A time-cost trade-off model by incorporating fuzzy earned value management: A statistical based approach

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Abstract. Time–cost trade-off problems (TCT) are well-known in project management contents. This approach normally applied for scheduling of a project especially where the project should be completed under predetermined deadline. This paper aims to extend time–cost trade-off (TCT) problems in order to provide a well-organized mechanism for both scheduling and rescheduling processes of a project. The proposed mechanism includes project scheduling which concerns with the TCT problem, monitoring of project performance during execution phase using earned value management (EVM), and also predicting project future performance through statistical modeling. Once the predicted values of project performance indicate the necessity for rescheduling, the initial TCT problem is modified to determine a new mode for the execution of the project. In the proposed model, several options have been considered with specific time and cost for an individual activity where options indicate different execution methods (EM) for the implementation of the whole project. Furthermore, due to vagueness and impreciseness associated with data of real case projects, the time and cost behavior of each option presumed as fuzzy numbers. The proposed control mechanism can help project managers to take the advantage of a comprehensive model to schedule, control, and reschedule a project through the life cycle. An illustrative case is then presented to successfully demonstrate the application of the proposed approach.

Keywords: Time–cost trade-off, fuzzy logic, earned value, fuzzy regression, rescheduling

1. Introduction

Since 1950, the critical path method (CPM) is extensively employed as an efficient approach in project scheduling. However, in many cases, it seems that it is desired for project managers to deliver the project earlier than the scheduled date. They may achieve this goal by employing additional resources such as equipment or additional human resources. However, using more resources leads to increase in the project total cost. The implementation of a project through an efficient approach with predetermined completion time and cost (deadline) is one of the important issues which planners may face with. This issue is commonly known as time–cost trade-off (TCT) problem. Generally, the TCT models can be categorized into two types: deterministic and non-deterministic models. The traditional models mainly focus on deterministic environments [1–3]. However, in reality, many unknown variables may affect the project objectives and deliverables. For instance, unpredictable changes in weather condition can be considered as an affecting variable. Hence, scheduling in non-deterministic environment is more practical compared to deterministic area. The non-deterministic scheduling models are constructed on the basis of two distinctive approaches: probabilistic based
Goldrat challenged the assumption of using deterministic scheduling and introduced his probabilistic model for the TCT problem [4]. In other words, historical time-cost data is used to obtain probability density functions for time and cost parameters [5]. In many situations, probability distribution functions (to obtain time or cost functions) may be unknown or just partially known. In such cases, using probability theory may not be helpful enough. Feng et al., Zheng et al. and Abbasnia et al. applied the fuzzy set theory to address this inefficiency [6–8]. In their models, the random values of time and cost have been selected according to the predefined cut line in fuzzy numbers. Thus, the fuzzy numbers replaced with the relevant crisp ones. In other study, Eshtehardian et al. employed the fuzzy concept in their TCT model and their obtained results are presented as fuzzy numbers [9]. Chen and Tsai also provided a TCT analysis of project networks in fuzzy environment [10]. Recently, the published researches mostly have focused on the integration of the other affecting issues such as the quality of outcomes, resource leveling and resource constrained with the traditional TCT problem. For instance, Amiri et al., Mungle et al., Zhang and Xing, and Zhang et al. discussed the time-cost-quality trade-off in construction projects [11–14]. Kim et al. studied the effect of potential quality lost cost in time-cost optimization problem [15]. Moreover, Ghodousi et al. incorporated the resource leveling problem in their TCT model [16].

The advantages of the proposed model can be mentioned from different aspects. Firstly, both input and output types in the model are uncertain (non-deterministic), which can adopt model with the real-life circumstances more than already existing models with deterministic data. Secondly, to the best of authors' knowledge, the scopes of all researches in the literature just limited to the scheduling process and have not been applied during project execution phase, while trade-off among the project completion time, cost, and the uncertainty of real situations are significant factors for decision makers through the whole life cycle of a project [17]. This tendency is to find an appropriate balance between time and cost performance which becomes more challenging when the project managers wish to make a balance between these factors (time, cost) during execution, specially, when the project should be re-scheduled due to its weak performance. To overcome this issue, There are some well-known techniques in project management research area such as the earned value management (EVM) which mainly focus on monitoring the time and cost performance of projects in order to keep such balance in projects to be completed timely and on the budget [18]. However, such technique is kept definitely separated from the TCT problem. Here, the authors attempt to develop the novel idea of integrating the scheduling process, using a time-cost-trade-off model, with the project performance assessment, through EVM, in order to present a comprehensive model. This effort improves the decision making process for a project manager. In another word, the proposed model bridges the gap between the scheduling and rescheduling process to present an integrated model which is applicable through the whole life cycle of a project. The rest of paper is arranged as follows: A comprehensive explanation about the proposed model is presented first. To show performance of the proposed model in practical projects, an illustrative case is then demonstrated and finally discussion and conclusion are provided, respectively.

2. Model formulation and interpretation

The problem under consideration in this paper can be stated through two individual phases. In the first phase, deals with the TCT problem in project scheduling process. (see Phase 1 in Fig. 1). After that, in the second phase, attempts to handle the TCT problem through the implementation of a project using EVM technique. The Fig. 1 illustrates the interaction between the above mentioned phases in a project. Furthermore, the following subsections are organized according to the presented flow diagram in Fig. 1.

2.1. Phase 1: TCT problem and scheduling process

The following subsections cope with the first phase of the proposed model including construction of TCT model in the scheduling process:

2.1.1. Definition of “option” for an activity and “execution method” for a project

Generally, options are choices by which an activity can be administered. These choices vary in resource consumption and duration. It goes without saying that they also differ in the required budget. For instance, an activity can be done by one worker in certain duration; also it can be done with two workers significantly faster but obviously needs more budget. Any combination of resources and duration is considered as an option for an activity and each defined execution
method of a project includes all individual activities of that project. For example, if a project consists of 5 activities and each activity has three options for the execution, there will be 243 available EM for this project (i.e. $3^5 = 243$).

Different execution methods are defined as $EM_r$, where $r$ varies from 1 to $m$, and $m$ indicates number of all EMs for the execution of a project. Figure 2 is depicted as an example to make the definition of EM clear.

2.1.2. Fuzzy behavior of activities

Initially, Lotfi Zadeh introduced fuzzy sets and theory [19]. Fuzzy theory describes vagueness in systems where uncertainty increases due to fuzziness rather than randomness. In many cases, it is difficult to define the time and cost of activities deterministically. In fact,
in real projects, many unknown variables affect the time and cost of activities [20]. To deal with such unknown variables, fuzzy theory can be successfully applied [6–8].

Generally, it is assumed fuzzy numbers may have different shapes, such as triangular, trapezoidal or Gaussian shapes. However, it makes basic fuzzy arithmetic easier if the fuzzy numbers are defined in triangular and trapezoidal shapes (for an overview see e.g., [10, 21–23]). In this paper, time and cost of activities and other defined parameters of the model are considered as triangular fuzzy numbers. To introduce a typical triangular fuzzy number, let $M$ be a triangular fuzzy number: $M = (m_l, m_c, m_r)$. The membership function of $M$ is defined as follows:

\[
\mu_M(x) = \begin{cases} 
\frac{(x - m_l)}{(m_c - m_l)} & m_l \leq x \leq m_c \\
\frac{(m_r - x)}{(m_r - m_c)} & m_c \leq x \leq m_r \\
0 & \text{otherwise}
\end{cases}
\]  

(1)

Let us suppose $q \geq 0$ is a real number and $\bar{A}$ and $\bar{B}$ are two triangular fuzzy numbers. Some basic arithmetic of fuzzy numbers is presented as follows [19]:

\[
\bar{A} = (a_l, a_c, a_r) \quad & \bar{B} = (b_l, b_c, b_r) \\
\bar{A} + \bar{B} = (a_l + b_l, a_c + b_c, a_r + b_r) \\
\bar{A} - \bar{B} = (a_l - b_l, a_c - b_c, a_r - b_r) \\
\bar{A} \times \bar{B} = (a_l \times b_l, a_c \times b_c, a_r \times b_r) \\
\bar{A}/\bar{B} = (a_l/b_l, a_c/b_c, a_r/b_r)
\]

There are different approaches in the literature introduced for the purpose of converting fuzzy numbers to crisp ones [22, 24]. Among these methods, the graded mean integration representation (GMIR) has been used in this paper due to its simplicity and accuracy [25]. The GMIR for a triangular fuzzy number such as $M = (m_l, m_c, m_r)$ is calculated as below:

\[
\text{GMIR}_M = \frac{(m_l + 4 \times m_c + m_r)}{6}
\]

(6)

larger GMIR means a larger fuzzy number. For instance, let us assume $\bar{A}$ and $\bar{B}$ as two triangular fuzzy numbers. $\bar{A}$ is greater number than $\bar{B}$ if

\[
\text{GMIR}_A > \text{GMIR}_B.
\]

2.1.3. Project completion time and cost

Network of a project can consist of different paths. All possible paths should be considered to determine the completion time for each EM. Let us symbolize the set of all paths in $r$th EM by $\text{Path}_r = \{p_i | p = 1, 2, 3, \ldots n\}$, where $n$ represents the total number of paths. The longest path introduced as critical path (CP) should be regarded as completion time of each EM:

\[
\text{Completion time for } r\text{th execution method } \quad \bar{T}_{EM} = \max_{\text{All } p_i \in \text{Path}_r} \bar{T}_{p_i}
\]

(7)

where $i_p, n_p$ and $\bar{T}_{p_i}$ define an activity, total number of activities and completion time of $p$th path on the $r$th execution method, respectively. The following equation is also introduced to calculate total cost of each EM:

\[
\bar{BAC}_EM = \sum_{i_p} \bar{T}_{EM} \times \bar{C}_D 
\]

(8)

\[
+ \max \left[0, \bar{E}_{EM} - \bar{T}_{EM} \right] \times \bar{C}_P
\]

where $\bar{C}_P, \bar{C}_D, \bar{E}_D$ represent the project delay penalty cost, daily incentive reward, daily indirect cost, and planned completion time, respectively. Moreover, $\bar{C}_D$ symbolizes cost of $d$th activity on $r$th execution method. Note that all of these parameters are assumed as triangular fuzzy numbers.

2.1.4. Calculation of the best EM

To deal with the TCT problem, the most appropriate execution method, introduced as best EM, is determined in this section. Here, the word "appropriate" carries some degree of vagueness, since the best EM may differ in various kind of project depending on project characteristics. To cope with such impreciseness, a common approach in the literature is to consider weights as coefficients in order to determine the significance of project completion time or cost in calculation procedure of best EM [10]. In the proposed model, the weights are introduced as linguistic terms, while fuzzy triangular numbers are assigned to each of these terms (see Table 1). The decision makers in the project determine the value of these weights prior to the project start date. Subsequently, the weights are modified based on the project performance in rescheduling process. Concerning the TCT problem, the best EM is determined as follows:

\[
\bar{A}_{max} = \max \{\bar{A}_1, \bar{A}_2, \bar{A}_3, \ldots, \bar{A}_n\}
\]

(9)

For each EM $\bar{A}_i$, the total cost or time is calculated as follows:

\[
\bar{BAC}_{EM} = \sum_{i} \bar{T}_{EM} \times \bar{C}_D 
\]

(10)

\[
+ \max \left[0, \bar{E}_{EM} - \bar{T}_{EM} \right] \times \bar{C}_P
\]

(11)

where $\bar{C}_P, \bar{C}_D, \bar{E}_D$ represent the project delay penalty cost, daily incentive reward, daily indirect cost, and planned completion time, respectively. Moreover, $\bar{C}_D$ symbolizes cost of $d$th activity on $r$th execution method. Note that all of these parameters are assumed as triangular fuzzy numbers.

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by comparing the earned value (EV) against actual cost (AC). The fuzzy-based illustration of this index is presented in the following equation:

$$\hat{CPI} = \frac{EV_l}{AC} \frac{EV_c}{AC} \frac{EV_r}{AC} = (CPI_l, CPI_c, CPI_r)$$

The primary use of SPI is to evaluate the project schedule performance. The fuzzy illustration of this index is determined as below:

$$\hat{SPI} = \left(\frac{ES}{AD}, \frac{ES}{AD}, \frac{ES}{AD} \right)$$

ES and AD indicate the earned schedule and actual duration of the project, respectively. The EVM mainly concerns with the past performance of a project, however, the presented model attempts to apply the previous trend of a project performance for assessment of project future behavior. Here, statistical modeling is employed in the following section to predict the EVM indices at the end of the project.

2.2.2. Statistical trend and modeling procedure

Statistical modeling has been widely employed as an efficient method for the prediction purposes. As one of the first studies in this research area, Covach et al. considered regression technique and presented 12 independent models to estimate budget at completion, i.e. (EAC) [33]. Due to imprecision and vagueness available in real cases, other researches focused on employing fuzzy sets in the regression modeling. Primarily, Tanaka et al. introduced linear regression analysis with fuzzy model [34]. Subsequently, other studies focused not only on the development of the fuzzy regression models (see e.g., [35–37]) but also concentrated on the application of the fuzzy regression in different systems (see e.g., [21, 38]). One of the great advantages of the fuzzy regression, likewise the basic regression, is the prediction of the future values of variables [21]. The proposed model utilizes the fuzzy regression to predict the $\hat{SPI}$ and $\hat{CPI}$ at the end of a
project regarding the past values of these indices. The predicted $\tilde{SPI_t}$ and $\tilde{CPI_t}$ provide project managers with the opportunity to be aware of the final status of the cost and schedule performance and deciding whether rescheduling of the project is required or not.

In the proposed model, the fuzzy regression modeling presented by Yang and Lin has been employed in prediction process [37]. The main arithmetic of this fuzzy regression model is demonstrated as below:

$$\tilde{y} = \hat{\beta}_0 + \hat{\beta}_i \tilde{x}_i + \tilde{e}_i \quad i = 1, 2, \ldots, n \tag{12}$$

Equation (12) presents the basic functional form of the employed fuzzy regression in which $\tilde{y}$, $\tilde{\beta}_0$, $\tilde{\beta}_i$, and $\tilde{x}_i$ are the fuzzy response variable, fuzzy parameters and the $i$th fuzzy predictors variable, respectively. Let us define $\tilde{y}_i$, $\tilde{x}_i$, $\tilde{\beta}_i$, $\tilde{\beta}_0$ as the following triangular fuzzy numbers. The following procedure is presented to find the values of $\tilde{\beta}_0$, $\tilde{\beta}_i$:

$$\tilde{y}_i = (y_i, y_{ic}, y_{ir})$$

$$\tilde{x}_i = (x_i, x_{il}, x_{ir})$$

$$\tilde{\beta}_i = (\tilde{\beta}_{ii}, \tilde{\beta}_{ic}, \tilde{\beta}_{ir})$$

$$\tilde{\beta}_0 = (\tilde{\beta}_{0l}, \tilde{\beta}_{0c}, \tilde{\beta}_{0r})$$

The estimates for the parameters are obtained as follows:

$$\tilde{\beta}_i = \frac{\sum_{i=1}^{n} x_i y_i - n \tilde{x}_i y_0}{\sum_{i=1}^{n} x_i^2 - n \tilde{x}_i^2} \quad \tilde{\beta}_0 = \tilde{y}_0 - \tilde{\beta}_i \tilde{x}_i \tag{13}$$

$$\tilde{\beta}_{ii} = \frac{n \tilde{x}_{il} y_{ic} - n_{il} \tilde{x}_i y_0}{\sum_{i=1}^{n} x_i^2 - n \tilde{x}_i^2} \quad \tilde{\beta}_{ic} = \tilde{y}_i - \tilde{\beta}_{ii} \tilde{x}_i \tag{14}$$

$$\tilde{\beta}_{ir} = \frac{n \tilde{x}_{ir} y_{ir} - n_{ir} \tilde{x}_i y_0}{\sum_{i=1}^{n} x_i^2 - n \tilde{x}_i^2} \quad \tilde{\beta}_{0c} = \tilde{y}_i - \tilde{\beta}_{ir} \tilde{x}_i \tag{15}$$

where

$$\tilde{x}_i = \frac{\sum_{i=1}^{n} x_i y_i}{n}, \quad \tilde{x}_{il} = \frac{\sum_{i=1}^{n} x_{il} y_i}{n}, \quad \tilde{x}_{ir} = \frac{\sum_{i=1}^{n} x_{ir} y_i}{n}$$

$$\tilde{y}_i = \frac{\sum_{i=1}^{n} y_i}{n}, \quad \tilde{y}_{ic} = \frac{\sum_{i=1}^{n} y_{ic}}{n}, \quad \tilde{y}_{ir} = \frac{\sum_{i=1}^{n} y_{ir}}{n}$$

The employed approach for the prediction of $\tilde{SPI_t}$ and $\tilde{CPI_t}$ are presented as follow:

$$\tilde{SPI_t} = \tilde{\beta}_0 + \tilde{\beta}_i \times (\tilde{SPI}_{t-1}) \quad \tag{16}$$

$$\tilde{CPI_t} = \tilde{\beta}_0 + \tilde{\beta}_i \times (\tilde{CPI}_{t-1}) \quad \tag{17}$$

where $\tilde{\beta}_0$, $\tilde{\beta}_i$ and $\tilde{\beta}_{ii}$ are the response variables, fuzzy parameters and the predictor variables and illustrate the values of these indices at time $T$. Eventually, the value of the response variables are predicted using fuzzy regression models (See Equations (16 and 17)). If $T-1$ is supposed as the current date then $T$ will be the next date which can be the following milestone or the measurement period. In order to predict the indices values of next time periods, each response variable should be replaced with the predictor variable. Therefore, to obtain the final values of the $\tilde{SPI_t}$ and $\tilde{CPI_t}$, this process should be repeated until $T$ will be equal to the completion time of the project.

2.2.3. The interpretation of the earned value indicators

Subsequent to prediction of fuzzy-based $\tilde{SPI_t}$ and $\tilde{CPI_t}$ at the completion time of the project, these predicted indices require to be interpreted (see Figs. 3 and 4).

The values of $\tilde{CPI_t}$ and $\tilde{SPI_t}$ should be compared with the ideal conditions (i.e. when the indices are equal to unity) to reveal the project status. Figures 3 and 4 make such comparison possible. For instance, if the values of $\tilde{CPI_t}$ and $\tilde{SPI_t}$ should be compared with the ideal conditions (i.e. when the indices are equal to unity) to reveal the project

Fig. 3. Different scenarios related to $\tilde{CPI_t}$.
status. Figures 3 and 4 make such comparison possible. For instance, if \( \left( \overline{SPI} \right)_T = (0.7, 0.8, 0.95) \) and \( \left( \overline{CPI} \right)_T = (0.96, 1.05, 1.2) \), then it can be mentioned that \( \left( \overline{SPI} \right)_T \) follows the scenario "I" that means "the project is behind schedule" (see Fig. 4). Additionally, considering Fig. 3, it can be concluded that \( \left( \overline{CPI} \right)_T \) follows the scenario "III" and that means "the project is approximately moving as budgeted" (see Fig. 3).

### 2.2.4. Rescheduling process

The final statuses of the cost and schedule performance of the project are determined in the previous section. If the final status of \( \left( \overline{CPI} \right)_T \) and \( \left( \overline{SPI} \right)_T \) indicate the project performance is behind the budget or schedule, it will be necessary to revise the initial plan and reschedule the project. The following procedure is presented for the rescheduling process of the project. The proposed rescheduling process is highly intertwined with the presented procedure for the scheduling process. That means the utilized equations for scheduling process are also applied in the re-scheduling process. However, some amendments are inevitable to make the incorporation between scheduling and rescheduling processes possible. The required amendments are obtained in the following subsections.

### 2.2.5. The recalculation of the weights factors

The new weights obtained in the following equations should be replaced with the weights used in Equation (9):

\[
\tilde{k}_1 = \frac{\left( \overline{CPI} \right)_T}{\left( \overline{CPI} \right)_T + \left( \overline{SPI} \right)_T} \quad (18)
\]

\[
\tilde{k}_2 = \frac{\left( \overline{SPI} \right)_T}{\left( \overline{CPI} \right)_T + \left( \overline{SPI} \right)_T} \quad (19)
\]

where \( \left( \overline{CPI} \right)_T \) and \( \left( \overline{SPI} \right)_T \) are the predicted values at the completion time of the project. The results of Equations (18) and (19) are not directly replaced with the predetermined weights. Initially, they should be compared with the assigned fuzzy numbers of Table 1 using GMIR method. The results of such comparison determine the new values of weights (see Table 2).

### 2.2.6. Modifying the execution methods

As mentioned previously, the rescheduling process is performed during the project execution. In the rescheduling process, the activities which had been started or completed, should be removed from the defined execution method. Figure 5 is provided as an example to illustrate how the EM should be modified in the rescheduling process. The project depicted in Fig. 5 consists of 5 activities. The first row indicates the activity numbers and the second row demonstrates the selected options for each activity. The initial EM for the execution of the project is EM\(_1\) = 32231. This project is an ongoing project in which the activity number 1 is completed and the activity number 2 started previously and has not finished yet. According to the proposed
approach for the modification of the EM, the modified EM project is $E_{M^I} = 231$.

2.2.7. Recalculate the best EM

The new illustration of EM shows that the selected options for the implementation of the remained activities (i.e. not started activities). Similar to the scheduling process, all available options for each remained activity should be considered for generating of new EMs. In the rescheduling process, the completion time and the budget of the project should be calculated again for each new EM using Equations (7) and (8). Eventually, Equation (9) is employed to determine the best EM for the remained activities. Note that the best EM is determined in this step using the novel values of the weights.

3. An illustrative case

In this section, a practical case is given to illustrate the application of the proposed model. This case study is related to a construction project. The related data of this project is obtained up to 15 days after initiation. Table 3 illustrates available options and their related duration and cost of the activities. Moreover, the precedence of each individual activity is provided in this table. Mean-while, Table 4 provides the parameters which should be utilized in Equation (8) for determining of the project total cost. The authors agree with the point that just one example cannot provide any solid evidence to show the application of the proposed model in all kind of projects and do not intend to replace the proposed model with traditional approaches. But, they attempt to provide project managers with a new model that can be utilized effectively in dealing with rescheduling process. Therefore, just one illustrative case is enough to demonstrate how the proposed model can be applied in a real case project.

3.1. The numerical results

The proposed model divides the project into two phases from the scheduling points of view: first phase takes place prior to the start date of the project and second one occurs during the execution of the project.

Table 3

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</tbody>
</table>
Concerning the first phase, there are 3888 potential EMs for the execution of this project. In the scheduling process, Equations (7) and (8) are employed to determine the completion time and the cost of each EM and finally Equation (9) is utilized to calculate the best EM. It is important to note that the project managers initially determine the weights utilized in Equation (9). In this illustrative case, the weights with the same importance degree are considered in the scheduling process. The completion time and cost of best EM is acquired in Table 5.

3.1.1. The prediction of project performance
According to the second phase, the cumulative \( \hat{S}_{P}T \) and \( \hat{CPI} \) up to day 15th are calculated using Equations (10) and (11):

\[
\hat{S}_{P}T = (0.54, 0.62, 0.68) \\
\hat{CPI} = (0.59, 0.64, 0.73)
\]

The fuzzy regression models for the prediction of \( \hat{S}_{P}T \) and \( \hat{CPI} \) are presented in Table 6. These values resulted from the fuzzy regression models should be evaluated using different scenarios introduced in Figs. 3 and 4. According to Fig. 3, the status of the predicted \( \hat{S}_{P}T \) is related to the scenario “I” which means “the project is approximately behind the budget” from the cost control points of view. Furthermore, as Fig. 4 indicates, “the project is behind the schedule” from the schedule points of view. The predicted \( \hat{CPI} \) and \( \hat{S}_{P}T \) indicate that the project has faced with problems from both cost and schedule aspects. Therefore, the project managers should reschedule the initial plan and choose another appropriate EM for the remained activities of the project.

### Table 5
The characteristics of the best EM determined in scheduling process

<table>
<thead>
<tr>
<th>Assigned fuzzy number to ( \tilde{y}_{1} )</th>
<th>Assigned fuzzy number to ( \tilde{y}_{2} )</th>
<th>Completion time</th>
<th>Completion Cost</th>
<th>Best EM</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.4, 0.5, 0.6)</td>
<td>(0.4, 0.5, 0.6)</td>
<td>162,75, 79</td>
<td>184,250, 192,270</td>
<td>204,625</td>
</tr>
</tbody>
</table>

Such challenge clearly shows the significance of employing the proposed model which integrates the scheduling rescheduling procedure.

3.1.2. The recalculation of the weights
Prior to the recalculation of the weights, \( \tilde{k}_{1} \) and \( \tilde{k}_{2} \), should be obtained using Equations (18) and (19).

\[
\tilde{k}_{1} = \frac{\hat{CPI}}{\frac{\hat{CPI}}{\hat{S}_{P}T} + (\hat{S}_{P}T)_{T}} = (0.32, 0.65, 1.17)
\]

(20)

\[
\tilde{k}_{2} = \frac{\frac{\hat{S}_{P}T}{\hat{CPI}}}{\frac{\hat{S}_{P}T}{\hat{CPI}} + \hat{CPI}} = (0.17, 0.35, 0.42)
\]

(21)

Eventually, \( \tilde{k}_{1} \) and \( \tilde{k}_{2} \) have to be compared with fuzzy numbers in the first column of Table 2. The outcome of this comparison results in the new values of weights (see Table 7).

3.1.3. Modifying the execution methods
In this step, the completed activities or the activities started prior to the initiation of the rescheduling process are removed from the initial EMs. Furthermore, new EMs ought to be generated for the remained activities of the project. In this project, the activity number 1 is started and has not been finished since the project start date comes to the present time (i.e. 15 days after the initial start date).

### Table 7
The attainment of the novel fuzzy weights

<table>
<thead>
<tr>
<th>Weights</th>
<th>The old value of the weights</th>
<th>Assigned linguistic terms to the novel values of weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega_{1} )</td>
<td>(0.4, 0.5, 0.6)</td>
<td>High (0.65, 0.8, 0.9)</td>
</tr>
<tr>
<td>( \omega_{2} )</td>
<td>(0.4, 0.5, 0.6)</td>
<td>Normal (0.4, 0.5, 0.6)</td>
</tr>
</tbody>
</table>

### Table 6
Assigned regression models to the schedule and cost performance index

\[
\begin{align*}
(\hat{S}_{P}T)_{T} & = (0.17, 0.35, 0.42) \\
(\hat{CPI})_{T} & = (0.11, 0.15, 0.19) + (0.32, 0.41, 0.45) \\
(\hat{S}_{P}T)_{T} & = (0.43, 0.46, 0.52) \\
(\hat{CPI})_{T} & = (0.54, 0.58, 0.63)
\end{align*}
\]
is 61 days. Adding 61 to 15 makes the total time of the remaining time of project in the new EM 8, some interesting results will be obtained. The crisp values of the completion time and completion cost in the new EM are 163200.171500.185800.6. The crisp value of the completion time is even shorter than the initial EM value of 166900.173000.186800. So, the new EM has been selected with more emphasis on achieving the timely completion of the project because the project suffers severely from a weak schedule performance. The crisp value of the completion cost is $192992. The weights are considered equal in the initial scheduling which means the importance of the cost and schedule performance of the project is considered equally (both have "high degree of significance", see Table 2). After 15 days from the commencement, the performance trend of the project can be easily observed. The modified weights are then used in a way to put further focus on the weak aspect of the project. The modified weights values replaced with the previous existing ones using the predicted outputs of the project performance. The new weight values replace the previous existing weights. The predictions can greatly assist project managers to reveal the project performance trend of the project can be easily observed. Therefore, the new weights put the higher significance on obtaining the new EM with lower duration. The new EM presented in Table 8 is attained attempting to achieve this purpose. Comparing Tables 5 and 8, some interesting results will be obtained. The crisp value of the remaining time of project in the new EM is 61 days. Adding 61 to 15 makes the total time of the project in the new EM, i.e. 76 days, which is even longer than duration of the initial EM. However, it should not be ignored the weak schedule performance of the project up to data has its negative influence to increase the total time of the project. The proposed model attempts to decrease such effects, however it cannot be removed, completely.

### 5. Conclusion

In this paper, a systematic model is presented to integrate the TCT problem and the project performance during project execution phase. The model can select the best known execution method where different potential options are available for execution during project implementation. Decision makers may involve in the process of selecting the best EM by assigning different fuzzy-based weights to project total time and cost. In addition to the above advantages, the proposed approach controls pre-specified EM from the beginning to the present time and also predicts the project performance from both cost and schedule viewpoints using EVM and fuzzy regression simultaneously. Also, the performance trend of the project can be easily observed. The answers to the above questions will be accessible by using the fuzzy regression approach (see Table 6). In the process of rescheduling, if it is required, the eventual status of the project performance and make appropriate alerts for rescheduling of the initial plan. The modified weights are then used in a way to put further focus on the weak aspect of the project performance. The numerical results of the illustrative case confirm this point of view and exhibit the model attempts to find out the new EM by further focusing on decreasing the total time of the project.

It is suggested to apply the model to the projects with multi optional activities such as R&D projects. The other possible improvement of the proposed model is employing the concept of parallel activities, so-called parallel funding, which mainly used in preventive risk strategies.

### References


### Table 8

The characteristics of the best EM determined in rescheduling process.

<table>
<thead>
<tr>
<th>Completion time</th>
<th>Completion cost</th>
<th>Best EM</th>
</tr>
</thead>
<tbody>
<tr>
<td>(51,62,72)</td>
<td>(163200.171500.185800)</td>
<td>EM = 212111</td>
</tr>
</tbody>
</table>

Decision makers may involve in the process of selecting the best EM by assigning different fuzzy-based weights to project total time and cost. In addition to the above advantages, the proposed approach controls pre-specified EM from the beginning to the present time and also predicts the project performance from both cost and schedule viewpoints using EVM and fuzzy regression simultaneously. Also, the performance trend of the project can be easily observed. In the next step, Figs. 3 and 4 have been employed to interpret these fuzzy-based outputs. Such interpretations can greatly assist project managers to reveal the eventual status of the project performance and make appropriate alerts for rescheduling of the initial plan. In the process of rescheduling, if it is required, the new weight values replaced with the previous existing ones using the predicted outputs of the project performance. The modified weights are then used in a way to put further focus on the weak aspect of the project performance. The numerical results of the illustrative case confirm this point of view and exhibit the model attempts to find out the new EM by further focusing on decreasing the total time of the project.

It is suggested to apply the model to the projects with multi optional activities such as R&D projects. The other possible improvement of the proposed model is employing the concept of parallel activities, so-called parallel funding, which mainly used in preventive risk strategies.

### References