Joint Subcarrier Pairing and Power Allocation for DF-Relayed OFDM Cognitive Systems

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Abstract—In this paper, the problem of resource allocation in OFDM based decode-and-forward cognitive radio (CR) system is considered. The objective is to maximize the CR system throughput by optimizing the subcarrier pairs and powers. The interference introduced to the primary system should not exceed the prescribed interference temperature limit. The resources are optimized jointly by applying the dual decomposition technique to get asymptotically optimal solution. In addition, a suboptimal algorithm that performs the resources allocation in two phases is proposed. By the suboptimal algorithm, the subcarrier pairing is first performed considering the channels quality as well as the interference introduced to the primary system. Afterwards, the powers are allocated in the second phase by applying a waterfilling-like algorithm. The suboptimal algorithm reduces significantly the computational complexity with small performance degradation. The simulation results confirm the efficiency of the proposed algorithm.

I. INTRODUCTION

Most of current frequency bands have been already allocated and it will be very hard to find vacant bands for the emerging wireless systems or services. Cognitive radio (CR) has been developed in order to solve the spectrum scarcity problem by allowing a group of secondary users (SUs) to use the vacant channels left by the licensed users, also called primary users (PUs). Multicarrier communications has been suggested as a candidate for the CR systems [1].

Relays (R) are used to assist the source (S) to destination (D) transmission in order to increase the system coverage and achievable capacity. The resource allocation problem in non-cognitive OFDM based relay system has been widely studied. In [2], the optimal power allocation was given in order to minimize the system outage probability in dual-hop and in multi-hop relays. Lin et al. proposed in [3] two efficient sub-optimal resource allocation algorithms in OFDMA-based decode-and-forward (DF) relay systems. In [4], optimal power allocation for DF-Relayed dual hop OFDM system is derived by fixing the subcarrier pairs under two different transmission protocols. In [5], joint subcarrier pairing and power allocation algorithm is proposed under total and individual power constraints. However, in case of cognitive systems, the different resources should be distributed adequately in order to maximize the capacity of the CR without causing harmful interference to the primary system. An overview of the cooperative communications in cognitive scenario has been presented in [6]. In [7], a power allocation algorithm in DF-OFDM based CR system has been proposed. Under the assumption of perfect subcarrier matching in the S-R and R-D links, the authors treated the optimization problem in S and R individually. The algorithm performance degrades significantly if the relay has to forward the receiving message on the same subcarrier, i.e. there is no subcarrier pairing.

In this paper, DF-relayed OFDM based CR system is considered. The subcarrier pairs and powers are optimized jointly in order to maximize the total system throughput without causing excessive interference to the primary system. The main contributions of this paper are as follows: 1) The resource allocation problem is formulated for the DF relaying strategy taking the interference constraints into consideration. Then, the original problem is reformulated so that the dual decomposition technique can be used to find the subcarrier pairs and powers jointly. 2) Due to the high computational complexity of the optimal solution, an efficient suboptimal algorithm is presented to perform subcarrier pairing and power allocation in two different phases making the proposed power allocation algorithm suitable when the system doesn’t allow the subcarrier pairing. The rest of this paper is organized as follows. Section II gives the system model while the problem is formulated and the optimal power allocation is derived in Section III. The sub-optimal scheme is presented in Section IV. Selected numerical results are discussed in Section V. Finally, Section VI concludes the paper.

II. SYSTEM MODEL

In this paper, an OFDM-based relay CR will be considered. As shown in Fig. 1, the CR relay system coexists with the
primary system in the same geographical location. Due to the existence of an obstacle or a large distance, there is no direct link between S and D so that S tries to communicate with D through R. The CR system’s frequency spectrum is divided into \( N \) subcarriers each having a \( \Delta f \) bandwidth. It is assumed that the CR system can use the inactive PU bands provided that the total interference introduced to the PU band does not exceed the maximum interference power that can be tolerated by PU, \( I_{th} \). The relay is assumed to be half-duplex, thus receiving and transmitting in two different time slots. In the first time slot, S transmits to R over the \( j \)th subcarrier while in the second time slot R decodes the message, re-encodes it and then forwards it to D over the \( k \)th subcarrier which may not be the same as \( j \) and they form the subcarrier pair \((j,k)\).

Assume that \( \Phi_i \) is the power spectrum density (PSD) of the \( i \)th subcarrier in S or R. The expression of the PSD depends on the used multicarrier technique. If an OFDM based CR is assumed, the PSD of the \( i \)th subcarrier can be written as [8]

\[
\Phi_i(f) = P_i T_s \left( \frac{\sin \pi f T_s}{\pi f T_s} \right)^2 \tag{1}
\]

where \( P_i \) is the total transmit power emitted by the \( i \)th subcarrier and \( T_s \) is the symbol duration. The mutual interference introduced by the \( i \)th subcarrier to PU, \( I_i(d_i, P_i) \), is the integration of the PSD of the \( i \)th subcarrier across the PU band, \( B \), and can be expressed as [8]

\[
I_i(d_i, P_i) = \int_{d_i-B/2}^{d_i+B/2} G_i \Phi_i(f) \, df = P_i \Omega^i \tag{2}
\]

where \( d_i \) is the spectral distance between the \( i \)th subcarrier and the PU band. \( G_i \) denotes the channel gain between the \( i \)th subcarrier and the PU band while \( \Omega^i \) denotes the interference factor of the \( i \)th subcarrier to the PU band.

By the same way, the interference power introduced by PU signal into the band of the \( j \)th subcarrier is [8]

\[
J_j = \int_{d_j-\Delta f/2}^{d_j+\Delta f/2} Y_j \Upsilon(e^{j\omega}) \, d\omega \tag{3}
\]

where \( \Upsilon(e^{j\omega}) \) is the power spectrum density of PU signal and \( Y_j \) is the channel gain between the \( i \)th subcarrier and the PU signal.

III. Problem Formulation and Optimal Solution

The transmission rate of the \( j \)th subcarrier in the source coupled with \( k \)th subcarrier in the relay, \( R(j,k) \), can be evaluated as follows

\[
R(j,k) = \frac{1}{2} \min \left\{ \log_2 \left( 1 + \frac{P_{jSR} H_{jSR}}{\sigma^2_{AWGN}} \right), \log_2 \left( 1 + \frac{P_{kRD} H_{kRD}}{\sigma^2_{AWGN}} \right) \right\} \tag{4}
\]

where \( P_{jSR}(P_{kRD}) \) is the power transmitted over the \( j \)th\((k \)th\) subcarrier in the S-R(R-D) link while \( H_{jSR}(H_{kRD}) \) is the \( j \)th\((k \)th\) subcarrier fading gain over S-R(R-D) link.

\[
\sigma^2_R(j,k) = \sigma^2_{AWGN} + J_{j(k)} \quad \text{where } \sigma^2_{AWGN} \text{ is the variance of the additive white Gaussian noise (AWGN) and } J_{j(k)} \text{ is the interference introduced by the PU signal into the } j \text{th}(k \text{th}) \text{ subcarrier which is evaluated using (3) and can be modeled as AWGN as described in [9].}
\]

To make the analysis more clear and without loss of generality, the noise variance is assumed to be constant for all subcarriers, i.e. \( \sigma^2 = \sigma^2_{AWGN} \).

Our objective is to maximize the CR system throughput by optimizing the subcarrier pairing and powers while guaranteeing that the instantaneous interference introduced to the primary system is below the maximum limit. Therefore, the optimization problem can be formulated as follows

\[
\begin{align*}
\max_{P_{jSR} > 0, P_{kRD} > 0, t_{j,k}} & \sum_{j=1}^{N} t_{j,k} R(j,k) \\
\text{s.t.} & \sum_{k=1}^{N} P_{jSR}^i \Omega_{SP}^i \leq I_{th} \\
& \sum_{k=1}^{N} P_{kRD}^k \Omega_{RD}^k \leq I_{th} \\
& \sum_{k=1}^{N} t_{j,k} \leq 1, \forall j; \sum_{j=1}^{N} t_{j,k} \leq 1, \forall k
\end{align*}
\]

where \( N \) denotes the total number of subcarriers while \( I_{th} \) is the interference threshold prescribed by PU. \( \Omega_{SP}^i \) and \( \Omega_{RD}^k \) are the \( j \)th\((k \)th\) subcarrier interference factor to the PU band from S and R respectively. The subcarrier pairing constraint ensures that each subcarrier in S is paired with only one subcarrier in R where \( t_{j,k} \in \{0,1\} \) is the subcarrier pairing indicator, i.e. \( t_{j,k} = 1 \) if the \( j \)th subcarrier in S is paired with the \( k \)th in R and zero otherwise. S will be in charge of the resources allocation where all the instantaneous fading gains are assumed to be perfectly known. Note that the channel gains between the CR system parts (S, R and D) can be obtained practically by the classical channel estimation techniques while the channel gains between the CR system and the PU can be obtained by estimating the received signal power from the primary terminal when it transmits, under the assumptions of pre-knowledge on the primary transmit power levels and the channel reciprocity [10].

From (4), the maximum capacity over a given subcarrier pair \((j,k)\) can be achieved when

\[
P_{jSR}^i H_{jSR}^i = P_{jRD}^k H_{jRD}^k \tag{6}
\]

Therefore, the power allocated at R can be expressed as function of the power at S as follows

\[
P_{kRD}^k = \frac{P_{jSR}^i H_{jSR}^i}{H_{jRD}^k} \tag{7}
\]

Hence, the optimization problem in (5) can be re-written as
follows
\[
\max_{P_{SR}^j > 0, t_{j,k}} \sum_{j=1}^{N} \sum_{k=1}^{N} \frac{1}{2} t_{j,k} \log \left( 1 + \frac{P_{jSR}^j H_{SR}^j}{a^2} \right) \\
\text{s.t.} \sum_{j=1}^{N} \sum_{k=1}^{N} P_{jSR}^j \Omega_j \leq I_{th}; \\
\sum_{j=1}^{N} \sum_{k=1}^{N} t_{j,k} \frac{P_{jSR}^j H_{SR}^j}{H_{RD}^j} \Omega_k \leq I_{th} \\
\sum_{j=1}^{N} \sum_{k=1}^{N} t_{j,k} \leq 1, \forall j; \sum_{j=1}^{N} \sum_{k=1}^{N} t_{j,k} \leq 1, \forall k
\] (8)

Finding the optimization variables \(P_{jSR}^j\) and \(t_{j,k}\) in (8) is a mixed binary integer programming problem. There are \(N!\) subcarrier pairing possibilities and hence the complexity is prohibitive for large number of subcarrier. The problem in (8) is satisfying the time sharing condition described in [11] and hence, the duality gap of the problem is negligible as the number of subcarrier is sufficiently large regardless of the convexity of the problem. The solution obtained by the dual method is asymptotically optimal [11].

The dual function \(g(\beta, \mu)\) of the problem (8) can be written as follows
\[
g(\beta, \mu) \triangleq \max_{P_{SR}^j > 0, t_{j,k}} L \left( P_{SR}^j, t_{j,k}, \beta, \mu \right) \\
\text{s.t.} \sum_{k=1}^{N} t_{j,k} \leq 1, \forall j; \sum_{j=1}^{N} t_{j,k} \leq 1, \forall k
\] (9)

where the Lagrangian is
\[
L \left( P_{SR}^j, t_{j,k}, \beta, \mu \right) = \sum_{j=1}^{N} \sum_{k=1}^{N} \frac{1}{2} t_{j,k} \log \left( 1 + \frac{P_{jSR}^j H_{SR}^j}{a^2} \right) \\
+ \beta \left( I_{th} - \sum_{j=1}^{N} P_{SR}^j \Omega_j \right) + \mu \left( I_{th} - \sum_{j=1}^{N} \sum_{k=1}^{N} t_{j,k} \frac{P_{jSR}^j H_{SR}^j}{H_{RD}^j} \Omega_k \right)
\] (10)

where \(\beta\) and \(\mu\) are the dual variables associated with the interference constraint at S and R respectively. Therefore, the dual problem can be written as
\[
\min_{\beta \geq 0, \mu \geq 0} g(\beta, \mu)
\] (11)

The subgradient method can be used to solve the dual problem with guaranteed convergence. Assume that \(P_{SR}^{i+1}\) and \(t_{j,k}^{i+1}\) are the optimal solutions of the dual function at a given dual points \(\beta\) and \(\mu\), then, the dual variables at the \((i+1)th\) iteration are updated as
\[
\beta^{i+1} = \beta^{i} - \delta(i) \left( I_{th} - \sum_{j=1}^{N} P_{SR}^{i} \Omega_j \right) \\
\mu^{i+1} = \mu^{i} - \delta(i) \left( I_{th} - \sum_{k=1}^{N} \sum_{j=1}^{N} t_{j,k}^{i} \frac{P_{jSR}^{i} H_{SR}^i}{H_{RD}^i} \Omega_k \right)
\] (12)

where \(\delta(i)\) can be updated according to the nonsummable diminishing step size policy.

In order to find the optimal solution of the dual function at a given dual variables \(\beta\) and \(\mu\), the dual function in (9) can be rewritten as follows
\[
g(\beta, \mu) = \max_{P_{SR}^j > 0, t_{j,k}} \left[ \sum_{j=1}^{N} \sum_{k=1}^{N} t_{j,k} D \left( P_{jSR}^j, k \right) + I_{th} (\beta + \mu) \right] \\
\text{s.t.} \sum_{j=1}^{N} t_{j,k} \leq 1, \forall k; \sum_{k=1}^{N} t_{j,k} \leq 1, \forall j
\] (13)

where
\[
D \left( P_{jSR}^j, k \right) = \frac{1}{2} \log \left( 1 + \frac{P_{jSR}^j H_{SR}^j}{\sigma^2} \right) - \beta P_{jSR}^j \Omega_j - \mu \frac{P_{jSR}^j H_{SR}^j}{H_{RD}^j} \Omega_k
\] (14)

Hence, the optimal power allocation and subcarrier pairing can be found in two steps as follows

A. Optimal Power Allocation for a Given Subcarrier Pair

Assume that \((j, k)\) is a valid subcarrier pair and is already matched. Hence, (13) is decomposed into \(N\) independent power allocation subproblems. The optimal power allocation can be determined by solving the following subproblem
\[
\max_{P_{jSR}^j} D \left( P_{jSR}^j, k \right) \\
\text{s.t.} \ P_{jSR}^j \geq 0
\] (15)

for every \((j, k)\) pair. Solving (15) for the optimal power we can find
\[
P_{jSR}^* = \left[ \frac{1}{\beta \Omega_j + \mu \frac{H_{SR}^j}{H_{RD}^j} \Omega_k} - \frac{\sigma^2}{H_{SR}^j} \right]^{+}
\] (16)

where \([x]^+ = \max(0, x)\).

B. Optimal Subcarrier Pairing

Substituting the optimal power allocation expression in (20) into (13) to eliminate the power variable, we obtain the corresponding dual function
\[
g(\beta, \mu) = \max_{t_{j,k}} \left[ \sum_{j=1}^{N} \sum_{k=1}^{N} t_{j,k} D \left( P_{jSR}^j, k \right) + I_{th} (\beta + \mu) \right] \\
\text{s.t.} \sum_{j=1}^{N} t_{j,k} \leq 1, \forall k; \sum_{k=1}^{N} t_{j,k} \leq 1, \forall j
\] (17)

By defining the \(N \times N\) profit matrix \(D = [D \left( P_{jSR}^j, k \right)]\) for every \((j, k)\) pair, the objective in (17) can be maximized by picking elements from the matrix \(D\) such that the sum of profits is as large as possible. This is a linear assignment problem that can be solved efficiently by the Hungarian method with a complexity of \(O \left( N^3 \right) \) [12].

IV. PROPOSED SUB-OPTIMAL ALGORITHM

Suppose that the complexity of the dual variables update in (12) is polynomial in the number of dual variables, i.e. \(2^n\) and the number of iterations required to converge is \(M\). Then, by including the computational complexity of the Hungarian method, the asymptotically optimal solution derived in the previous section has a complexity of \(O \left( 2^n MN^3 \right) \) which
may not be efficient with high number of subcarriers [11]. In order to solve the problem efficiently, we propose in this section a suboptimal algorithm by which the resource allocation problem is solved in two phases. In the first phase the subcarrier pairs are determined while the power is allocated to the different pairs in the second phase. The detailed description of the proposed suboptimal scheme follows.

### A. Proposed Subcarrier Pairing Algorithm

The optimal subcarrier pairing strategy in non-cognitive OFDM based system is achieved by ordering the subcarriers in S and R according to their signal to noise ratio (SNR) and pair the subcarriers with same order together. This strategy is not optimal in CR system due to the existence of the interference constraints. Therefore, any suggested pairing scheme should consider the channel quality as well as the interference constraints. Therefore, any suggested pairing scheme should be suboptimal by which the resource allocation problem is solved in two phases. In the first phase the subcarriers in S and R are ordered according to their signal to noise ratio (SNR) and pair the subcarrier in S with the th subcarrier in R. Note that the allocated powers according to (18) are only used to allocate the k th subcarrier in S(R) is

\[
P^k_{SR}(P^k_{RD}) = \frac{I_{th}}{N\Omega_{SP}(\Omega_{RP})}\]  

(18)

Afterwards, the subcarriers in S and R are ordered according to the product of the powers by the channel gains, i.e. \( P^i_{SR}(P^i_{RD}) \times H^i_{SR}(H^i_{RP}) \). Then, every subcarrier in S will be matched with the subcarrier with the same order in R. Note that the allocated powers according to (18) are only used to find the different pairs. The optimal power allocation will be derived next based on the subcarrier pairing information.

### B. Proposed Power Allocation Algorithm

By applying the subcarrier pairing algorithm and ordering the subcarrier in S and R, the subcarrier index in S and R can be changed from \( j \) and \( k \) to \( i \) for notation simplicity, i.e. the \( j \) th subcarrier in S is paired with the \( i \) th subcarrier in R. Therefore, the power optimization problem can be written as follows

\[
\max_{P^i_{SR} > 0} \sum_{i=1}^{N} \frac{1}{2} \log_2 \left( 1 + \frac{P^i_{SR}H^i_{SR}}{\sigma_i^2} \right) \\
\text{s.t.} \sum_{i=1}^{N} P^i_{SR} \Omega^i_{SP} \leq I_{th}; \\
\sum_{i=1}^{N} P^i_{SR} \frac{H^i_{SR}}{H^i_{RD}} \Omega^i_{RP} \leq I_{th};
\]  

(19)

The above problem is a convex optimization problem. Applying the KKT conditions and solving for the optimal power, we can get

\[
P^*_{SR} = \left[ \frac{1}{\eta \Omega_{SP} + \gamma \frac{H^i_{SR}}{H^i_{RD}} \Omega_{RP}} - \frac{\sigma_i^2}{H^i_{SR}} \right]^+
\]  

(20)

where \( \eta, \gamma \) are the non-negative Lagrange multipliers. Solving for multiple Lagrange multipliers is still computationally complex. We further develop a computationally efficient algorithm to allocate the different powers by applying a waterfilling-like solution. Starting by ignoring the second constraint corresponds to the interference in the second time slot, the following problem can be formulated

\[
\max_{P^i_{SR(T1)} > 0} \sum_{i=1}^{N} \frac{1}{2} \log_2 \left( 1 + \frac{P^i_{SR(T1)}H^i_{SR}}{\sigma_i^2} \right) \\
\text{s.t.} \sum_{i=1}^{N} P^i_{SR(T1)} \Omega^i_{SP} \leq I_{th}
\]  

(21)

where \((T1)\) stands for optimization under the interference constraint in the first time slot only. Following the analysis given in [9], we get

\[
P^i_{SR(T1)} = \left[ \frac{1}{\lambda_1 \Omega_{SP}} - \frac{\sigma_i^2}{\Omega_{SR}} \right]^+
\]  

(22)

where \( \lambda_1 \) is the non-negative Lagrange multiplier. Similarly, ignoring the interference constraint in the first time slot will lead to the following optimization problem

\[
\max_{P^i_{SR(T2)} > 0} \sum_{i=1}^{N} \frac{1}{2} \log_2 \left( 1 + \frac{P^i_{SR(T2)}H^i_{SR}}{\sigma_i^2} \right) \\
\text{s.t.} \sum_{i=1}^{N} P^i_{SR(T2)} \frac{H^i_{SR}}{H^i_{RD}} \Omega_{RP} \leq I_{th}
\]  

(23)

where \((T2)\) stands for optimization under the interference constraint in the second time slot only. The solution can be given as follows

\[
P^i_{SR(T2)} = \left[ \frac{1}{\lambda_2 \Omega_{SR} \Omega_{RP}} - \frac{\sigma_i^2}{\Omega_{SR}} \right]^+
\]  

(24)

where \( \lambda_2 \) is the non-negative Lagrange multiplier.

We can start by assuming that the maximum power that can be allocated to each subcarrier \( P_{SR}^{max} \) is determined according to the interference in the first time slot only, i.e. \( P_{SR}^{max} = P_{SR(T1)} \). Afterwards, the interference in the second time slot is tested to check whether \( \sum_{i=1}^{N} \left( \frac{P_{SR}^{max} \Omega^i_{SR} \Omega^i_{RP}}{H^i_{RD}} \right) \leq I_{th} \) holds or not. If the relation holds, then the solution is found where \( P_{SR}^{max} = P_{SR}^{max} \). Otherwise, the power should be distributed according to the interference in the second time slot only given that the power allocated to each subcarrier is lower than or equal to \( P_{SR}^{max} \). Hence, the following problem should be solved

\[
\max_{P_{SR}^{max}} \sum_{i=1}^{N} \frac{1}{2} \log_2 \left( 1 + \frac{P_{SR}^{max} \Omega^i_{SR} \Omega^i_{RP}}{H^i_{RD}} \right) \\
\text{s.t.} \sum_{i=1}^{N} \frac{P_{SR}^{max} \Omega^i_{SR} \Omega^i_{RP}}{H^i_{RD}} \leq I_{th} \\
0 \leq P^i_{SR} \leq P_{SR}^{max}
\]  

(25)

The former problem can be solved efficiently by using the concept of the “cap-limited” waterfilling [13]. Given the initial
solution evaluated by (24), the channels that violate the maximum power $P_{\text{max}}^i$ are determined and upper bounded with $P_{\text{max}}^i$. The total interference $I_{th}$ is reduced by subtracting the interference induced by the powers assigned so far. At the next step, the algorithm proceeds to successively applying of (24) over the subcarriers that did not violate the maximum power $P_{\text{max}}^i$ in the last step. These procedures are repeated until the allocated power $P_{SR(\text{max})}^i$ doesn’t violate the maximum power $P_{\text{max}}^i$ in any of the subcarriers in the new iteration.  

Hence, the complexity of the proposed power allocation algorithm has a sorting complexity, i.e. $O(N \log N)$. Steps 2 and 7 of the algorithm execute the "cap-limited" waterfilling with a complexity of $O(N \log N)$ [13]. Steps 6 has a complexity of $O(|S| \log |S|) \leq O(N \log N)$. Hence, the complexity of the proposed power allocation algorithm is $O(N \log N)$. Additionally, the proposed subcarrier pairing algorithm has a sorting complexity, i.e. $O(N \log N)$ and hence the overall computational complexity of the proposed subcarrier pairing and power allocation algorithm is $O(N \log N)$.  

V. SIMULATIONS RESULTS  

The simulations are performed under the scenario given in Fig.1. An OFDM system of $N = 64$ subcarriers is assumed. The values of $T_s$, $\Delta f$, and $\sigma_i^2$ are assumed to be 4 ps, 0.3125 MHz and $10^{-6}$ respectively. The channel gains are outcomes of independent Rayleigh distributed random variables with mean equal to 1. All the results have been averaged over 1000 iterations.  

Algorithm 1 Power Allocation Algorithm  

1) Initialize $N = \{1, 2, \cdots , N\}$, $I_{\text{residual}} = 0$ and $S = \emptyset$.  
2) Sort $\{\psi_i = \frac{P_i^\text{opt} \sigma_i^2}{H_{SR}^i}, i \in N\}$ in decreasing order with $k$ being the sorted index. Find the $P_{\text{max}}^i$ as follows:  
   a) $\psi_{\text{sum}} = \sum_{i \in N} \psi_i$, $\lambda_1 = |N| / (I_{th} + \psi_{\text{sum}})$, $n = 1$.  
   b) while $\lambda_1 > \psi_{k(n)}^{-1}$ do  
      $\psi_{\text{sum}} = \psi_{\text{sum}} - \psi_{k(n)}$, $N = N \setminus \{k(n)\}$,  
      $\lambda_1 = |N| / (I_{th} + \psi_{\text{sum}})$, $n = n + 1$  
   end while  
   c) Set $P_{SR(T1)}^i = \left[\frac{1}{N_i \Omega_{SP}^i} - \frac{\sigma_i^2}{P_{SR}^i}\right]^+$  
3) Let $P_{\text{max}}^i = P_{SR(T1)}^i$  
4) if $\sum_{i = 1}^N (P_i^\text{max} H_{SR}^i \Omega_{RP}^i) / H_{RD}^i \leq I_{th}$  
   Let $P_{SR}^i = P_{\text{max}}^i$ and stop the algorithm.  
end if  
5) Solve (25) and find the set $S \subseteq N$ where $P_{SR(\text{max})}^i = P_{\text{max}}^i$.  
6) Evaluate $I_{\text{residual}} = I_{th} - \sum_{i = 1}^N P_{SR(\text{max})}^i \Omega_{SP}^i$, set $N = S$, $I_{th} = I_{\text{residual}} + \sum_{i \in S} P_{SR(\text{max})}^i \Omega_{SP}$ and apply again only steps (2 – 3) to update $P_{\text{max}}^i$.  
7) Solve again (25) and set $P^*_{SR} = P_{SR(\text{max})}^i$.  

Starting by assuming that the CR system doesn’t allow the subcarrier pairing, i.e. the data transmitted by S over a given subcarrier in the first time slot will be forwarded by R over the same subcarrier in the second time slot. Fig. 2 plots the average capacity of the CR system vs. the interference threshold. Since the subcarrier pairing is not allowed, optimal algorithm means that the powers are allocated using (20) while proposed means the power allocation algorithm presented in Section IV-B. Individual algorithm applies the method proposed in [7] which allocates the powers in S and R independently. One can observe that the CR system throughput increases with the interference constraint as the CR system become able to use more power on the different subcarriers. Moreover, Fig. 2 depicts the near optimal performance of the proposed power allocation while showing the limited performance of the individual power allocation in comparison with the optimal solution. Fig. 3 plots the average capacity of the CR system vs. the interference constraint when the subcarrier pairing is allowed. Optimal algorithm pairs the subcarriers and allocates the powers according to the dual problem solution presented in Section III while proposed applies the algorithms described in Sections IV-A and IV-B. The subcarrier pairing with the individual algorithm is performed by multiplying the resulted powers by the channel gains and then match the different subcarriers in order to try satisfying (6). It can be noted that the performance of the system can be enhanced by allowing the subcarrier pairing. Moreover, Fig. 3 shows the efficiency of
using the proposed low computational complexity suboptimal algorithm. Additionally, the proposed algorithm outperforms the SNR pairing strategy used in non-CR systems which reveals the importance of the consideration of the interference constraint in the subcarrier pairing in CR systems.

VI. CONCLUSION

In this paper, the problem of resource allocation in decode and forward OFDM based CR system has been tackled. The objective was to maximize the system throughput of the CR system subject to the interference introduced to the primary system. The problem is solved by using dual decomposition technique in order to determine jointly the subcarrier pairs and powers. Due to the high computational complexity of the optimal scheme, a low computational complexity suboptimal algorithm is presented. The proposed suboptimal algorithm performs the resource allocation in two steps. In the first step, the subcarriers are paired according to their channel quality as well as the interference that can be induced to the primary system. Afterward, the powers are allocated to the different pairs by optimizing jointly the powers in the source and the relay. Its shown that the suboptimal algorithm has a near optimal performance with much less complexity and its observed that the proposed scheme outperforms the SNR based and individual schemes presented in the literature. We are currently working on the extension of the proposed system by considering more interference and power constraints as well as considering multiple relay nodes.

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