Abstract—Wireless Sensor Networks (WSNs) have had remarkable advances in the past couple of decades due to their fast growth and flexibility. In order to supervise an area, hundreds or thousands of sensors can be established and collaborate with each other in the environment. The sensors’ sensed and collected data can be delivered to the base station. Energy optimisation is crucial in WSN’s efficiency. Organising sensor nodes into small clusters helps save their initial energy and thus increases their lifetime. Also, the number and distribution of Cluster Heads (CHs) are fundamental for energy saving and flexibility of clustering methods. Avoid Near Cluster Heads (ANCH) is one of the most recent energy-efficient clustering algorithms proposed for WSNs in order to extend their lifetime by uniform distributing of CHs through the network area. In this manuscript, we suggest an analytical approach to model the energy consumption of the ANCH algorithm. The results of our comprehensive research show a 95.4% to 98.6% accuracy in energy consumption estimation using the proposed analytical model under different practical situations. The suggested analytical model gives a number of indications concerning the impact of different factors on the energy depletion pattern of the ANCH clustering algorithm.

Keywords—Wireless Sensor Networks, ANCH, Energy Efficiency, Clustering, Analytical Model.

I. INTRODUCTION

A Wireless Sensor Network (WSN) is a network of tiny and on-board battery operated sensors with limited power of processing and radio data transmission. They can collect and send their sensed data to a base station to monitor a remote area and perhaps to send the collected data to a remote centre. WSNs can be employed in different applications, because of their low-cost and adaptable nature, including healthcare, emergency response, weather forecasting, commercial, smart traffics, surveillance and volcanic earthquakes [1]–[6]. Moreover, WSNs can be used in an ad-hoc manner and in harsh environments in which a human presence is hard or impossible [7], [8].

Energy efficiency is essential for WSN lifetimes because there is usually no opportunity for battery replacement or recharging. Therefore, developing energy-efficient algorithms is of high importance in WSNs. A large amount of research has been conducted over the past few years to optimise the energy consumption in this area [9]–[12].

Clustering is a widely accepted approach for organising a high number of sensors spread over a large area in an ad-hoc manner [13]. This is useful when we consider that in most cases, neighbouring sensors sense similar data. If each sensor directly sends its data to the base station using long-distance transmission, its energy drains quickly. Moreover, this might also lead to some other issues, such as traffic congestion and data collision. In clustering, all sensors are grouped into a number of clusters and in each cluster, one sensor is elected as the Cluster Head (CH). Transmitting data in each cluster from each sensor to its CH is accomplished using short-distance radio transmission, whereas transmitting data from CHs to the base station is carried out using long-distance radio transmission. In addition, CHs can combine and aggregate similar transmitted data to reduce the length of the messages passed. In this way, CHs consume more energy than other sensors and, therefore, periodic CH election across the network is a suitable approach to balance energy consumption among sensor nodes [14].

The appropriate number and size of the clusters is essential for increasing the network lifetime. For a low number of clusters, a large amount of the energy is consumed to send data from Cluster Members (CMs) to CHs. On the other hand, if the number of clusters is high, a large number of the CHs will be elected and consequently a large number of nodes will operate using long-distance transmission to communicate with the base station. Therefore, a trade-off should be made between these two factors to optimise energy consumption across the network [15].

Over the past few years, a number of clustering algorithms have been proposed. Hence, it is critical that when proposing a new algorithm, we specify its scope and evaluate it with accurate modelling of the underlying organisation and communication mechanisms. Clearly, after using such models, a comprehensive understanding of the factors that affect the potential performance of a network emerges and this makes it easier to evaluate different algorithms and select the best one for practical implementations. Employing physical experiments is impractical for a large number of configurations and running a network simulator for a large number of configurations takes an unacceptable amount of time. Analytical modelling, in contrast, offers a cost-effective and versatile tool that can help to assess the merits of an algorithm [16], [17].

Avoid Near Cluster Heads (ANCH) is a new energy efficient clustering algorithm proposed recently for WSNs to prolong...
network lifetimes by uniformly distributing the CHs [12].
In this manuscript, an analytical model for predicting the
energy consumption of ANCH is proposed. The model details
the factors affecting and analyses energy consumption under
various operational conditions. The accuracy of the proposed
model is evaluated using simulation.

The remainder of this manuscript is organised as follows.
In Section II, related work is discussed. The ANCH clustering
algorithm is briefly presented in Section III. The proposed
analytical model of ANCH and its validation are presented in
Section IV and Section V, respectively. Finally, Section VI
contains our concluding remarks and future work.

II. RELATED WORK

Over the past few years, a number of clustering algorithms
for WSNs have been proposed such as Low Energy Adaptive
Clustering Hierarchy (LEACH) [9], Hybrid Energy-Efficient
Distributed (HEED) [18], Low Energy Adaptive Clustering
Hierarchy with Sliding Window and Dynamic Number of
Nodes (LEACH-SWDN) [19], ANCH [12], and Activity-
aware ANCH (A-ANCH) [20]. One of the most popular
clustering algorithms for WSNs is LEACH. LEACH is popular
not only for its simplicity, but also for the idea of rotating CHs
to efficiently balance energy consumption among nodes [9].
The lifetime of the LEACH is divided into a number of rounds
in which every round includes a set-up and a steady phase.
In set-up phase, clusters are organised, while in steady phase,
sensed data are transferred from sensors to CHs. Election of
CHs is carried out in a distributed manner and independent
from each other. CHs collect sensed data from sensors and
aggregate and forward them to the base station in each round.
To decide whether to become a CH, each sensor chooses a
random number which is a number between 0 and 1, in
each round. A sensor decides to be a CH in a round if its
generated random number is less than the threshold below [9]:

\[ T(n) = \begin{cases} 
\frac{p}{r \bmod \frac{p}{r}} & \text{if } n \in G \\
0 & \text{otherwise}
\end{cases} \tag{1} \]

where \( n \) is the number of each sensor in the network, \( r \) is the
current interval’s round number, \( p \) is a predefined percentage
of CHs in each round, \( \frac{p}{r} \) is called the interval, and \( G \) is the
set of sensors which have not yet been CHs in the current
interval. An example of CHs and CMs formed after a set-up
phase in a round according to the LEACH algorithm is shown in
Figure 1.

In LEACH, \( p \) percent of sensors are elected as CHs on
average in each round and also each sensor is elected as a
CH only once in an interval. The probability of each node
becoming a CH is \( p \) in the first round. This probability is
\( \frac{p}{1-p} \) for the second and third rounds and
finally this probability reaches 1 for the last round. Thus all
of the remaining sensors will be CHs in the last round. Each
sensor that has decided to be a CH in a round, advertises itself
to all of the other sensors throughout the network. Using the
received signal’s strength, all non-CH sensors can find the
closest CH to themselves and join that cluster as a CM.

A centralised version of LEACH, called LEACH-C, is
proposed by the authors of LEACH, Heinzelman et al. [21],
in which each sensor knows its position in the network and
consequently the number and position of CHs are selected in
an optimum manner per round. It has been demonstrated
that LEACH-C shows up to 40% greater performance than
LEACH. Another centralised version of LEACH is Base sta-
tion Controlled Dynamic Clustering Protocol (BCDCP), which
was proposed by Muruganathan et al. [22]. BCDCP uses
multi-level clustering so that each CH serves an approximately
equal number of sensors. In the BCDCP algorithm all CMs
send their sensed data to their CHs and CHs conduct a CH-
to-CH multi-hop routing to deliver their data to a higher
level CH, which is selected randomly. Finally, only one CH
communicates directly with the base station. It has been shown
that BCDCP in this way outperforms LEACH and LEACH-C.

Two distributed variations of LEACH have been proposed
by Nam et al. [23] and Handy et al. [24]. In the first algorithm,
a number of messages are exchanged between sensors to
find out their position across the network. Thus, more equal
clusters are shaped by choosing appropriate CH positions.
They compared their algorithm with LEACH and LEACH-C
and their experiments showed that their algorithm outperforms
LEACH but not LEACH-C, which is a centralised algorithm.

On the other hand, Handy et al. [24] believe that a stochastic
CH election might not lead to an efficient energy consumption
model in a network. They therefore involved the sensor’s
residual energy when computing the threshold \( T(n) \) as an
effective parameter in deciding whether or not to stand as

\[ T(n) = \begin{cases} 
\frac{p}{r \bmod \frac{p}{r}} & \text{if } n \in G \\
0 & \text{otherwise}
\end{cases} \tag{1} \]
a CH. Their experiments showed that their algorithm can prolong network lifetime by up to 30%.

Heterogeneity of sensors energy in the network has been treated by Smaragdakis et al. [25]. They proposed the Stable Election Protocol (SEP) algorithm, in which a number of sensors are equipped with more energy than others to prolong the stable time of the network, which is when all the network’s sensors are alive. SEP is a distributed algorithm and sensors are not aware of other sensors’ residual energy. Their experiments showed that SEP can increase the stable time of the network and this increment depends on the percentage and the initial energy levels of the network’s powerful sensors.

Two centralised fuzzy algorithms for WSNs have been proposed by Gupta et al. [26] and Ran et al. [27]. They assumed that each sensor knows its position in the network and sends its local information to the base station. Appropriate sensors are then elected by the base station to act as CHs. In both algorithms, three parameters are used as the algorithm input. In [26] these parameters are sensor residual energy, sensor density, and sensor centrality in the clusters whereas in [27] the sensor centrality is replaced by distance of sensors from the base station. The performance evaluations of both studies showed the superiority of their algorithms over the LEACH clustering algorithm.

Wireless Sensor Network Clustering with Artificial Bee Colony (WSNCABC) has been proposed by Karaboga et al. [28]. In WSNCABC each sensor calculates its distance from other sensors using message transferring advertisement and sends this information to the base station. The base station then selects CHs using an Artificial Bee Colony algorithm and broadcasts them back to the network. Finally, each sensor joins the closest elected CH. Their performance study showed that the proposed algorithm considerably outperforms the LEACH clustering algorithm, by up to 70% in some scenarios, when the base station is very close to the edge of the network. However, as there is a large amount of communication between sensors and the base station in the set-up phase, WSNCABC appears to be inefficient when the base station is located far away from the network area.

HEED [18] is a distributed clustering algorithm for WSNs which takes into account sensors residual energy and communication cost during CH election. In HEED, the transmission power of every node is set to a constant value and each sensor considers other nodes as its neighbouring nodes if they are within its transmission range. Moreover, two neighbouring sensors, which are within the transmission range of each other, are not elected as CHs simultaneously, trying to uniformly distribute CHs across the network. In order to find its CH, each sensor launches an iterative procedure in each round. A node nominates itself as a CH if it has either no neighbouring node or has not received a CH advertisement.

ANCH also, similar to HEED, takes the advantage of uniform distribution of CHs in order to achieve optimised, or close to, network energy consumption. Nevertheless, it has a few key advantages over HEED. Firstly, the set-up phase overhead of ANCH is much less than that of HEED because HEED executes a procedure to find neighbouring sensors. Also, in this phase, each sensor in HEED executes a complicated iteration including some message passing to select its CH. Secondly, by the end of each iteration in HEED, a node elects itself as a CH if no other CH advertisement has been received. Thus, in many rounds, the number of formed clusters is much more than that of the ANCH algorithm where all sensors receive CH advertisement if there exists at least one CH in the network. Finally, ANCH and LEACH are two scalable algorithms both with processing time and message exchange complexity of O(1) and O(N), respectively [29]. Whereas, HEED has O(N) complexity for both processing time and message exchange complexity [30], [31].

In order to design an energy efficient algorithm for WSNs, it is important to make a trade-off between different parameters involved in a specific application to ensure that the optimum configuration has been applied to maximise the network lifetime. In particular, it is quite critical to balance the energy costs of individual nodes in order to obtain the best overall network energy cost. Simulation study of the effects of different parameters on the performance of a network under various network circumstances is difficult because of the time consuming feature of these kinds of tools. Analytical modelling, in contrast, is beneficial as it offers a cost-effective tool to estimate the network energy consumption accurately within an acceptable amount of time. Therefore, in addition to the research on proposing efficient algorithms for WSNs, a number of studies have also been conducted to develop analytical models [15]–[17], [21], [32].

The first analytical model for the LEACH algorithm has been proposed by Heinzelman et al. [21]. In this study, it has been shown that the energy consumption in a network is proportional to the square of transmission distance in clusters. This can be obtained for each sensor using the following expression:

\[ E \left[ d^2_{toCH} \right] = \rho \int_{\theta=0}^{2\pi} \int_{r=0}^{M} r^3 \, dr \, d\theta = \frac{M^2}{2k\pi} \]  

where \( E \left[ d^2_{toCH} \right] \) is the expected square distance of sensors from their CH, \( \rho = \frac{1}{r^2} \) and is called sensors’ density, \( k \) is the number of clusters, and \( M \) is one side of the network area.

However, some non-realistic assumptions have been made when developing the model; the area of all clusters are disc-shaped with radius \( r \), all clusters are assumed to be formed equally, and also the area of the network is covered by these \( k \) non-overlapping clusters.

In [33], Bandyopadhyay and Coyle have proposed a mathematical model for hierarchical clustering algorithms for WSNs. They assumed that the sensors are very simple and all sensors transmit at a fixed power level. Their model analytically suggests the number of CHs at each level of clustering. They conducted a set of experiments to show the optimum number of CHs in different levels of hierarchy in dense networks, with up to 25,000 nodes. Nevertheless, their proposed model is not
general enough due to a number of unrealistic assumptions on the fixed power level transmitting ability of nodes.

In [15], Islam et al. have argued that a complete analytical model to find the best configuration on LEACH is beneficial to prolong network lifetimes. They therefore proposed a complete mathematical model for LEACH to find the optimum number of CHs. Their experiments showed the reasonable accuracy of their model against original LEACH in order to compute the optimal number of CHs. Although their model takes into account all steps of the LEACH algorithm, it does not show a considerable advantage over the model presented in [21].

The issues raised in [21] have been addressed in another model proposed by Choi et al. [34]. This model can estimate the energy consumption and determine the optimal number of clusters to minimise the energy spent in the network for the LEACH algorithm. In this research, it has been shown that the energy consumption in a network is proportional to the summation of square of transmission distance in clusters. This can be obtained for each cluster using the following equation:

\[
E \left[ \sum_{\text{node} \in \text{cluster}(j)} d_{i \text{CM}}^2 \right] = 2\pi \lambda_{CM} \int_{0}^{\infty} r^3 \exp(-\lambda_{CH} \pi r^2) dr
\]

where \( E \left[ \sum_{\text{node} \in \text{cluster}(j)} d_{i \text{CM}}^2 \right] \) is the expected sum of the square distance of sensors from their CH, \( \lambda_{CM} \) is the density of the CMs in the network area and is given by \( \frac{N - k}{M^2} \), \( \lambda_{CH} \) is the density of the CHs in the network area and is given by \( \frac{\lambda}{\pi r^2} \), \( N \) is the number of nodes, \( k \) is the number of clusters, and \( M \) is one side of network area.

Using a simulation software, their experiments showed that their model has over 80% accuracy with the LEACH algorithm and is considerably superior to [21].

Details of the ANCH algorithm are described in the next section.

III. THE ANCH CLUSTERING ALGORITHM

The proper position of CHs is essential in the energy efficiency of clustering algorithms. This has been neglected in the LEACH algorithm and consequently there might be some CHs which are located too close or too far from each other. In either case, some waste of energy might occurred for data transferring from sensors to the base station.

To overcome this, the ANCH algorithm tries to uniformly distribute CHs across the network as much as possible. To do so, a parameter \( d \) is defined as the closeness depending on the region size and also network density. If two CHs are found too close to each other in a particular round, closer than \( d \), one of them should stand as the CH. Thus once the first CH is selected following normal LEACH procedure, the next potential CH checks its distance from the first CH before advertising itself to other sensors as a CH. If the distance is less than \( d \), it cancels its decision to be a new CH in the current round and remains a CH candidate for the future rounds. The same procedure is applied for all of the potential CHs in the current round.

This action is applied and can be considered as the first attempt to shape clusters with about the equal size. Figure 2 demonstrates an example of CH selection according to the first step of the ANCH algorithm.

Fig. 2: An example of CHs and CMs in first step of ANCH algorithm. CHs are shown by red diamonds and CMs are shown by black circles.

Also, at the end of each interval, the number of the CHs is most likely more than the optimum amount. Thus, the number of clusters probably increase and more CHs have to send their packets to the base station by using long-distance communication. Figure 3 demonstrates an example of CHs and CMs at the end of each interval.

Since all sensors have to be a CH in an interval, all of the remaining sensors will be elected as CHs in the last round regardless of their closeness to each other.

A further improvement in ANCH is also obtained by considering the optimum number of CHs in the network. Imagine \( p \) is the optimum percentage of CHs among all sensors. So far in the ANCH algorithm, apart from the intervals’ last rounds, the number of selected CHs in each round is most likely less than \( p \) percent. This is because a number of potential CHs might cancel their decision of being a CH due to their close position to other CHs. Therefore, the number of clusters would be less than the optimum number suggested in the LEACH algorithm. This leads to the bigger cluster size and more energy consumption over the intra-cluster transmission.

On the other hand, in the last rounds of intervals, the percentage of CHs would be much more than the optimum
Fig. 3: An example of CHs and CMs at the end of each interval in the ANCH algorithm. CHs are shown by red diamonds and CMs are shown by black circles.

number \( p \). Consequently, the number of clusters might increase and more CHs have to send their data to the base station directly using long-distance transmission. This issue is addressed in the ANCH algorithm by increasing the threshold \( T(n) \) and consequently increasing the number of potential CHs in each round. As a result, in every round more than \( p \) percent of sensors will be nominated as CHs, on average, to become closer to the optimum value, \( p \), after dropping a number of them because of the closeness issue. After setting the new threshold, close to \( p \) percent of sensors are eventually selected as the CHs in every round which are more uniformly distributed compared with LEACH. The new threshold, \( T'(n) \), in ANCH is defined as follows:

\[
T'(n) = T(n) + (1 - T(n)) \times a. \tag{4}
\]

\( T(n) \) is the threshold value of the LEACH algorithm \cite{9} and \( a \), the add-on coefficient, is a constant, whose value depends on the network configuration and also on the closeness value, \( d \). This value plays an essential role in the ANCH algorithm’s efficiency.

The ANCH algorithm significantly improves network energy consumption and, consequently, prolongs the network lifetime compared with the LEACH algorithm. An example of the positions of CHs and CMs in ANCH is shown in Figure 4. Comparing this arrangement with the one presented in Figure 1 reveals more uniform distribution of CHs in the ANCH algorithm.

The analytical model of the ANCH clustering algorithm is proposed in the next section.

IV. ANCH ANALYTICAL MODELLING

In this section, our proposed analytical model for the energy consumption in the ANCH clustering algorithm is presented. Using the model, a comprehensive understanding of the factors affecting the performance of a network emerges. Since a clustering approach is employed in the ANCH algorithm, the total network energy consumption can be derived when the energy consumed by one cluster is calculated.

Let us assume that \( N \) sensor nodes are randomly distributed in a \( M \times M \) area and the number of clusters, on average, is \( k \) during the lifetime of the network. As a result, there are \( \frac{N}{k} \) sensors, on average, per cluster with \( \left( \frac{N}{k} - 1 \right) \) sensors as CMs and also one node as the CH.

The energy required for a CM to send its data to a CH can be calculated using the following expression \cite{9}:

\[
E_{CM} = l E_{elec} + l \epsilon_{amp} d^{2}_{toCH} \tag{5}
\]

Also, for all nodes in a cluster, this energy can be calculated as follows:

\[
E_{Cluster} = l E_{elec}(k - 1) + l \epsilon_{amp} E \left[ \sum_{nodes \in Cluster} d^{2}_{toCH} \right] \tag{6}
\]

where \( l \) is the length of messages, \( E_{elec} \) is the transmission electronics, \( \epsilon_{amp} \) is the transmission amplifier, \( d_{toCH} \) is the distance between a CM and its CH, and \( E \left[ \sum d^{2}_{toCH} \right] \) is the expected sum of square distance of CMs from their CH. Except for \( E \left[ \sum d^{2}_{toCH} \right] \), all other parameters in (6) are known with
constant values. Therefore, by calculating $E \left[ \sum d_{toCH}^2 \right]$ we are able to calculate all the energy spent in the network.

$E \left[ \sum d_{toCH}^2 \right]$ can be calculated using the following expression for LEACH [34]:

$$E \left[ \sum_{node \in \text{cluster}(j)} d_{toCH}^2 \right] = 2\pi \lambda_{CM} \int_0^\infty r^3.P \{ (r,j) \in \text{cluster}(j) \} dr$$  \hspace{1cm} (7)$$

where $\lambda_{CM}$ represents the density of the CMs in the network and is given by $\frac{N-K}{r^2}$. $P \{ (r,j) \in \text{cluster}(j) \}$ is the probability of a sensor node becoming a member of cluster $j$. The distance between the node and the head of cluster $j$ is also represented by $r$. According to [35], $P \{ (r,j) \in \text{cluster}(j) \}$ can be derived from the palm distribution as follows:

$$P \{ (r,j) \in \text{cluster}(j) \} = \exp \left\{ -\lambda_{CH}\pi r^2 \right\} \hspace{1cm} (8)$$

where $\lambda_{CH}$ represents the density of the CHs in the network and is given by $\frac{K}{r^2}$. In ANCH, the distance between any two CHs is not less than $d$. Each cluster area is divided into two different parts, which are treated separately in our model. The first part is the circular area with the radius of $d/2$ from the CH. All sensors in this area securely belong to that cluster. The second area covers those sensors whose distance from the current CH is more than $d/2$. For the first part, (7) with the probability $P \{ (r,j) \in \text{cluster}(j) \} = 1$ can be used. Thus, the expected sum of the square distance of CMs, located in the first part of the cluster area, from their CH can be obtained using the following expression:

$$E \left[ \sum_{node \in \text{cluster}(j)} d_{toCH}^2 \right] = 2\pi \lambda_{CM} \int_0^{d/2} r^3 dr$$  \hspace{1cm} (9)$$

On the other hand, all sensors whose distance from other CHs is less than $d/2$ are secure members of other CHs and are not members of the current CH. Thus, $P \{ (r,j) \in \text{cluster}(j) \} = 0$ for those nodes. Consequently, the value of (7) for those nodes is 0. To calculate the second part of the cluster area, we must subtract the cluster areas whose nodes’ distance from a CH is less than $d/2$.

The second part of each cluster area can be calculated by

$$E \left[ \sum_{node \in \text{cluster}(j)} d_{toCH}^2 \right] = 2\pi \lambda_{CM} \int_{(d/2)^3 \sqrt{\pi}}^\infty r^3.\exp \left\{ -\lambda_{CH}\pi r^2 \right\} dr$$  \hspace{1cm} (10)$$

In the above expression, $R_1$ can be calculated as follows:

$$\pi R_1^2 = k\pi \left( \frac{d}{2} \right)^2 \Rightarrow R_1 = \left( \frac{d}{2} \right) \sqrt{k} \hspace{1cm} (11)$$

Using (9) and (10), the first and second parts of each cluster area can be merged. Thus, the expected sum of the square of each CM from its CH can be obtained from the following expression:

$$E \left[ \sum_{node \in \text{cluster}(j)} d_{toCH}^2 \right] = 2\pi \lambda_{CM} \left[ \int_0^{d/2} r^3 dr + \int_{(d/2)^3 \sqrt{\pi}}^\infty r^3.\exp \left\{ -\lambda_{CH}\pi r^2 \right\} dr \right]$$  \hspace{1cm} (12)$$

Also we know that

$$\int_0^{d/2} r^3 dr = \left( \frac{d^4}{64} \right) \hspace{1cm} (13)$$

and we can imagine

$$u = \lambda_{CH}\pi \hspace{1cm} (14)$$

The expression (10) can be simplified to the following expression:

$$\int_{(d/2)^3 \sqrt{\pi}}^\infty r^3.\exp \left\{ -ur^2 \right\} dr = 2\pi \lambda_{CM} \hspace{1cm} (15)$$

Consequently, expression (12) can be simplified to the following expression:

$$E \left[ \sum_{node \in \text{cluster}(j)} d_{toCH}^2 \right] = 2\pi \lambda_{CM} \hspace{1cm} (16)$$
The first and second parts of the network areas can be shown in Figure 5. In this figure, the inner circle shows the first part of each cluster in which \( P\{ (r,j) \in \text{cluster}(j) \} = 1 \). The area between inner and outer circles demonstrates the first part of the other clusters in which \( P\{ (r,j) \in \text{cluster}(j) \} = 0 \). The area beyond the outer circle shows the second part of the current cluster in which \( P\{ (r,j) \in \text{cluster}(j) \} = \exp\{ -\lambda_{CH} \pi r^2 \} \).

The accuracy of the proposed analytical model for ANCH is evaluated in the next section.

V. MODEL VALIDATION

The accuracy of the described analytical model has been verified by comparing it with simulation results. Extensive validation experiments have been performed for several combinations of cluster size, network dimension, different values of closeness, density of sensors in the network, and the number of messages which are sent from CMs to their CHs during the steady phase, called \( M\text{Numbers} \). In order to select parameter \( a \), different values including \( a = 0.02, 0.05, 0.15, 0.25, ... , 0.75 \) have been considered and the most effective value has been selected. Each simulation scenario is run for 100 different randomly generated topologies and the average results are presented. In our experiments, the sensors’ inner computational procedures do not consume energy: all of their energy is used for message passing only. The energy model in all of our experiments is precisely the same as the one employed in [9].

Moreover, \( d = 15 \) metres and the initial energy of each node is 10 J. Finally, the number of clusters in this experiment varies from 4 to 15 clusters. The results are presented in Figures 6 and 7. In these figures, the horizontal axis shows the number of clusters where the vertical axis represents the total consumed energy. Figure 6 shows the accuracy of our model for three different networks with different numbers of nodes, \( N = 50, 100, \) and 200, when \( M\text{Numbers} \) is considered to be 25. Also, Figure 7 shows the accuracy of our model for three different \( M\text{Numbers} \), \( M\text{Numbers} = 25, 50, \) and 100 messages per round, when the number of nodes is \( N = 100 \).

96.3% accuracy in Figure 6 and 97.1% in Figure 7 show that the simulation results closely match those predicted by the analytical model.

In the second experiment, we aim at observing the impact of network size on our analytical model. Different network dimensions from 10 to 100 metres are examined while the value of \( d \) is 30% of one dimension. Moreover, the initial energy of each node is 10 J and the number of clusters, \( k \), is 5. These are depicted in Figures 8 and 9, highlighting that the proposed model on average presents an accuracy of 95.4% and 98.6%, respectively. Figure 8 shows the accuracy of our model for three different networks with different numbers of nodes, \( N = 50, 100, \) and 200, when \( M\text{Numbers} \) is considered to be 25. Also, Figure 9 shows the accuracy of our model for three different \( M\text{Numbers} \), \( M\text{Numbers} = 25, 50, \) and 100 messages per round, when the number of nodes is \( N = 100 \).

In the third experiment, we aim at observing the impact of the closeness parameter, \( d \), on our analytical model. Different closeness values from 5 to 25 metres are examined where the network area is considered to be 50 \( \times \) 50 square metres and the base station is 100 metres away from the network’s edge. Moreover, the initial energy of each node is 10 J and
the number of clusters is 5. This is depicted in Figures 10 and 11, highlighting very close agreement between the model and the simulation in these figures, with 95.8% and 95.6% similarity on average. Figure 10 demonstrates the accuracy of the proposed model for three different networks with different numbers of nodes, $N = 50, 100, \text{ and } 200$, when $M_{\text{Number}}$ is considered to be 25. Also, Figure 11 shows the accuracy of our model for three different $M_{\text{Numbers}}$, $M_{\text{Number}} = 25, 50, \text{ and } 100$ messages per round, when the number of nodes is $N = 100$.

In the fourth experiment, we aim at observing the impact of network density on our analytical model. In this experiment, different numbers of sensors, from 40 to 500, are examined. Moreover, the network area is $50 \times 50$ square metres when the base station is 100 metres away from the network’s edge, $d = 15$ metres, the initial energy of each node is 10 J, and the number of clusters is 5. The results are presented in Figure 12 for three different configurations, $M_{\text{Number}} = 25, 50, \text{ and } 100$.

These results show a close agreement, an accuracy of 95.4% on average, between the proposed model and simulation results.

Finally, in the last experiment, we aim at observing the impact of steady phase duration on our analytical model by varying the number of $M_{\text{Numbers}}$ from 5 to 1000 messages...
Fig. 11: Accuracy of the model comparing against simulation results for three values of \( M_{\text{Number}} \), \( M_{\text{Number}} = 25, 50, \) and 100 messages per round.

Fig. 12: Accuracy of the model comparing against simulation results for three values of \( M_{\text{Number}} \), \( M_{\text{Number}} = 25, 50, \) and 100 messages per round.

Fig. 13: Accuracy of the model comparing against simulation results for three networks with different numbers of nodes, \( N = 50, 100, \) and 200.

Overall, our extensive validation study shows the credible accuracy of our proposed analytical model in predicting the total energy spent by the ANCH algorithm. Using the proposed model, a number of factors have been revealed. First, the energy consumed by the ANCH algorithm is almost insensitive to the optimum number of clusters, \( k \), proposed by the LEACH algorithm. This is due to the important role of the add-on coefficient, \( a \), in balancing the energy consumption of each cluster. By increasing the value of \( k \), the optimum value of \( a \) is also increased to protect the network from forming a large number of clusters with smaller numbers of nodes in each cluster and hence to avoid wasting energy. Concomitantly, the optimum value of \( a \) is also decreased to block the negative effects of smaller numbers of clusters.

In the same way, the energy consumed by the ANCH algorithm is almost insensitive to the closeness parameter. This is again due to the balancing role of the add-on coefficient, \( a \). By increasing the value of the closeness parameter, the optimum value of \( a \) is also increased to increase the number of potential CHs, avoiding smaller numbers of clusters. It also prevents the formation of large numbers of clusters when the closeness value is decreased.

VI. Conclusions and Future Work

Energy efficiency is essential in the design of practical WSNs. One of the most recent suggested distributed energy-efficient algorithms for them is the ANCH algorithm. ANCH extends the network lifetime by uniform distributing CHs throughout the network environment. In this manuscript, an analytical model for ANCH has been presented which demonstrates the impact of different factors in ANCH energy consumption and which can estimates its energy consumption in different situations. Our comprehensive research has shown a 95.4% to 98.6% accuracy in energy consumption estimation using the suggested analytical model compared to simulations. Also, the suggested analytical model has shown that the energy depletion of the ANCH algorithm is unresponsive to
the closeness parameter and the number of clusters due to the adjusting role of the add-on coefficient in optimising the total energy depletion of clusters. In our future work, we are going to adapt the ANCH algorithm and its analytical model for mobile sensors and evaluate them using other simulation softwares.

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REFERENCES


