A variational method for vessels segmentation: algorithm and application to liver vessels visualization

M. Freiman\textsuperscript{a} and L. Joskowicz\textsuperscript{a} and J. Sosna\textsuperscript{b}

\textsuperscript{a}School of Engineering and Computer Science, The Hebrew University of Jerusalem, Israel.
\textsuperscript{b}Dept. of Radiology, School of Medicine, Hadassah Hebrew Univ. Medical Center, Israel.

ABSTRACT

We present a new variational-based method for automatic liver vessels segmentation from abdominal CTA images. The segmentation task is formulated as a functional minimization problem within a variational framework. We introduce a new functional that incorporates both geometrical vesselness measure and vessels surface properties. The functional describes the distance between the desired segmentation and the original image. To minimize the functional, we derive the Euler-Lagrange equation from it and solve it using the conjugate gradients algorithm. Our approach is automatic and improves upon other Hessian-based methods in the detection of bifurcations and complex vessels structures by incorporating a surface term into the functional. To assess our method, we conducted with an expert radiologist two comparative studies on 8 abdominal CTA clinical datasets. In the first study, the radiologist assessed the presence of 11 vascular bifurcations on each dataset, totaling of 73 bifurcations. The radiologist qualitatively compared the bifurcations segmentation of our method and that of a Hessian-based threshold method. Our method correctly segmented 88% of the bifurcations with a higher visibility score of 82%, as compared to only 55% in the Hessian-based method with a visibility score of 33%. In the second study, the radiologist assessed the individual vessels visibility on the 3D segmentation images and on the original CTA slices. Ten main liver vessels were examined in each dataset. The overall visibility score was 93%. These results indicate that our method is suitable for the automatic segmentation and visualization of the liver vessels.

Keywords: segmentation, visualization, abdominal procedures

1. INTRODUCTION

Vessels segmentation of volumetric medical images has increasingly gained acceptance in current clinical practice. The segmentation can be used to generate three-dimensional visualizations and models of the vessels branching structure and its pathologies to support diagnosis and planning of surgical procedures. In particular, liver vascular structure segmentation and visualization is useful for tumor resection planning, liver regeneration after hepatectomy, and liver monitoring, among many others.

Liver vessels segmentation is known to be a challenging task due to the low contrast between the blood vessels and surrounding liver parenchyma, and the complex branching structure, morphology, and pathologies of the hepatic vessel system. Several methods for vessels segmentation have been proposed in the past decade. These methods can be classified into: 1) histogram based methods;\textsuperscript{1,2} 2) semi-automatic region growing based methods;\textsuperscript{3} 3) topology based methods;\textsuperscript{4} 4) level-set based methods;\textsuperscript{5,6} and 5) geometrical model based methods.\textsuperscript{7–9} Hybrid approaches, such as combining histogram and topological analysis, have also been proposed.\textsuperscript{10}

The advantages of geometric model-based methods are that they are less sensitive to intensity variations inside the vessels and can detect small, low-contrast vessels. They fit a tubular model to each voxel according to its neighborhood. Frangi et al\textsuperscript{7} and Sato et al\textsuperscript{8} segmented the vessels by multi-scale eigen-system analysis of the Hessian matrix. Each voxel is assigned a value that quantifies the similarity between the voxel neighborhood and an ideal tubular shape at a given scale. Viergever et al\textsuperscript{11} use the vesselness measure to control an anisotropic
Figure 1. Illustration of segmentation results on (a)-(i) synthetic images from Krissian et al paper.9 (a)-(c) Maximum Intensity Projections (MIP) of the original volumes; (d)-(f) MIP of Frangi’s7 Hessian-based vesselness measure, and (g)-(i) MIP with our method.

Several discrete optimization methods that combine both geometrical and intensity information were proposed. Slabaugh et al12 used a graph-cut approach with ellipsoid shape prior to segment vessels from medical images. Homann et al13 used the vesselness measure proposed by Frangi et al7 as an additive shape term in a graph-cut based optimization framework for liver vessels segmentation. The main disadvantage of these methods is that they require significant user interaction to initialize the algorithm.

In this paper, we present a new variational-based optimization method for vessels segmentation from Computed Tomography Angiography (CTA) images. Our method incorporates Hessian-based geometrical vesselness measure, surface normals, and gradients magnitudes into a functional. The functional quantifies the distance between the desired segmentation image and the original image based on surfaces normals and regions of interest. It is then minimized within a variational framework by solving the resulting Euler-Lagrange equation. The solution is the desired segmentation map. Our approach is fully automatic and improves upon Hessian-based vesselness measure methods. Since it uses the surface properties – gradients magnitudes and directions – in addition to the vesselness measure, it can cope with bifurcations, complex vessel structures, and pathologies (Fig. 1). It also handles the low contrast between vessels and the surrounding liver parenchyma, and the intensity variations inside the vessels due to the contrast agent flow. We demonstrate our method on the liver vascular structure, although it can be used for other vascular structures.
2. METHOD

Within the variational framework, the segmentation task is defined as an energy minimization problem. The input image is interpreted as a discretized version of a continuous function \( v : V \rightarrow \mathbb{R} \), where \( V \) is a subset of \( \mathbb{R}^3 \). The goal is to automatically construct a new function \( u : V \rightarrow \mathbb{R} \), defined over the same domain, which identifies the regions of interest (the vessels) and discards the rest (background). The search for \( u \) is formulated as an optimization problem over the space of the twice differentiable functions from \( V \) to \( \mathbb{R} \). Specifically, the goal is to find a function \( u \) that minimizes the energy function:

\[
E(u) = \int_V F(u, \nabla u, x) \, dx
\]

where \( F \) is a functional (defined below), \( \nabla u = (u_x, u_y, u_z)^T \) is the gradient of \( u \), and \( u = 0 \) on the boundary of \( V \). The energy functional quantifies the distance function between the input image and the resulting segmentation map.

To find the image function \( u \) that minimizes Eq. 1, we minimize the Euler-Lagrange equation:

\[
\frac{\partial F}{\partial u} - \frac{d}{dx} \frac{\partial F}{\partial u_x} - \frac{d}{dy} \frac{\partial F}{\partial u_y} - \frac{d}{dz} \frac{\partial F}{\partial u_z} = 0
\]

derived from Eq. 1 by solving a finite differences approximation of it with the conjugate gradients method. The resulting function \( u \) is the desired segmentation map.

We begin by describing the vesselness measure we used to detect the regions of interest in the image, and then define the functional used for the segmentation.

2.1 Vesselness measure

We use a Hessian-based measure to detect the regions in which vessels are present at a given scale. The measure describes the geometric structure of vessels by analyzing the eigensystem of the \( 3 \times 3 \) Hessian matrix \( H \) of each voxel from the volumetric image. For bright vessels on a dark background, the three eigenvalues \(|\lambda_1| \leq |\lambda_2| \leq |\lambda_3|\) and their related eigenvectors describe the structure of the vessel. The eigenvector associated with \( \lambda_1 \) is the direction along the vessel, and the eigenvectors associated to \( \lambda_2 \) and \( \lambda_3 \) are perpendicular to it.

In particular, we use Frangi et al. vesselness measure from:

\[
M(\sigma) = \begin{cases} 
0 & \lambda_2 > 0 \text{ or } \lambda_3 > 0, \\
\left(1 - \exp\left(-\frac{R_A^2}{(2a)^2}\right)\right) \left(\exp\left(-\frac{R_B^2}{(2b)^2}\right)\right) \left(1 - \exp\left(-\frac{S^2}{(2c)^2}\right)\right) & \text{otherwise},
\end{cases}
\]

where

\[
R_A = \frac{|\lambda_2|}{|\lambda_3|} \quad R_B = \frac{|\lambda_1|}{\sqrt{|\lambda_2\lambda_3|}} \quad S = \sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}
\]

and \( \sigma \) is the scale at which the measure is computed. \( R_A \) differentiates between plate and line-like structures, \( R_B \) measures the deviation from blob-like structures, and \( S \) differentiates between foreground (vessel) and background (noise). Constants \( a \), \( b \) and \( c \) are predefined weights determining the influence of \( R_A \), \( R_B \) and \( S \). The vesselness measure value is close to 1 for voxels with tube-like structures, and close to 0 otherwise. The vesselness measure is computed for each voxel and for each scale. Finally, for each voxel, the maximal value among the different scales is set to the vesselness measure for the voxel. Fig. 1(d-f) illustrates the vesselness measure on several synthetic images. While this measure performs well on centers of the vessels, it provides poor results on bifurcations, and a nearby vessels.
2.2 Energy functional

The functional \( F \), which describes the relation between the original image and the desired result, plays a key role in the variational framework. We define it as a weighted linear combination of three terms:

\[
F = \alpha F_{\text{str}} + \beta F_{\text{tan}} + \gamma F_{\text{ind}}
\]  

(4)

The first term \( F_{\text{str}} \) describes the fidelity of the resulting image \( u \) to the structures of interest in the original image \( v \). It is defined as:

\[
F_{\text{str}} = u^2 (v_{\text{str}} - c)^2 \left( \frac{1}{|\nabla v|^2 + \epsilon} \right)
\]

(5)

where \( v_{\text{str}} \) is the vesselness response as described by Frangi et al.\(^7\) and \( c \) is a constant value that discriminates between vessels and background. Since \( u^2 \) is multiplied by a term that rapidly increases as \( v_{\text{str}} \) gets further away from \( c \), a solution that minimizes the integral in Eq. 1 must be as close to zero as possible in such regions. On the other hand, in regions where \( v_{\text{str}} \) is close or equal to \( c \), the solution \( u \) can take any value without increasing the integral. Consequently, \( u \) is zero almost everywhere except for voxels along the vessels in the original image. Since the vesselness measure yields relatively low values near the vessels edges even though the gradients have high values, we multiply this term by the inverse of the gradient magnitude to obtain high values along the vessels edges.

The second term \( F_{\text{tan}} \) ensures that \( F \) has tangential directions similar to those in the original image:

\[
F_{\text{tan}} = |\nabla u \times \nabla v|^2
\]

(6)

This term is smallest when the difference of the tangential directions between the resulting and original images is minimized. It is designed to cope with complex, non-vessel structures, such as bifurcations, narrowings, and aneurisms, which are filtered out by the vesselness measure.

The third term \( F_{\text{ind}} \) is a regularization term that requires the resulting image voxels \( u \) to have a value \( u_{\text{ind}} \neq 0 \) whenever possible. Specifically, it “pulls” \( u \) towards \( u_{\text{ind}} \) in regions where the original image contains vessels structures. It is defined as:

\[
F_{\text{ind}} = (\delta + (1 - \delta)|\nabla v|)(u - u_{\text{ind}})^2
\]

(7)

The combination of these three terms eliminates undesired regions and “pulls” the structures of interest defined by the vesselness measure to fit to the vessels in the original image, as illustrated in Fig. 1(a-f). An additional advantage of the functional is that the Euler-Lagrange equation derived from it can be approximated with the finite differences method by a set of linear equations that can be readily solved.

3. EXPERIMENTAL RESULTS

To evaluate our method, we obtained 8 clinical abdominal CTA datasets with a total of 73 liver vessel bifurcations. The datasets were acquired by a 16 slice MDCT (Philips Medical Systems, Best, the Netherlands) with the following parameters: 2mm slice thickness, 1mm increment, energy 240 mAs, 120 KvP. Scanning was performed 60 seconds after the injection of 100cc intravenous contrast agent. The datasets have 72-110 slices (122-150mm) at 1mm increments and 2mm thickness, with window lengths of 117-193 pixels (200-300mm). Most of the images included pathologies, such as tumors, metastases, and cysts of different shapes and sizes. The datasets were preprocessed as suggested by Sato et al.\(^8\) by manually segmenting the liver and resampling them by linear interpolation.

We performed two vessel segmentations of the datasets: one with a Hessian-based filter with user-defined thresholding as proposed by Sato et al.\(^8\) and one with our method. For both methods, we used the same vesselness measure,\(^7\) with parameters set to \( a = b = 0.5 \) to provide equal weighting of plate and blob-like structures, and \( c = 0.0001 \) for high sensitivity to low-contrast vessels. The energy functional parameters (Eq. 1) were set to \( \alpha = 0.8, \beta = 0.3 \) and \( \gamma = 0.1 \) to emphasize the vesselness measure over the similarity between the surfaces
Figure 2. Example of the segmentation results obtained with the Hessian-based method and with our method: (a)-(c) 2D axial views. The segmentation is shown superimposed on the original CTA slices; (d)-(e) 3D visualization. The arrows point to bifurcations and vessel surface regions which appear as disconnected or non-existent in the Hessian-based method and are fully visible with our method. (f)-(i) Zoom-in details of two bifurcations. (j)-(m) Zoom-in details of a vessel.
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<th>Num. of bifurcations</th>
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<th>Score</th>
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<td>Our method</td>
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<td>119/146</td>
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Table 1. Experimental results of 8 datasets with 73 bifurcations using our method and Hessian-based filtering. For each dataset, we report the number of bifurcations, the number of detected bifurcations with each method, and a visibility score indicating the quality of the segmentation. The score is the sum of the individual bifurcation scores: 0 for not detected, 1 for partially detected, and 2 for fully detected, where detection is determined from the segmentation.

normals in the original image and the segmentation results. Thus, only surfaces close to vessels will appear in the final image. Fig. 1(g-l) shows an example of the vessel segmentation results for both methods on synthetic images. The running time of our method was 53-137 secs on a Pentium 4 3GHz processor with 1GB of memory.

We conducted two evaluation studies with an expert radiologist on 3D images generated from the vessels segmentations of each dataset. In the first study, the radiologist assessed the presence of 11 vascular bifurcations, including hepatic and portal venous bifurcations. The radiologist qualitatively compared the bifurcations segmentation of our method and that of Hessian-based filter method (Table 1). We defined a quantitative-qualitative visibility score indicating the quality of the segmentation, compared to the ideal visibility of the structures. The score is the ratio between sum of the individual bifurcation scores: 0 for not detected, 1 for partially detected, and 2 for fully detected and the ideal score (2 for each bifurcation).

Our method correctly segmented 88% of the bifurcations with a visibility score of 82%, as compared to only 55% in the Hessian-based method with a visibility score of 33%. Fig. 2 illustrates the performance of our method compared to that of Hessian-based filter method.

In the second study, the radiologist assessed the individual vessels visibility on the 3D segmentation images and on the original CT slices. Ten main liver vessels were examined in each dataset: Left Portal Vein (LPV), Right Anterior Portal Vein (RAPV), Right Posterior Portal Vein (RPPV), Main Portal Vein (MPV), Left Hepatic Vein (LHV), Middle Hepatic Vein (MHV), Right Hepatic Vein (RHV), Right Hepatic Artery (RHA), Main Hepatic Artery (MHA), and Left Hepatic Artery (MHA). Each one of them was scored as in the first study: LPV (14/15), RAPV (12/14), RPPV (16/14), MPV (16/16), LHV (14/13), MHV (14/14), RHV (2/2), RHA (2/8) and LHA (2/2). Our method successfully segmented the RPPV and LHV vessels despite their very low contrast in the original CTA images. Fig. 3 summarizes the clinical analysis results of the proposed method. Each column describes the visibility of each part of the vessels in the original CT (white), and as visualized by the proposed method (black). The overall visibility score of the vessels segments was 93%.

4. CONCLUSION

We have presented a new variational method for vessels segmentation and showed its application to liver vessels segmentation. The proposed method aims to overcome the limitations of the Hessian-based methods in bifurcations, complex vessels structures, and pathologies by incorporating surface normals coupled with Hessian-based vesselness information into a variational framework. The advantages of our method are that it is fully automatic, that it produces high-quality segmentations, that it is efficient, and that it performs better than existing Hessian-based methods. Two clinical evaluation studies on 8 abdominal CTA datasets by an expert radiologist show that our method successfully segments liver vessels structure and their bifurcations.
Liver vessels visibility

Figure 3. Comparison between the visibility of liver vessels in the original CT slices (white) and in the 3D visualization (black) produced by the proposed method. Eight clinical cases were evaluated. In each case, the following vessels were judged: the Left Portal Vein (LPV), the Right Anterior Portal Vein (RAPV), the Right Posterior Portal Vein (RPPV), the Main Portal Vein (MPV), the Left Hepatic Vein (LHV), the Middle Hepatic Vein (MHV), the Right Hepatic Vein (RHV), the Right Hepatic Artery (RHA), the Main Hepatic Artery (MHA), and the Left Hepatic Artery (MHA). Each one of them was graded as: invisible (0), partially visible (0.5), or fully visible (1). Each column describes the sum of scores over all datasets.

In the future, we plan to evaluate our method on more liver datasets and test it on other vascular structures, such as the lung and the brain.

REFERENCES


