Performance Modeling of Blocking Probability in Multihop Wireless Networks

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ABSTRACT
Ad-hoc network communication requires efficient routing protocols to overcome the problems associated with unpredictable and dynamically changing topologies, which are mostly triggered by nodes mobility and non-existence of base stations and central controllers. We propose a performance evaluation model for blocking probability of multihop calls in ideal macrocell environment conditions using the Markovian process. The proposed model which computes the end-to-end blocking probability and establishes a relation between blocking probability and the system's throughput is implemented through extensive computer simulation. Simulation results allow for an accurate evaluation of the network performance in the presence of impairments caused by call blocking.

INDEX TERMS
Ad-Hoc Networks, System Throughput, Call Blocking, Markovian Processes

I. INTRODUCTION
Call blocking is a barrier in signal transmission that impairs or influences communication. In cellular networks, blocking occurs when a base station (BS) fails to allocate a free channel to a mobile user. In [1], two types of blocking in cellular networks have been distinguished. They include new call blocking (blocking of new calls) and handoff or handover blocking (blocking of on going calls due to users mobility).

For new calls, before communication is initiated, a mobile user (MU) must first obtain a channel from one of the closet base stations (i.e., that which clearly hears it). The MU is granted the channel if it is available, otherwise, the call is blocked (i.e., if all channels are busy). On successful channel assignment, the MU either completes the call or enters another cell before completing the call and finally releases the channel. This process of moving into another BS (cell) while a call is still in progress is referred to as handoff. During the handoff process, the MU demands a handshake from the nearest (new) BS for channel allocation. If the new cell has no free channel then the handshake process is incomplete and the handoff call is blocked. This type of blocking is referred to as handoff blocking. An analysis of blocking probability and dynamic channel assignment in cellular networks has been extensively covered in [2-4].

A network of wirelessly communicating mobile nodes that does not require a fixed infrastructure or BS is referred to as an ad-hoc network. Ad-hoc networks can be used to connect wireless devices such as laptops, palmtops and personal digital assistants (PDAs), for the purpose of communication and resource sharing. The absence of fixed infrastructure results in frequent dynamically changing and unpredictable topologies of the wireless network and thus requires an efficient, distributed and dynamic routing protocol to ensure economical bandwidth availability, assignment, usage and avoidance of co-channel interference. Most proposed routing protocols are usually evaluated with discrete event simulation [5-7].

A multihop wireless network is a cooperative ad-hoc network where data streams are transmitted over multiple wireless hops to reach the destination [8]. Multihop topologies depend on the nodes transmission radius and can adjust to varying transmission power. In a multihop wireless network, calls hop through various links before reaching the destination. This imposes additional complexity as non-interfering mobile nodes must be assigned channels on the wireless links along the transmission path. One major limitation of multihop networks is limited frequency: a situation where the spectrum may need to be shared amongst applications and com-
munication media (channels), which requires medium access protocol that is critical for performance constraints on transmission, caused by limited battery energy transmission regulators and data delays that do not allow all sensors in the network design to communicate with the sink. Apart from these limitations, there also exist some challenges in multihop networks. One major challenge is the device failure, which has necessitated the self-organizing architecture.

Though proposed routing protocols are usually evaluated using discrete event simulation, this paper shall develop and simulate a mathematical model that can evaluate end-to-end blocking probabilities. The result of this work could be used to determine the optimum bandwidth, power requirement and network user density and could be refined for other factors that are relevant to the development of a viable ad-hoc network. The channel assignment policy in multihop networks is often represented as a Markovian process. Figure 1 shows the state transition model of progressive calls at a node for an infinite number of calls.

In Figure 1, 
\[ \lambda_n \] is the arrival rate for calls to be relayed at node \( n \).
\[ P_{n,k} \] is the probability that a call request is accepted (i.e., granted at node \( n \))
\[ \mu_n \] is the call holding time.

The Markov property makes a process easier to analyze since we do not have to recall the complete past trajectories. In the next section, we provide background literature on call blocking and the modeling of routing protocols.

**II. BACKGROUND**

Call blocking probabilities computation in wireless code division multiple access (WCDMA) networks using several modifications of the well-known Kaufman-Roberts (K-R) recursion; has been well studied in [9-10]. Special peculiarities of these networks such as soft blocking and activity factors have been incorporated into the K-R recursion to finally form a recurrent formula that achieves approximate but efficient computation of the system state probabilities and blocking probabilities. The proposed recursion corresponds to the Erlang Multi-rate Loss Model (EMLM) used for analyzing wired networks with random (Poisson) arriving calls.

In [11] an end-to-end call blocking probability computation algorithm for dynamic connections in multicast networks has been developed. Their algorithm enables them to study the accuracy of previous results based on Reduced Load Approximation (RLA). They further extend the model to include background traffic, permitting mixed traffic networks.

A model, computing call blocking probabilities for both uplink and downlink directions, has been developed in [12]. The model exploits the Delbrouk’s algorithm [13], and provides a more general case of call arrival process different from Poisson or quasi-random distributions. In [14] a model that analyzes the tradeoff between blocking and dropping probabilities in multi-cell CDMA systems in the presence of elastic traffic has been developed. The authors build on their previous model [15] extending the model to accommodate state dependent soft blocking and capturing of call sessions drop.

In [8], a multihop wireless network with a connection-oriented traffic model and multiple transmission channels that can be re-used spatially is considered. In their paper, the blocking probability of a call requesting channel assignment depends on the channel assignment scheme and the transmission radius of the nodes that affect the network link structure. They analyze blocking probability for a wireless line and grid networks and explore the tradeoff between the transmission radius and probability of blocked calls.

**A. Modeling Routing Protocols**

Routing protocols for ad-hoc networks need to overcome the problems associated with dynamic and unpredictable topologies arising from mobile nodes mobility and the absence of base stations and central controllers. With an analytical method, we could evaluate the end-to-end blocking probability required to generate faster results for routing protocol comparison and network optimization. In ad-hoc networks, a remote mobile node interconnec-
tion is achieved via peer level multi-hopping. Each intermediate node must behave as both a router for the network as well as a host. The network model consists of N nodes indexed by n and therefore has \( R = \frac{N(N-1)}{2} \) node pairs with each pair being indexed by r. Each node pair has M routes between them consisting of intermediate nodes for the delay of information over a multihop path. The routes are ordered from best to worst according to their suitability which is decided by the particular routing protocol under evaluation. The routes have \( n \) indexes and the \( m^{th} \) route for the \( r^{th} \) node pair is \( (r, m) \) or equivalently \( r_m \) and consists of \( h_{rm} \) intermediate nodes. Calls are Poisson processes with mean arrival rate \( 1/\lambda_r \) and mean holding time \( 1/\mu_r \). If no route exists, the call is rejected.

In section IV, we will focus on the development of an analytical model for voice routing protocols. To simplify the route determination procedure, we assume that the available routes and their suitability are determined by the traffic parameter of the offered traffic. Routes between each node pair are listed from shortest to longest. Both the route length and the congestion level at each intermediate node are considered and the route selected is the shortest route that has the least congestion on its most congested intermediate node. Suppose we define the most congested node on route \( (r, m) \) as \( K_{rm} \); the route selected becomes the route that minimizes the cost function,

\[
W_1 Q(K_{rm}) + W_2 h_m
\]

where \( W_1 \) and \( W_2 \) are weighting factors and \( Q(K_{rm}) \) is the number of calls being received by the most congested node on route \( (r,m) \).

III. PROBLEM STATEMENT

Ad-hoc wireless networks are “special” cases of wireless networks that have no fixed backbone infrastructure. This property though adds flexibility and makes the network rapidly deployable, some significant technical challenges still remain. Some of the numerous challenges include effective routing, medium access, power management, security and quality of service (QoS) issues [16-17]. As the nodes communicate over wireless links, each node contends with the highly erratic nature of wireless channels as well as interference from other transmitting nodes. These factors make it more challenging a problem to optimize data throughput while satisfying the required users QoS.

Owing to mobility of the nodes and interference among nodes, great difficulties arise in the implementation of ad-hoc networks due to frequent route changes. The high packet loss rates and frequent topological changes causes instability within the transport layer, thereby limiting the amount of traffic the network can support. Three well-known problems in ad-hoc networks include the lack of reliable packet delivery due to interference and mobility of nodes, limited bandwidth due to channel restrictions and limited node lifetime due to small battery size [18-19]. For traffic flows in mobile ad-hoc networks, more researches are required to offer reliable transport services as well as guaranty good QoS. Since ad-hoc networks are not limited by capacity, due to its dynamic nature; good scalability of the QoS architecture is important. Previous research shows that network scalability issues are hardly ever measured [20].

Multihop routing distinguishes mobile ad-hoc networks from infrastructure-based networks, such as cellular wireless networks and Wireless Local Area Networks (WLAN), where one-hop routing is adopted between mobile hosts and base stations or access points. Intuitively, wireless multihop routing would be more complex than the one-hop counterpart [21], since the traffic flow would have to deal with the access contention issue in every hop along its path. Considering an environment with frequent topological changes and wireless channels with a high bit-error-rate (BER), some routing mechanisms exhibit poor performance due to reliability and efficiency. Most existing ad-hoc routing protocols build and utilize only a single route for each pair of source and destination nodes. Due to node mobility, node failures, and the dynamic characteristics of the radio channel, links in a route may become temporarily unavailable, making the route invalid. The overhead of finding alternative routes may be high and extra delay in packet delivery may be introduced. This paper, therefore, studies how blocked calls impinge on the performance of multihop wireless networks. To achieve this goal, we propose a blocking probability model that will enhance network stability and evaluate the performance of the network in a more efficient manner.
IV. METHODOLOGY

Let us consider an iterative process which is used to determine the steady state distribution of the network, where four sets of unknowns are to be solved simultaneously using three related mappings. The unknowns are defined as follows: The arrival rate of calls at node \( n \) is \( \nu_n \). The steady state probability that node \( n \) is receiving \( k \) calls simultaneously is \( P_n(k) \), where \( 0 < k < a \). The fourth unknown is \( q_{rm} \), denoting the probability that route \( (r,m) \) will be selected for a call between nodes of node pair \( r \). In the first mapping, the values of \( a_n \) and \( q_{rm} \) are held fixed to find \( \nu_n \). For the second mapping, the value of \( \nu_n \) is held fixed to calculate \( P_n(k) \) and \( a_n \). Finally, in the third mapping, the value of \( P_n(k) \) is held fixed to calculate \( q_{rm} \).

A. Mapping 1: \( a_n, q_{rm} \rightarrow \nu_n \)

The arrival rate of calls to be relayed at a node is as a result of multiple routes between various node pairs that would be using the node as an intermediate node. Therefore the arrival rate at node \( n \) for calls to be relayed is:

\[
\nu_n = \sum_{(r,m)} \lambda_r q_m \prod_{i \neq n} A_i \quad (2)
\]

where

\[
I[x \in X] = 1 \text{ if } x \in X \text{ and } I[x \in X] = 0 \text{ if } x \notin X
\]

B. Mapping 2: \( \nu_n \rightarrow P_n(k), a_n \)

The processing gain \( G \) of a Direct Sequence CDMA (DS-CDMA) system is defined as the ratio of the transmission bandwidth \( W \) to the transmission bit rate \( B \). This gives a measure of how much the base-band signal has been spread. The energy per bit received by the node from each of the \( k \) users is denoted by \( \epsilon_b \). We then define the one-sided spectral noise density received at a node as \( n_0 \) and the total acceptable interference density as \( i_0 \). For a call to be accepted: Thermal noise + Interference < Allowance

\[
\text{Thermal noise + Interference < Allowance}
\]

i.e.

\[
N_0W + kB \epsilon_b \leq I_0W
\]

\[
kB \epsilon_b < W(I_0 - N_0)
\]

Dividing through by \( I_0B \) gives

\[
k \epsilon_b < G(I - \eta)
\]

where \( \eta = N_0 / I_0 \). Let \( Z_k = kE_b / I_0 \) which represents the signal to noise ratio (SIR). It can be shown that \( Z_k \) is a Gaussian process with mean

\[
Z_k = kC_m + E_n[k] C_{n2}
\]

and variance

\[
\sigma_{Z_k}^2 = \kappa c
\]

where

\[
E_n[k] = \frac{\nu_n}{\mu_n} a_n
\]

is the expected number of calls in progress at node \( n \) and \( C_m, C_{n1}, C_{n2}, C_{n3}, C_{n4} \) are constants. The state of a node \( n \) can be defined as \( \delta_n(k), 0 < k < \infty \), where \( k \) is the number of users communicating with the node. Two possible state transitions are most likely to occur:

1. \( s_n(k) \rightarrow s_n(k-1) \) at rate \( k\mu \) and
2. \( s_n(k) \rightarrow s_n(k+1) \) at rate \( E_{nk} \nu \)

where

\[
E_{nk} = P_r \{ \text{call request is accepted given k call in progress at node n} \}
\]

\[
= P_r \{ Z_k \leq G(1-\eta) \}
\]

\[
E_{nk} \text{ is given in [22] as}
\]

\[
E_{nk} = \int_0^{G(1-\eta)} \frac{1}{\sqrt{2\pi}\sigma Z_{k-1}} e^{\frac{(Z-Z_{k-1})^2}{2\sigma^2 Z_{k-1}}} dZ
\]

which simplifies to

\[
E_{nk} = \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{G(1-\eta) - Z_{k+1}}{\sqrt{2\sigma Z_{k+1}}} \right)
\]

The error function \( \text{erf} \) is given by

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt
\]

The birth-death model in Figure 1 is then solved using the following standard difference - differen-
tial equations:
\[ P_n(k) = -(\nu_n E_{n+k} + \mu_n) P_n(k) + (k + 1) \mu_n P_n(k - 1) \quad \text{for } 1 < k < \infty \] (12)

and
\[ P_n(0) = -\nu_n + \mu_n P_n(1) \] (13)

The sum of the steady state probabilities is one, i.e.,
\[ \sum_{k=0}^{\infty} P_n(k) = 1 \] (14)

By defining the following variables, the steady state distribution can be found:
\[ a_n(k) = \frac{(\nu_n E_{n+1}) (\nu_n E_{n+2}) \cdots (\nu_n E_{n+k-1})}{(\mu_n) (2\mu_n) (3\mu_n) \cdots (k\mu_n)} \quad k > 0 \] (15)

\[ a_n(0) = 1 \] (16)

Therefore the steadystate distribution is given by:
\[ P_n(k) = a_n(k) P_n(0), \quad 1 < k < \infty \] (17)

\[ P_n(0) = \frac{a_n(0)}{\sum_{k=0}^{\infty} a_n(k)} \] (18)

The steady state probability that a call will be accepted by intermediate node \( n \) is then given by
\[ a_n = \sum_{k=0}^{\infty} P_n(k) [E_{n,k}] \] (19)

**C. Mapping 3:** \( P_n(k) \rightarrow q_m \)

The most congested node on a route \( (r,m) \) can be obtained from the expected number of calls being routed on the node as follows:
\[ K_{rm} = \arg \max_{n \in (r,m)} [E_n[k]] \] (20)

Let the probability that there are at most \( k \) calls in progress at node \( n \), be defined as
\[ t_n(k) = \sum_{i=0}^{k} P_n(i) \] (21)

Equation (21) can be rewritten as
\[ t_n(k) = \sum_{i=0}^{k} P_n(k) = P_n(k) P_n(0) \] (22)

with \( P_n(0) = \frac{a_n(0)}{\sum_{k=0}^{\infty} a_n(k)} \)

and
\[ P_n(k) = a_n(k) P_n(0), \quad 1 \leq k < \infty \] (23)

Substituting equations (18) and (19) into (23) yields.
\[ t_n(k) = v_n \left[ \frac{1}{2} \sum_{i=0}^{k} \left( \frac{G(1-\eta) - Z_{i+1}}{\sqrt{2\sigma_{i+1}}} \right) \right] \] (24)

Equation (24) can be compressed as follows:
\[ y = v_n \left[ \frac{1}{2} \sum_{i=0}^{k} \left( \frac{G(1-\eta) - Z_{i+1}}{\sqrt{2\sigma_{i+1}}} \right) \right] \]

then
\[ t_n(k) = \frac{y}{k\mu_n} a_n(0) \sum_k y \]

\[ = \frac{y}{k\mu_n} a_n(k) (y_0 + y_1 + \ldots + y_k) \]

\[ = \frac{y}{k\mu_n} a_n(0) \]

\[ = \frac{a_n(0)}{k^2} \mu_n \] (25)

The probability that a route \( (r,m) \) will be selected from \( M_r \) routes between node pair \( r \), as having the capability of admitting calls and minimizing the cost function, is given by
\[ q_m = \sum_{k=1}^{M_r} t_k \left( \sum_{i=0}^{M_r} \frac{W_i (h_i - h_m) + k}{W} \right) \] (26)

**D. Blocking Probability**

The end-to-end blocking probability of calls for node pair \( r \) is given by
\[ B_r = 1 - \sum_{m=1}^{M_r} q_m \prod_n a_n(n(r,m)) \] (27)
Expanding equation (27) by using $q_{r_m}$ and $a_n$ from equations (26) and (19) respectively gives

$$B_r = 1 - \sum_{k=1}^{\infty} p_{r_m}(k) \left( E_k \right)_{1}^{W_1} h_{1/n} \left( W_1 (h_2 - h_n) + k \right) \Pi a_n(k)$$ \hspace{1cm} (28)

where

$$E_{km} = \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{G(1 - n) - Z_{k+1}}{\sqrt{2} \sigma_{z_{k+1}}} \right)$$

and

$$p_{r_m} = \nu_m \left[ \frac{1}{2} + \frac{1}{2} \text{erf} \left( \frac{G(1 - n) - Z_{k+1}}{\sqrt{2} \sigma_{z_{k+1}}} \right) \right] \left( a_n(0) \right) \sum_{k=1}^{\infty} a_n(k)$$

E. Relationship with Throughput

We now formulate a relationship between blocking probability and the system’s throughput. This relationship, which is given in equation (29) will enable us to evaluate the system’s performance in the presence of blocking.

$$Th = \lambda d_{rate} (1 - B_r)$$ \hspace{1cm} (29)

where $\lambda$ is the arrival rate and $d_{rate}$ represents the data rate.

V. SIMULATION, RESULTS AND ANALYSIS

A. Simulation Input Parameters

Table 1 shows the simulation settings (input parameters and their respective values). These values are empirical data measured from a macrocell (urban) environment and portray our research domain.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady state distribution at node 0 ($a(0)$)</td>
<td>1</td>
</tr>
<tr>
<td>Active callers (K)</td>
<td>5-80</td>
</tr>
<tr>
<td>Mean holding time ($1/\mu$)</td>
<td>105s</td>
</tr>
<tr>
<td>Processing gain (G)</td>
<td>32 dB</td>
</tr>
<tr>
<td>Call node (n)</td>
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</tr>
<tr>
<td>Arrival rate of calls at node n ($\nu_n$)</td>
<td>0.1-2.5</td>
</tr>
<tr>
<td>Arrival rate of incoming calls ($\lambda$)</td>
<td>0.1</td>
</tr>
<tr>
<td>Weight factor 1 ($W_1$)</td>
<td>3</td>
</tr>
<tr>
<td>Weight factor 2 ($W_2$)</td>
<td>4</td>
</tr>
<tr>
<td>Number of routed nodes ($h_1$)</td>
<td>8</td>
</tr>
<tr>
<td>Number of intermediate nodes ($h_n$)</td>
<td>5</td>
</tr>
<tr>
<td>SIR</td>
<td>5 dB</td>
</tr>
<tr>
<td>Route time ($t_{route}$)</td>
<td>0.05 s</td>
</tr>
<tr>
<td>Data rate ($d_{rate}$)</td>
<td>(500, 250, 125) kb/s</td>
</tr>
</tbody>
</table>

Table 1. Model input parameters and values

Figure 2 shows the input form that accepts data for the simulation. Using Visual Basic 6.0 programming toolkit, a simulator was developed to implement the proposed model. The simulator is generic and can be adapted for the simulation of other propagation environments, such as microcells and picocells. It accepts the model input parameters and computes the end-to-end blocking probability with outputs that shows the various parameter relationships that enables a performance evaluation of the Markovian model.

B. Interpretation of simulation results

Figure 3 shows a stationary distribution of calls in progress. The graph shows that the higher the interference (from neighboring nodes) in the network, the poorer the system performance. Hence a constraint should be placed on the amount of calls the system can accommodate at a time.

A graph of blocking probability vs. calls in progress is shown in Figure 4. As expected, the rate of blocked calls increases as the number of calls increases. This can be attributed to insufficient resources to cater for all the calls accessing the network.

In Figure 5, the throughput is plotted as a function of call arrival rate. In this graph, the dependence of throughput on data rate is well displayed. From the graph, we observe that throughput increases as the achievable rate of data transfer increases. In other words, the system is able to provide response to users even when the conditions of load are varied from low to high.
VI. FURTHER RESEARCH
So far, we have considered voice communication and mobility of users within a single cell. The next direction of this research will address two issues namely, data communication and mobility of users across cell boundaries. This extension will enable the performance evaluation of handover blocking for voice and data communications.

VII. CONCLUSION
In cellular networks, blocking occurs when a base station has no free channel to allocate to mobile calls. Hence, dynamic resource management algorithms are required to manage call admission and channel allocation. The design of these algorithms have always posed difficulties, mostly in multihop “ad-hoc” networks, where relaying nodes and communication needs are primarily accessible between nodes within the same network. This activity leads to dynamic and unpredictable changes in the topology of the system.

This paper has appraised the effect of calls blocking on the performance of multi-hop networks and has presented a probabilistic model that is most suitable to handle blocked calls and system’s performance. We have also examined the various constraints that impair system’s performance due to blocking. These evaluation and performance measures will assist network designers to strategically decide on cell size and the number of channel frequencies allocated to each cell.

REFERENCES


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