Optimal and Suboptimal Resource Allocation For Two-Hop OFDM-Based Multi-Relay Cognitive Networks

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Abstract—We consider a cognitive relay network with multi-relay nodes where the cognitive network transmits over the unused spectrum bands originally allocated to the primary system. This paper presents an optimal and suboptimal schemes that optimize the subcarrier pairs, relay selection and power allocation so that the total system capacity is maximized while guaranteeing that the interference introduced to the primary system is not harmful. The optimal scheme adopts the dual decomposition technique to allocate the different resources. In order to reduce the computational complexity of the optimal scheme, a greedy suboptimal algorithm is proposed. The suboptimal algorithm jointly allocates the different resources by considering channels quality, interference to the primary system and relaying strategy. The performance of the different scheme is discussed through the numerical simulations.

I. INTRODUCTION

The rapid growth of wireless communications and the current inflexible spectrum policy make the spectrum scarcity one of the major problem. Cognitive radio (CR) has been proposed as a possible solution for this problem by enabling the secondary users (SU), also called unlicensed users, to periodically sense the frequency spectrum to detect and transmit over the unused spectrum bands left by the primary users (PU), also called licensed users. The CR should not disturb the operation of PU or negatively altering its performance and hence careful allocation of the different CR resources is required. To further improve the spectrum usage and enhance the CR performance, cooperative communications can be used so that the different relays collaborate to detect the open spectrum and assist the network transmission [1] [2].

Recently, resource allocation problem in cooperative CR systems has gained wide attention. Mietzner et al. developed in [3] a fully decentralized and a distributed feedback-assisted power allocation schemes to maximize the output signal to interference plus noise ratio (SINR) or minimize the overall transmit power subject to predefined SINR target. In [4], the CR network use the same spectrum of the primary network so that the transmission time and power of relay-assisted CR network is optimized to reduce its generated interference while still guaranteeing its quality-of-service (QoS) level. A joint relay selection and power allocation algorithm where the cognitive relay system is prevented from inducing severe interference to the primary system by limiting its maximum transmission power is presented in [5]. Guodong et al. proposed in [6] an algorithm to select the best transmit way between the network nodes. The algorithm can select direct, dual or diversity transmission based on the available spectrum as well as the maximum allowable transmission powers.

In this paper, an optimal and suboptimal resource allocation algorithms for an OFDM-Based relay assisted CR network are developed. The objective is to maximize the system capacity by jointly optimizing the subcarrier pairs, relay selection and power allocation. The sum of the allocated transmit powers at the source and the relay nodes should be lower than available power budgets and should not cause harmful interference to the primary system. The paper formulates the problem as a mixed integer programming problem. Although the obtained problem is not convex, but it satisfies the time sharing condition described in [7] and hence, the dual decomposition technique is used to find iteratively the optimal solution. In each iteration, the power levels are determined firstly for every relay and subcarrier pair. Afterwards, the best relay is selected for a given pair and finally the Hungarian method [8] is used to pair the subcarriers. The iterations are repeated until convergence. To get rid of the high computational complexity of the optimal solution, a greedy suboptimal algorithm is proposed. The suboptimal algorithm allocates jointly the different resources taking into consideration the channel qualities, interference to the primary system, individual power budgets and the limitations introduced from applying the DF relaying strategy. The suboptimal algorithm performs close to optimal with less computational complexity.

II. SYSTEM MODEL

In this paper, an OFDM-based cooperative CR system will be considered. As shown in Fig. 1, the CR system coexists with the primary system in the same geographical location. Due to the existence of an obstacle or a large distance, there is no direct link between the source and the destination so that the source tries to communicate with destination through \(M\) relays. The CR system’s frequency spectrum is divided into \(N\) subcarriers each having a \(\Delta f\) bandwidth. It is assumed that the CR system can use the inactive PU bands provided
that the total interference introduced to the PU band does not exceed the maximum interference power that can be tolerated by PU, $I_{th}$. The relays are assumed to operate in half-duplex mode with DF-protocol, thus receiving and transmitting in two different time slots. In the first time slot, the source transmits to the different relays while in the second time slot the relays decode the message, re-encode it and then forward it to the destination. The $j^{th}$ subcarrier in the source should be paired with only one subcarrier $k$ in the destination which may not be the same as $j$ to form the $(j, k)$ pair that should be assigned to only one relay $m$. The maximum total transmission powers that can be used in the source and the different relays are $P_{S}$ and $P_{R_{m}}$ respectively.

In OFDM systems, the mutual interference introduced by the $i^{th}$ subcarrier to PU, $I_{i}(d_{i}, P_{i})$, can be expressed as [9]

$$I_{i}(d_{i}, P_{i}) = \int_{d_{i}-B/2}^{d_{i}+B/2} G_{i}P_{i}T_{s} \left( \sin \frac{\pi f T_{s}}{\pi f T_{s}} \right)^{2} df \leq P_{i}\Omega_{i} \quad (1)$$

where $d_{i}$ is the spectral distance between the $i^{th}$ subcarrier and the PU band. $G_{i}$ denotes the square of the channel gain between the $i^{th}$ subcarrier and the PU band. $P_{i}$ is the total transmit power emitted by the $i^{th}$ subcarrier and $T_{s}$ is the symbol duration while $\Omega_{i}$ denotes the interference factor of the $j^{th}$ subcarrier to the PU band. By the same way, the interference power introduced by PU signal with power spectrum density $\gamma(e^{j\omega})$ into the band of the $i^{th}$ subcarrier is [9]

$$J_{i} = \int_{d_{i}-\Delta f/2}^{d_{i}+\Delta f/2} Y_{i}\gamma(e^{j\omega}) \, d\omega \quad (2)$$

where $Y_{i}$ is the square of the channel gain between the $i^{th}$ subcarrier and the PU signal.

### III. Problem Formulation and Optimal Solution

The transmission rate of the $j^{th}$ subcarrier in the source coupled with the $k^{th}$ subcarrier in the destination and assigned to the $m^{th}$ relay, $Rate_{m,(j,k)}$, can be evaluated as follows

$$Rate_{m,(j,k)} = \frac{1}{2} \min \left\{ \log_{2} \left( 1 + \frac{P_{S}H_{j,m}^{2}}{\sigma_{j,m}^{2}} \right), \log_{2} \left( 1 + \frac{P_{R_{m}}H_{j,k}^{2}}{\sigma_{j,k}^{2}} \right) \right\} \quad (3)$$

where $P_{S}H_{j,m}^{2}$ is the power transmitted over the $j^{th}(k^{th})$ subcarrier in the Source $- R_{m}(R_{m} - \text{Destination})$ link. $R_{m}$ means the $m^{th}$ relay. Moreover, $H_{j,m}^{2}$ is the square of the $j^{th}(k^{th})$ subcarrier fading gain over Source $- R_{m}(R_{m} - \text{Destination})$ link, $\sigma_{j,m}^{2}$ is the variance of the additive white Gaussian noise (AWGN) on the Source $- R_{m}(R_{m} - \text{Destination})$ link, and $J_{j,k}$ is the interference introduced by the PU signal into the $j^{th}(k^{th})$ subcarrier which is evaluated using (2) and can be modeled as AWGN as described in [10]. To make the analysis more clear and without loss of generality, the noise variance is assumed to be constant for all the subcarriers and users, i.e. $\sigma_{j,m}^{2} = \sigma_{j,k}^{2} = \sigma^{2}$.

Our objective is to maximize the CR system throughput by optimizing the subcarrier pairing, relays assignment and distributing the available power budgets in the source and the different relays among assigned subcarrier pairs so that the instantaneous interference introduced to the primary system is below the maximum limit. Therefore, the optimization problem can be formulated as follows

$$\max \quad P_{S} \sum_{m=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{N} \sum_{\pi_{m,(j,k)}} \pi_{m,(j,k)} \\text{Rate}_{m,(j,k)} \quad (4)$$

s.t.
- (C1: Source power constraint):
  $$0 \leq P_{S} \sum_{m=1}^{M} \sum_{j=1}^{N} P_{S,R_{m}} \leq P_{S} \quad (5)$$
- (C2: Relays individual power constraints):
  $$0 \leq P_{R_{m}} \quad \forall m \quad (6)$$
- (C3: Interference at the first time slot):
  $$0 \leq P_{j,k} \sum_{m=1}^{M} \sum_{\pi_{m,(j,k)}} \pi_{m,(j,k)} \Omega_{k,m} \leq I_{th} \quad (7)$$
- (C4: Interference at the second time slot):
  $$0 \leq P_{R_{m}} \sum_{k=1}^{N} \sum_{\pi_{m,(j,k)}} \pi_{m,(j,k)} \Omega_{k,m} \leq I_{th} \quad (8)$$
- (C5: Subcarrier pairing constraint):
  $$\sum_{k=1}^{N} \sum_{\pi_{m,(j,k)}} \pi_{m,(j,k)} \Omega_{k,m} \leq I_{th} \quad \forall k \quad (9)$$
- (C6: Relay Assignment constraint):
  $$\sum_{m=1}^{M} \sum_{\pi_{m,(j,k)}} \pi_{m,(j,k)} = \delta_{j,k} \quad \forall j,k \quad (10)$$

where $N$ denotes the total number of subcarriers while $I_{th}$ is the interference threshold prescribed by PU. $P_{S}$ and $P_{R_{m}}$ are available power budget in the source and the $m^{th}$ relay respectively. $\Omega_{j}$ and $\Omega_{k,m}$ are the $j^{th}(k^{th})$ subcarrier interference factor to the PU band from the source and the $m^{th}$ relay respectively. Note that the interference factor in the first time slot depends only on the source subcarrier index regardless of the assigned relay like the downlink case in the non-cooperative multicarrier based CR system [11]. In contrast, the interference factor in the second time slot depends on the destination subcarrier index as well as the assigned relay like the uplink case in the non-cooperative multicarrier based CR system [12]. The subcarrier pairing constraint ensures that each subcarrier in the source is paired with only one subcarrier in the destination where $\delta_{j,k} \in \{0, 1\}$ is the subcarrier pairing indicator, i.e. $\delta_{j,k} = 1$ if the $j^{th}$ subcarrier in the source is paired with the $k^{th}$ in the destination, and zero otherwise. Additionally, $\pi_{m,(j,k)}$ is the relay assignment indicator which...
equals to one when the pair \((j, k)\) is assigned to the \(m^{th}\) relay and zero otherwise. The source will be in charge of the resources allocation where all the instantaneous fading gains are assumed to be perfectly known. Remark that the channel gains between the CR system nodes can be obtained practically by the classical channel estimation techniques while the channel gains between the CR system and the PU can be obtained by estimating the received signal power from the primary terminal when it transmits, under the assumptions of pre-knowledge on the primary transmit power levels and the channel reciprocity [13].

From (3), the maximum capacity over the \((j, k)\) subcarrier pair which allocated to the \(m^{th}\) relay can be achieved when \(P_{SRm}^{j} H_{SRm}^{j} = P_{Rm}^{j} H_{Rm}^{j} D\). Therefore, the power allocated at \(R_m\) can be expressed as function of the power at \(S\) as

\[
P_{Rm}^{j} = P_{SRm}^{j} \frac{H_{SRm}^{j}}{H_{Rm}^{j} D}.
\]

Hence, the optimization problem in (4) can be re-written as follows

\[
\begin{align*}
\max_{P_{SRm}^{j} \geq 0, t_{j,k} \geq 0, \pi_{m(j,k)}} & \sum_{m=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{N} t_{j,k} \log \left( 1 + \frac{P_{SRm}^{j} H_{SRm}^{j}}{\sigma^2} \right) \\
\text{s.t.} & (C1), (C3), (C5), (C6) \\
\sum_{m=1}^{M} \sum_{j=1}^{N} \pi_{m(j,k)} t_{j,k} & \leq P_{Rm}^{j} ; \forall m \\
\sum_{m=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{N} \pi_{m(j,k)} t_{j,k} & \leq I_{th}.
\end{align*}
\]

Finding the optimization variables \(P_{SRm}^{j}, t_{j,k}\) and \(\pi_{m(j,k)}\) in (5) is a mixed binary integer programming problem. There are \((MN \times N!)\) subcarrier matching and relay assignment possibilities and hence the complexity is prohibitive for large number of subcarriers. The problem in (5) is satisfying the time sharing condition described in [7] and hence, the duality gap of the problem is negligible as the number of subcarrier is sufficiently large, i.e. \(N > 8\), regardless of the convexity of the problem. The solution obtained by the dual method is asymptotically optimal [7], [14], [15]. The solution presented in [14] will be modified to consider the cognitive network with DF relaying strategy.

The dual problem associated with the primal problem (5) can be written as

\[
\begin{align*}
\min_{\beta \geq 0, \gamma \geq 0, \lambda \geq 0, \mu \geq 0} & g(\beta, \gamma, \lambda, \mu) \\
\text{s.t.} & \beta \geq 0, \gamma \geq 0, \lambda \geq 0, \mu \geq 0
\end{align*}
\]

where \(\beta\) and \(\gamma\) are the dual variables associated with the power constraints at the source and the different relays respectively. Moreover, the dual variables \(\lambda\) and \(\mu\) are related to the interference constraints at the first and second time slots respectively. The dual function \(g(\beta, \gamma, \lambda, \mu)\) is defined as follows

\[
g(\beta, \gamma, \lambda, \mu) = \max_{P_{SRm}^{j} \geq 0, t_{j,k} \geq 0, \pi_{m(j,k)}} \mathcal{L}
\]

s.t. (C5), (C6)

where the Lagrangian \(\mathcal{L}\) is given in (8) at the top of the next page.

The dual function in (7) can be rewritten as follows

\[
g(\beta, \gamma, \lambda, \mu) = \max_{P_{SRm}^{j}, t_{j,k} \geq 0, \pi_{m(j,k)}} \left[ \begin{array}{c} \sum_{m=1}^{M} \sum_{j=1}^{N} \pi_{m(j,k)} t_{j,k} D + \\
\beta P_{S} + \sum_{m=1}^{M} \gamma_{m} P_{Rm} + I_{th}(\lambda + \mu) \end{array} \right]
\]

s.t. (C5), (C6)

where

\[
D \doteq \frac{D\left(P_{SRm}^{j}, k\right)}{D_{SRm}^{j}} = \frac{1}{2} \log \left( 1 + \frac{P_{SRm}^{j} H_{SRm}^{j}}{\sigma^2} \right) - \beta P_{S} - \gamma_{m} P_{Rm} + \lambda I_{th} + \mu \Omega_{m} - \sigma_{m} H_{SRm}^{j}
\]

\[\text{s.t.} \quad \beta \geq 0, \gamma \geq 0, \lambda \geq 0, \mu \geq 0.
\]

Therefore, the optimal relay assignment strategy is achieved by allocating the \((j, k)\) pair to the relay which maximizes the function \(D\left(P_{SRm}^{j}, k\right)\), i.e. \(\pi_{m(j,k)} = 1\) if \(m = \arg \max_{m} D\left(P_{SRm}^{j}, k\right)\) and zero otherwise. By performing this allocation, the best relay is determined for every possible subcarrier pair.

Once the power and relay allocation are determined for every subcarrier pair, the following dual function will be obtained

\[
g(\beta, \gamma, \lambda, \mu) = \max_{t_{j,k} \geq 0, \pi_{m(j,k)}} \left[ \begin{array}{c} \sum_{m=1}^{M} \pi_{m(j,k)} t_{j,k} D + \\
\beta P_{S} + \sum_{m=1}^{M} \gamma_{m} P_{Rm} + I_{th}(\lambda + \mu) \end{array} \right]
\]

s.t. (C5), (C6)

The problem in (14) is a linear assignment problem that can be solved efficiently by the Hungarian method with a complexity of \(O(N^3)\) [8].

The subgradient method can be used to solve the dual problem with guaranteed convergence. After finding the optimal
\[ \mathcal{L} = \sum_{m=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{N} \pi_{m}(j,k) t_{j,k} \log \left( 1 + \frac{P_{SRm} H_{SRm}^{k}}{\sigma} \right) + \beta \left( P_{S} - \sum_{m=1}^{M} \sum_{j=1}^{N} P_{SRm} \right) + \lambda \left( I_{th} - \sum_{m=1}^{M} \sum_{j=1}^{N} \pi_{m}(j,k) t_{j,k} \right) \]

solution, i.e. \( P_{SRm}^{j} \), \( \pi_{m}(j,k) \) and \( t_{j,k}^{*} \), of the dual function at a given dual points \( \beta, \gamma_{m}, \lambda \) and \( \mu \), the dual variables at the \((i + 1)^{th}\) iteration are updated as

\[
\begin{align*}
\beta^{(i+1)} &= \beta^{(i)} - \delta^{(i)}(P_{S} - \sum_{m=1}^{M} \sum_{j=1}^{N} P_{SRm}^{j}) \\
\gamma_{m}^{(i+1)} &= \gamma_{m}^{(i)} - \delta^{(i)}(P_{Rm} - \sum_{k=1}^{N} \sum_{j=1}^{N} t_{j,k}^{*} P_{SRm}^{j} H_{SRm}^{k}) \forall m \\
\lambda^{(i+1)} &= \lambda^{(i)} - \delta^{(i)}(I_{th} - \sum_{m=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{N} \pi_{m}(j,k) t_{j,k}^{*} P_{SRm}^{j} H_{SRm}^{k} \Omega_{k,m}) \\
\mu^{(i+1)} &= \mu^{(i)} - \delta^{(i)}(I_{th} - \sum_{m=1}^{M} \sum_{j=1}^{N} \sum_{k=1}^{N} \pi_{m}(j,k) t_{j,k}^{*} P_{SRm}^{j} H_{SRm}^{k} \Omega_{k,m})
\end{align*}
\]

where \( \delta^{(i)} \) is the step size that can be updated according to the nonsummable diminishing step size policy.

IV. SUBOPTIMAL ALGORITHM

In the optimal solution derived in the previous section, \((M + 3)\) dual variables are updated in every iteration. Using these values, \(MN\) function evaluations are performed to find the power allocation. Afterwards, \(M\) function evaluations are performed for every possible subcarrier pair where there are \(N!\) subcarrier matching possibilities. By including the computational complexity of the Hungarian method, the optimal solution derived in the previous section has a complexity of \(O(T(MN) + M(N!) + N^{3})\) where \(T\) is the number of iterations required to converge which is usually high [7]. In order to solve the problem efficiently, we propose in this section a suboptimal greedy algorithm by which the different system resources are allocated jointly with lower computational complexity than that of the optimal solution. The suboptimal algorithm takes into consideration the interference introduced to the primary system, the different channel qualities, the available power budgets and limitation introduced by using the DF-protocol.

Start by defining the sets \(A\) and \(B\) to include all the non-assigned subcarriers in the source and the destination sides respectively. Moreover, define the set \(M\) to contain all the relays in the network. In the source side, assume that available source power is distributed uniformly over the subcarriers, i.e. \(P_{uni}^{m} = \frac{P_{S}}{N}\), and also assume that the interference introduced to the primary system by every subcarrier is equal and hence form (1), the maximum allowable power that can be allocated to the \(j^{th}\) subcarrier is \(P_{max}^{m} = \frac{P_{S}}{M_{N}}\). Therefore, the allocated power to the \(j^{th}\) subcarrier in the source side is \(P_{SRm}^{j} = \min(P_{uni}^{m}, P_{max}^{m})\). The assigning procedures of a particular subcarrier \(j \in A\) are as follows

1) For every relay \(m \in M\), evaluate the rate \(RT_{j,m}\) achieved by allocating the subcarrier \(j\) to the \(m^{th}\) relay.

2) For every relay \(m \in M\) and subcarrier \(k \in B\), compute the required power to achieve a rate in the relay to destination link equal to that in the source to relay link, i.e. \(P_{rate}^{k,m} = (\frac{2^{\frac{R_{T,j,m}}{4\Delta}} - 1}{\sigma})^2\). Then, evaluate \(P_{uni,m}^{k,m} = \frac{P_{uni}^{m}}{M_{N}k}\) and \(P_{max,m}^{k,m} = \frac{P_{max}^{m}}{M_{N}k}\) where \([B]\) means the cardinality of the set \(B\). Afterwards, set \(P_{power,k,m} = \min(P_{rate}^{k,m}, P_{uni,m}^{k,m})\).

3) Find \(k^{*}\) and \(m^{*}\) satisfying \((k^{*}, m^{*}) = \arg \max_{k,m} (P_{power,k,m} H_{Rm}^{k})\). Set \(t_{j,k^{*}} = 1\), \(\pi_{m^{*},j,k^{*}} = 1\), and \(P_{Rm}^{k^{*}} = P_{power,k^{*},m^{*}}\), and update the \(m^{th}\) relay power budget as \(P_{Rm} = P_{Rm}^{k^{*}} - P_{power,k^{*},m^{*}}\).

4) Remove the subcarriers \(j\) and \(k^{*}\) from the sets \(A\) and \(B\) respectively and repeat the procedures until the set \(A\) is empty.

In the proposed scheme, every subcarrier in the source side requires no more than \((M + MN)\) function evaluations to be paired and assigned to the relay. Therefore, the complexity of the proposed algorithm is \(O(MN + MN^2)\).

V. SIMULATION RESULTS AND CONCLUSION

The simulations are performed under the scenario given in Fig.1. An OFDM system of \(N = 64\) subcarriers is assumed with \(M = 5\) relays. The values of \(T_{s}, \Delta f, \sigma^{2}\) are assumed to be \(4\mu s\) seconds, 0.3125 MHz and \(10^{-6}\) respectively. The channel gains are outcomes of independent Rayleigh distributed random variables with mean equal to \(1\). All the results have been averaged over 1000 iterations. In the simulations, \(Optimal\) and \(Suboptimal\) algorithms apply the dual decomposition technique presented in Sec. III and the proposed method presented in Sec. IV respectively. Additionally, \(SNR\) algorithm refers to the method by which the subcarriers and users are selected according to the \(SNR\) values while in \(Random\), the subcarriers are matched and assigned randomly. For both \(SNR\) and \(Random\), the powers are evaluated by solving (5) with the known \(t_{j,k}\) and \(\pi_{m}(j,k)\).

Fig. 2 shows the achieved capacity of the different algorithms vs. the interference threshold. One can note that the CR system capacity increases with the interference threshold as the CR system become able to use more power on the different subcarriers. Additionally, the throughput increases as expected with the increase of the available power budgets. However, the increment in the throughput by changing the available power form 0 dBm to 20 dBm is very small when the interference threshold is low since both systems use approximately the same amount of power to induce the maximum allowed interference to the PU. Moreover, the \(suboptimal\) algorithm with low computational complexity has
a near optimal performance and outperform SNR and random algorithms. In the low interference thresholds region, the SNR-based matching criteria applied in the non-cognitive system has limited performance in comparison with optimal because it doesn’t take the interference to the primary system into account. Furthermore, the gap between the optimal algorithm and the SNR algorithm is decreased with the interference threshold as the system behaves closer to the non-cognitive one.

Fig. 3 depicts the achieved capacities vs. the available power budgets in the source and the relays. The capacity of the different schemes increases with the power budgets up to certain power value after which the capacity becomes constant. This can be justified by that the system induces the maximum allowable interference by using this power value and hence any increase in the available power budget will not be used by the system and has no effect on the system performance. The CR system behaves as a non-cognitive one when the amount of the available power budgets are not able to induce the maximum allowed interference which justifies the good performance of the SNR algorithm in low power regions. This performance become limited in comparison with the optimal as the available powers increased.

To summarize, in this paper we have considered the resource allocation problem in multiple relay DF-OFDM based CR system. To maximize the achieved capacity while maintain the interference introduced to the primary system below a pre-specified threshold, the subcarrier pairing, relay assignment and power allocation are performed jointly using the dual decomposition technique. Due to the high computational complexity of the optimal scheme, a greedy suboptimal algorithm is presented. The suboptimal algorithm allocates the different resources jointly and achieve a near optimal performance with much less complexity. Moreover, the proposed scheme outperforms the SNR and Random based methods. We are currently working on extension of the proposed scheme by considering more interference constraints as well as channel estimation errors in the cognitive and primary links.