely and having the same amount of energy, the transmitted signal with the maximum decision variable from those given in (7). Assuming that the $m^{th}$ signal is transmitted, the probability of an error symbol detection can be expressed by

$$
P_e(E) = \Pr \left\{ \bigcup_{i=1}^{H_1} (\chi_m < \chi_{i1}) \bigcup_{i=1}^{H_2} (\chi_m < \chi_{i2}) \right\}.
$$  
(8)

The decision variables $\chi_m$, $\chi_{i1}$, and $\chi_{i2}$ may be interdependent on each other, which makes the exact evaluation of the symbol error probability (SEP) in (8) so difficult [4]. In analogy with the performance evaluation mechanism for coded systems [4], the SEP can be upper bounded as

$$
P_e(E) \leq H_1 \Pr \left( \sum_{l=1}^{(w-1)L} \left[ \left| \alpha_{ik} \sqrt{2E_c} + n_{il} \right|^2 - \left| n_{il} \right|^2 \right] < 0 \right) + H_2 \Pr \left( \sum_{l=1}^{wL} \left[ \left| \alpha_{ik} \sqrt{2E_c} + n_{il} \right|^2 - \left| n_{il} \right|^2 \right] < 0 \right),
$$  
(9)

where $\Pr(\sum_{l=1}^{(w-1)L} \left| \alpha_{ik} \sqrt{2E_c} + n_{il} \right|^2 - \left| n_{il} \right|^2 < 0)$ denotes the pairwise SEP that the transmitted signal is detected in error as one of $H_1$ nonorthogonal waveforms, which will be referred to as the nonorthogonal pairwise SEP, and $\Pr(\sum_{l=1}^{wL} \left| \alpha_{ik} \sqrt{2E_c} + n_{il} \right|^2 - \left| n_{il} \right|^2 < 0)$ represents the pairwise SEP that the transmitted signal is detected in error as one of the $H_2$ orthogonal signals, which will be referred to as the orthogonal pairwise SEP. The nonorthogonal and orthogonal pairwise SEPs, conditioned on the channel fading statistics, can be considered as special cases from the probability of error for binary orthogonal signals with noncoherent detection in multichannel AWGN systems [4]. The conditional pairwise SEPs, in terms of the general Marcum Q-function $Q(x, y)$ [5], [6], can be written as

$$
P_e(E | \gamma_{t_n}) = \frac{1}{2} \left\{ \frac{2}{L_1 - t} \sum_{l=1}^{L_1} \left[ 2L_n - 1 \right] \right\} \times Q_i \left( a \sqrt{\gamma_{t_n}}, b \sqrt{\gamma_{t_n}} \right) - Q_i \left( b \sqrt{\gamma_{t_n}}, a \sqrt{\gamma_{t_n}} \right),
$$  
(10)

where $a = 0, b = 1, n = 1$ or 2, $L_1 = (w-1)L$, and $L_2 = wL$. $P_1(E | \gamma_{t_n})$ is the nonorthogonal conditional pairwise SEP with
instantaneous combined SNR per symbol $\gamma_1 = \sum_{i=1}^{L} \gamma_i$, and $P_2(E|\gamma_{t_2})$ is the orthogonal conditional pairwise SEP with $\gamma_{t_2} = \sum_{i=1}^{L} \gamma_i$, where $\gamma_i = \alpha_i^2 E_c / N_0$ represents the received instantaneous SNR over the $i$th diversity branch.

The expression in (10) is useful because it enables evaluating the average pairwise error probability by using the alternative representations of the generalized Marcum Q-function [5]. Using these alternative representations in (10), and the fact that the fading envelopes are independent, the average pairwise SEPs become

$$\bar{P}_n(E|\{\gamma_i\}_{i=1}^{L}) = \int_0^\infty \int_0^\infty \cdots \int_0^\infty \prod_{i=1}^{L} \gamma_i^{\frac{1}{2}} d\gamma_2 \cdots d\gamma_L \times \prod_{i=1}^{L} \psi_i \left( \frac{1}{2} \right) d\varphi,$$

where $\xi \to 0^+$, $\psi_i(s)$ is the moment generating function (MGF) of $\gamma_i$, and $\bar{\gamma}_b$ is the average SNR, where $\bar{\gamma}_b$ is the average SNR per bit of $i$th diversity branch. The expression of $F(L_n;\xi;\varphi)$ is given by

$$F(L_n;\xi;\varphi) = \sum_{i=1}^{L_n} \left( \frac{2L_n - 1}{L_n - t} \right) \prod_{i=1}^{L_n} \left[ \left( \xi^{-1} - (\xi^2)^{+1} \right) \cos \left( (t - 1)(\varphi + \pi/2) \right) - \left( -\xi^2 + \xi^1 \right) \cos \left( t(\varphi + \pi/2) \right) \right].$$

Substituting (11) into (9) gives the final expression of the upper bound on the average SEP for MC-FSK signals transmitted over non-identically distributed fading channels.

$$\bar{P}_1(E) \leq H_1 \bar{P}_1(E|\{\gamma_i\}_{i=1}^{L}) + H_2 \bar{P}_2(E|\{\gamma_i\}_{i=1}^{L}).$$

The average BEP is related to the average SEP by

$$\bar{P}_b(E) = \frac{2^{k_s-1}}{2^{k_s}-1} \bar{P}_s(E).$$

It should be noted that an upper bound on the average BEP for $M$-ary FSK system can be obtained from that for MC-FSK system by substituting one instead of the number of orthogonal components per transmitted fading channel (i.e., $w = 1$). Using the fact that $H_1 = 0$ and $H_2 = 2^{k_s} - 1$ in the $M$-ary FSK system, the upper bound on the average BEP becomes

$$\bar{P}_b(E) \leq \frac{2^{k_s-1}}{2^{k_s}-1} \bar{P}_2(E|\{\gamma_i\}_{i=1}^{L})$$

where $\bar{P}_2(E|\{\gamma_i\}_{i=1}^{L})$ is given by (11) with $L_2 = L$.

VI. THEORETICAL BANDWIDTH EFFICIENCY

Using the minimum frequency separation (i.e., $\Delta f = 1/T_s$), MC-FSK system requires approximately a baseband bandwidth of $LT_s/T_c$ (Hz) to transmit $M$ signal waveforms over $L$ diversity channels, whereas $M$-ary FSK system requires at least $LM/T_s$ (Hz) of bandwidth to transmit the same number of signals. Therefore, the theoretical bandwidth efficiencies of MC-FSK and $M$-ary FSK systems, which are defined as the ratio of the bit rate $R_b = 1/T_b$ to $W$, are respectively, given by

$$\eta_{MC-FSK} = \log_2 M / Lw$$

and

$$\eta_{M-FSK} = \log_2 M / LM,$$

where $M = 2^{k_s}$, is the symbol size.

It is apparent from Table I that by changing the value of $w$ in the MC-FSK system, different number of orthogonal frequencies are required to represent all transmitted signals at a given value of $k_s$. Thus, the value of the combined diversity order $L_c$ can be obtained using different combinations of $w$ and $L$, and hence, MC-FSK system can provide different bandwidth efficiencies at the same diversity order (or equivalently power efficiency). For example, MC-FSK systems at $k_s = 7$ with the combinations $(w = 3, L = 3)$ and $(w = 4, L = 2)$ provides $L_c = 6$ combined diversity order but the later combination provides 16.25% more bandwidth efficiency than that provided by the former. This feature, which is not available with the $M$-ary FSK system, enables selecting the appropriate combination of $w$ and $L$ that assures the best bandwidth efficiency at a given required diversity order (power efficiency).

VI. NUMERICAL RESULTS

In this section, we compare the performance of the MC-FSK system with that of the FSK system. The performance comparison is based on evaluating both the bandwidth and power efficiencies.

Fig. 3 shows the average BEPs of the MC-FSK and $M$-ary FSK systems as functions of the average SNR per bit of first diversity path ($\bar{\gamma}_b$) with $k_s = 5$ over Nakagami-$m$ fading channels having fading parameter $m = 1$ and assuming uniform power delay profile ($\bar{\gamma}_m = \bar{\gamma}$) using $L$ and $w$ as parameters. From this figure we note that MC-FSK system with $L = 1$ approximately provides the same power efficiency of the $M$-ary FSK system having $L = w - 1$. This observation is clear by comparing between the curves (1,2), (3,4), and (5,6). Another observation is that the performance gap between each pair of the curves mentioned above is directly proportional to $w$, and it becomes almost negligible as $w$ increases, at which MC-FSK system requires more carrier frequencies $\nu$ to represent all data symbols, thereby increasing the system power efficiency but at the expense of its bandwidth efficiency. The effectiveness of the derived upper bound formula for the average BEP of MC-FSK system can be justified by noting that MC-FSK with $w = 41$ (curve 5) performs exactly similar to 32-ary FSK system (curve 6) especially for high SNRs.

Beside the power efficiency, the bandwidth efficiency is of prime importance in comparing the performance of communication systems specially over bandwidth-limited channels. For the case of $w = 3$ and $L = 1$ (combined diversity order of $L_c = 2$), MC-FSK system provides a bandwidth efficiency that is 64/15 times that of 32-ary FSK having $L = 2$ with comparable power efficiency (see curves 1 and 2 in Fig. 3). The power efficiency of MC-FSK system can be increased by increasing the value of $L$, which is negatively affecting the system bandwidth efficiency. For example, MC-FSK with $w = 3$ and $L = 3 (L_c = 6)$ provides a bandwidth efficiency that is 64/45, 96/45, and 128/45 times that of 32-ary FSK with $L = 2, L = 3, L = 4$, respectively, with better power efficiency in all cases (compare curve 7 with curves 2, 4, and 6 in Fig. 3).
In general, MC-FSK provides better power efficiency than that of M-ary FSK system whenever its combined diversity order is larger. Moreover, at same combined diversity orders, both MC-FSK and M-ary FSK systems approximately provide same power efficiency, but MC-FSK is considered as a more bandwidth efficient compared to M-ary FSK system.

Fig. 4 illustrates the effects of the fading parameter \( m \) and the exponentially decaying power delay profile \( (\gamma_0 = \bar{\gamma}_0 e^{-\rho(t-1)}, t = 1, 2, \ldots, L_n) \) on the average BEP performance of MC-FSK with \( k_s = 6, L = 1, \) and \( w = 3 \) and 4 over Nakagami-\( m \) fading system. Clearly, as \( m \) increases from 2 to 8, significant improvement on performance is achieved because that the severity of fading channels is inversely proportional to the value of \( m \). Moreover, MC-FSK system with \( w = 4 \) provides better power efficiency than that provided for the case when \( w = 3 \), where this improvement is more obvious for small \( m \). This can be explained by noting that MC-FSK system having \( w = 4 \) provides a combined diversity order \( L_c \) that is greater than that when \( w = 3 \), where the improvements achieved by increasing \( L_c \) are more noticeable for the case of sever fading channels (small \( m \)). Finally, compared with the system having uniform power delay profile (\( \rho = 0 \)), MC-FSK with exponentially decaying power delay profile (\( \rho > 0 \)) introduces non-negligible amount of degradation on the system performance, which obviously increases by increasing \( \rho \), \( L_c \) and/or \( m \). This degradation in system performance is due to the fact that for the case of channels with exponentially decaying power delay profile, the combined average SNR is smaller than that for the case of channels with uniform power delay profile.

VII. CONCLUSION

In this paper, we derived an expression for the average BEP of the MC-FSK signals over generalized fading channels. The derived expression was expressed in terms of MGF of the received combined SNR, and is applicable for the case of non-identically distributed fading branches. We compared, in terms of the power and bandwidth efficiencies considering the effects of the severity of channels and the exponentially decaying power delay profile, between the MC-FSK and M-ary FSK systems. The results showed that MC-FSK approximately provides same power efficiency, but better bandwidth efficiency, than that of M-ary FSK when their equivalent diversity orders are equal. Moreover, MC-FSK provides both better power and bandwidth efficiencies than those of M-ary FSK employing the same diversity level. In addition, MC-FSK can approximately provide the same power efficiency by using different combinations of the number of orthogonal components and the diversity branches, which enables flexibility in choosing the suitable combination that assures the best bandwidth efficiency at a given required average BEP, where this property can not be provided by M-ary FSK system.

REFERENCES