Using optical flow equation for particle detection and velocity prediction in particle tracking

Luca Shindler *, Monica Moroni, Antonio Cenedese

DICEA, Sapienza University of Rome, Via Eudossiana 18, 00184 Rome, Italy

**A B S T R A C T**

A new algorithm of particle identification suitable for particle tracking technique in fluid mechanics is proposed and tested with synthetic images specifically developed with different particle parameters. The new approach is based on the solution of the optical flow equation via a sum-of-squared-difference method. Particles are detected through the identification of corner features, where image intensity gradients are not null in two orthogonal directions. It is thus possible to identify low intensity and overlapped particles. Furthermore, the feature selection criterion is optimal by construction because it is based on the optical flow solution and therefore a good feature is the one that can be tracked well. This leads to the second advantage of the method, which is the possibility to obtain the local velocity, given by the approximate solution of the optical flow equation, that can be used as a predictor for the subsequent particle pairing step. The proposed algorithm is tested using synthetically generated and experimental images and demonstrates its ability to detect a great number of particles with high reliability in different cases analysed.

© 2012 Elsevier Inc. All rights reserved.

1. Introduction

Particle tracking velocimetry (PTV) is a widely used image analysis technique that is able to describe the properties of fluid flows via the reconstruction of particle paths. These are obtained by tracking tracer particles seeding the fluid under investigation. The particles are illuminated with an appropriate light source, such as lasers, and the scattered light is recorded with high quality digital cameras. PTV differs from cross-correlation techniques, in its ability to provide a Lagrangian description of motion, which is of fundamental importance for understanding turbulent mixing and scalar dispersion [1,2].

Once the single-exposure images recording the fluid motion are acquired, the two-dimensional PTV procedure involves three consecutive steps:

1. identification of each particle in the image space of the camera;
2. calculation of barycentre coordinates for each detected particle;
3. temporal matching of particle positions, so as to generate particle trajectories.

Moreover, in three-dimensional analysis the reconstruction of particle position in the object space is usually required [3,4].

Although these different phases are all of fundamental importance, the steps of particle identification and the subsequent barycentre calculation will crucially influence the accuracy and precision of the whole particle tracking system.
A higher accuracy of a PTV system can be achieved by increasing both the camera spatial and temporal resolution, although oversampling may create additional problems (e.g. slowly moving particles sampled with higher frequency generate errors in the trajectory reconstruction). At the same time the analysis of the acquired images has to guarantee comparable high accuracy through the development of suitable algorithms. Optimal algorithms for particle identification and barycentre calculation must address the following issues:

- **Overlapping particles**: the increase in the number of particle images determines an additional difficulty to distinguish particle centres.
- **Sub-pixel accuracy**: the interpolation of digital number (DN) values (grey levels) associated with the pixel coordinates of the identified particle determines the calculation of particle centres within an approximation of a fraction of a pixel.
- **Robustness to noise**: the noise of the acquired images, represented by variations of digital numbers not related to the object (particles) and due to signal amplification in the camera sensor, thermal noise, or detection of scattered light, will influence the accuracy of both particle identification and centre calculation algorithms.
- **Speed**: due to the use of high temporal resolution camera systems, the particle identification algorithms must meet the criteria of efficiency and speed.

The usual approach to particle identification in PTV systems is segmentation, which means dividing an image into meaningful regions. In PTV systems the meaningful regions are represented by particle images [5]. The simplest way to perceive segmentation is to use a single threshold value for the whole image. This method assumes that particle pixels have grey levels generally different than those of the background. Obviously, this is reliable only if a small number of particles with high luminosity characterises the images. This is unlikely due to random noise, low-intensity particles, overlapping particles and inhomogeneous illumination. A possible solution to overcome these problems is to vary the threshold across the image, by using a local threshold binarization [6]. This value is set based on the local grey level mean and standard deviation. Although this method is able to take into account spatial variations of illumination, the problem of overlapping particles cannot be solved. This problem could be solved by introducing an additional procedure that provides for the calculation of local maxima of grey level intensity, that is known with the name of blob splitting [7].

Improved results can be obtained by using the dynamic threshold binarization algorithm [8,9]. This method starts from a binarization of the whole image with a base threshold. Then adjacent pixels with intensity levels that exceed the base threshold are labelled together. The mean grey level of the labelled areas is then compared with the base threshold: if the difference between these two values does not exceed a preset contrast threshold, the procedure ends and no further operation is performed; otherwise, one brightness level is deducted from each pixel of this area. In the next step, the binarization and labelling procedures are repeated and the mean intensity level of every labelled area is checked again with the base threshold. This cycle is iterated until the difference between the mean brightness of each labelled area and the base threshold becomes lower than the contrast threshold.

Thresholding algorithms are not the only way to detect particle images. The particle-mask-correlation operator [10] and the improved Moravec operator [11] compute particle centres directly from the image without a binarization procedure. An advanced algorithm for particle identification is presented here. Although it was partially introduced by the authors in Shindler et al. [12], in this paper its performances are deeply investigated.

It is based on the solution of the optical flow equation using sum-of-squared-difference (SSD) of the pixel intensity levels between interrogation windows in two consecutive frames [13]. It is a corner detection algorithm, similar to the Moravec operator, but it differs from it because the identification of the feature is strictly related to the motion of the particle itself. This is why the conceived particle identification criterion is optimal and the corners detected are called good features to track [13]. Furthermore, the solution of the optical flow equation through the Lucas–Kanade algorithm [14] enables the calculation of the local velocity vector that can be used as a predictor for the successive step of particle pairing.

The proposed algorithm is tested using 2D synthetic images specifically developed for this work and 2D experimental data. The results are compared with those obtained by using thresholding algorithms.

2. Good features to track

A procedure of feature selection for particle detection should be based on the assumption of correspondence between particle and feature. Despite this, even if a region of a PTV image is rich in texture this might not be a particle, but a bad feature such as reflections or noise in the acquired image. Therefore the good feature associated to the particle should be the one that makes the tracking work best [13]. In this sense the solution of the optical flow equation can be a possibility. Under the hypothesis of brightness constancy constraint (i.e. a given fluid particle keeps the same brightness along its trajectory), the optical flow equation defines the conservation of the image intensity at time \( t \), \( I(x, t) \), with \( x = (x,y) \) the generic pixel coordinate [15]:

\[
\frac{\partial I}{\partial t} + u \frac{\partial I}{\partial x} + v \frac{\partial I}{\partial y} = \frac{\partial I}{\partial t} + \nabla^2 I \mathbf{U} = I_t + u I_x + v I_y = 0,
\]  

(1)
where

\[ \nabla l(x, t) = \begin{bmatrix} \frac{\partial l(x, t)}{\partial x} \\ \frac{\partial l(x, t)}{\partial y} \end{bmatrix} \]

is the pixel grey level gradient;

\[ l_t \]

is the partial derivative of image intensity with respect to \( t \);

\( \mathbf{U} = (u, v) \) is the unknown velocity vector.

Eq. (1) cannot be directly solved to determine the two unknown velocity components \((u, v)\) associated to a single pixel. This underdetermined problem can be solved if the equation is computed in a window \( W \), with the assumption that the velocity is locally constant. A least square optimisation can be adopted to estimate the displacement vector of the interrogation window between two consecutive frames.

The cost functional SSD over the window \( W \), between the image intensity at time \( t \), \( l_1 = l(x, t) \), and the image intensity at a following time \( t + \tau \), \( l_2 = l(x + \mathbf{U} \tau, t + \tau) \), can be written as [16]:

\[ \text{SSD}(\mathbf{U}) = \iint_W (l_2 - l_1)^2 dx. \]  

(2)

The linearisation of the Taylor expansion of \( l_2 \) (assuming the sampling frequency is sufficiently high) and the minimisation of the derivative of the SSD with respect to \( \mathbf{U} \), by setting it to zero, lead to the linear system [14, 17]:

\[
\begin{bmatrix}
\int_W l_x^2 \, dx & \int_W l_x l_y \, dx \\
\int_W l_y l_x \, dx & \int_W l_y^2 \, dx
\end{bmatrix}
\begin{bmatrix}
u \\
v
\end{bmatrix}
+
\begin{bmatrix}
\int_W l_x l_1 \, dx \\
\int_W l_y l_1 \, dx
\end{bmatrix}
= 0.
\]  

(3)

or more simply:

\[ \mathbf{G} \cdot \mathbf{U} + \mathbf{b} = 0, \]  

(4)

where \( \mathbf{G} \) is the Harris matrix [18] and \( \mathbf{b} \) is a two-dimensional vector, also called image mismatch vector [19].

A feature tracking technique [16] aims to calculate the velocity vector, that is given by:

\[ \mathbf{U} = -\mathbf{G}^{-1} \cdot \mathbf{b}. \]  

(5)

The existence of a solution depends on the invertibility of the matrix \( \mathbf{G} \), which is defined only by the grey levels of the image \( l \).

Matrix \( \mathbf{G} \) is invertible if its eigenvalues, \( \lambda_1 \) and \( \lambda_2 \), are not null. If both \( l_x \) and \( l_y \) are locally different from zero, then the trace and the determinant of \( \mathbf{G} \) are positive. Therefore, both eigenvalues are always real and positive, and system (3) can be solved [12].

Furthermore, the two eigenvalues define if the feature and the correspondent particle are good to track. The noise requirement implies that both eigenvalues must be large, while according to the conditioning requirement they cannot differ by several orders of magnitude. Since the grey level variations are bounded by the maximum allowable pixel value, if the smaller eigenvalue is sufficiently large to overcome the noise criterion, the matrix \( \mathbf{G} \) is usually well-conditioned [13].

2.1. The feature extraction procedure

The first step of the feature extraction algorithm is to compute \( \mathbf{G} \) in a discrete way for each pixel of the image with a \( 3 \times 3 \) pixel window (\( W \)), where the partial derivatives are calculated with a central difference scheme. For the tracking prediction step, via the solution of system (3), a larger window should be used in order to handle large motions [19]. The side length of this window should be at least twice the displacement of the feature between two consecutive frames and can be approximated with a priori knowledge of the mean displacement of the flow. Once the Harris matrix is computed, the second step consists in determining the minimum eigenvalue, for each image pixel, and then rejecting the corners that do not satisfy the condition:

\[ \min(\lambda_1, \lambda_2) > \lambda_T, \]  

(6)

where \( \lambda_T \) is a predefined threshold. This value is set as a percentage of the maximum among all the minimum eigenvalues \( \{\lambda_{\text{min}}\} \):

\[ \lambda_T = C \max\{\lambda_{\text{min}}\}, \]  

(7)

where \( C \) is a user-defined parameter, expressed as percentage value. The choice of the value of \( C \) is related to the noise present in the image: the higher the noise, the higher should be the percentage. For a medium noisy image typical values of \( C \) can range from 0.005 to 0.01.

The definition of \( \lambda_T \) corresponds to introducing a threshold level embedded in the single particle identification: the particle exists if two sufficiently high grey level gradients along two orthogonal directions are present. In this way, dark particles or overlapping particles can be identified [12]. Among pixels fulfilling the threshold constraint, only the local maxima of \( \{\lambda_{\text{min}}\} \) in \( 3 \times 3 \) neighbourhoods are retained [19].
Despite this local search, a single particle could be associated with more than one feature. This aspect needs to be considered. So, first of all, a minimum distance, $d_{\text{min}}$, between the features, depending on the seeding density, is introduced. This distance should be the minimum length for recognising overlapped particles and should guarantee that each particle is associated to one feature only.

Once the good feature is identified, it is necessary to calculate the coordinates of the particle barycentre, which is used in the tracking procedure. Therefore, the local maximum of grey levels, within a square region of side $2L + 1$ around the integer position of the feature, is found first. In order to reduce overlapping windows the following constraint is imposed:

$$L \leq d_{\text{min}}. \quad (8)$$

This local maximum represents the grey level peak of the grey level distribution of the particle image. The barycentre is then calculated through the best fitting method developed for small particles, i.e. by two 1D Gaussian functions [2]. If $x_p$ is the horizontal integer coordinate of the local maximum intensity, the $x$-coordinate, $x_0$, of the particle barycentre can be determined with sub-pixel accuracy [12]:

$$x_0 = x_p + \frac{\ln(x_p - 1, y_p) - \ln(x_p + 1, y_p)}{2\ln(x_p - 1, y_p) - 4\ln(x_p, y_p) + 2\ln(x_p + 1, y_p)} \quad (9)$$

where $y_p$ is the vertical integer coordinate in pixels of the local maximum intensity. The $y$-coordinate, $y_0$, of the barycentre, will be defined analogously.

Despite the introduction of the minimum distance $d_{\text{min}}$, large particles may have two or more features associated. To overcome this problem, a minimum distance, $d_{\text{min}}$, is imposed discarding barycentres located within the lowest intensity pixels. This step guarantees that the algorithm will also work well with heterogeneous particle size distributions. Finally, as previously stated, once the good features are identified, system (3) can be completely solved. In particular, in order to handle large motions, maintaining at the same time a high local accuracy, an iterative pyramidal implementation can be used [19]. The estimated velocity field thus obtained can be used to enforce the particle tracking scheme and to avoid ambiguity in the particle matching phase.

Fig. 1. Procedural scheme of good features extraction algorithm. (a) First frame synthetic image; (b) minimum eigenvalue image (8-bit); (c) good features to track detected for the first frame; (d) velocity field obtained by solving system (3) using two subsequent images of the VSJ data.
This approach has been used in the past [20–22] through cross-correlation analysis of sub-regions of consecutive images acquired. The advantage of using the optical flow approach is the possibility of obtaining at the same time, the particle position and the related velocity predictor, with a subsequent considerable reduction in time consumption and with an increased spatial resolution.

Fig. 1 shows the procedural scheme of the proposed algorithm applied to two consecutive synthetic images provided by the visualization society of Japan (VSJ) [23], relative to a jet-impinging on a wall, which is a good measure for all PTV algorithms [4]. The velocity field (Fig. 1d) is obtained via the iterative pyramidal implementation, described in detail by Bouguet [19], with an integration window, $W$, with side length of 21 pixels.

3. Test cases and discussion

In order to analyse the performances of the proposed algorithm, hereinafter referred to as feature extraction (FE), synthetic images with different characteristics were generated and used as test cases. The advantage of using synthetic images is the possibility to compare the original particle positions with those detected by the measurement technique. The images generated are single exposure snapshot of 256 × 256 pixels at 8-bit greyscale. The procedure to create images is described in the appendix. Since the particle positions were known in advance, the performance of the feature extraction algorithm was evaluated through the reliability, $E_r$, defined as the number of correct particles that are reconstructed by the particle identification algorithm ($n_{pci}$) divided by the total number of detected particles ($n_{pi}$):

$$E_r = \frac{n_{pci}}{n_{pi}}$$

(10)

It is assumed that a particle is correctly identified if the difference between the measured and generated particle position is less than 1/5 of a pixel. Not all the particles can be actually detected due to the thickness (5 pixels) of the simulated laser-light sheet, which is thinner than the constant depth (20 pixels) of the fluid volume in which the particles are distributed (cfr Appendix).

The performances of the FE method were compared with two binarization algorithms, the global threshold (GT) binarization based on the Otsu method [24], and the dynamic threshold (DT) binarization [9]. In order to ensure a fair comparison, all the particle identification methods employ the same algorithm for barycentre calculation: the 1D Gaussian estimator, described in the previous section, which shows in these tests a sub-pixel accuracy with zero mean and standard deviation always less than 0.1 pixel. Furthermore, the base threshold for the dynamic threshold binarization coincides with the global threshold detected with the Otsu method.

Four different tests were carried out varying four image parameters:

- Particle image density, $\rho$ (particles per pixel, ppp), defined as the ratio between the number of particles within an image, $n_p$, and the image size in pixels.
- Particle image size, expressed with the mean radius, $r_p$ (pixel), of each particle image;
- Background mean noise, $F$ (DN), which is set as a constant grey level for each pixel in the image;
- Background standard deviation noise, $\sigma_g$ (DN), applied to the image using an additive Gaussian noise.

Specifications of the image parameters are reported in detail in the appendix. To obtain an overall evaluation of the proposed method, the algorithm speed has been tested by varying the particle image density.

A final test was conducted by using two consecutive real images of a ventricular flow supplied by Querzoli et al. [25].

3.1. Particle image density

In the simulations of this test, the number of tracer particles was varied from 125 ($\rho \approx 2 \times 10^{-3}$ ppp) to 16,000 ($\rho \approx 2.4 \times 10^{-1}$ ppp) within a domain of 256 × 256 pixels. The mean radius of particles, the mean and standard deviation noise were set constant to 3 pixels, 10 DN and 1 DN, respectively. The number of particles per image and the parameters

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$n_p$</th>
<th>Base threshold (DN)</th>
<th>Contrast threshold (DN)</th>
<th>$C$</th>
<th>$d_{min}$ = $L$ (pixel)</th>
<th>$d_{min}$ (pixel)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{1-1}$</td>
<td>125</td>
<td>49</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>$S_{1-2}$</td>
<td>250</td>
<td>49</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>$S_{1-3}$</td>
<td>500</td>
<td>52</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>$S_{1-4}$</td>
<td>1000</td>
<td>64</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$S_{1-5}$</td>
<td>2000</td>
<td>69</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$S_{1-6}$</td>
<td>4000</td>
<td>87</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$S_{1-7}$</td>
<td>8000</td>
<td>115</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$S_{1-8}$</td>
<td>16,000</td>
<td>134</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1 Number of generated particles and principal parameters of the identification algorithms (global threshold, dynamic threshold, feature extraction) applied for the simulation series $S_1$.
used for the different algorithms are given in Table 1. As the background noise remains unchanged, the base threshold level of both global and dynamic thresholding should be set constant for all the simulations. Despite this, with the higher seeding densities the resultant size of a large amount of the binarized particle images may fall into one pixel (K. Ohmi, written communication, 2011). In order to avoid this, the base threshold was defined with the Otsu method and was increased as the particle density grows, due to a modification of the grey level distribution within the image. The minimum distance between features, $d_{fmin}$, was reduced to take into account the variation of the inter-particle mean distance, while the minimum distance between particles, $d_{pmin}$, was kept the same, due to the constant mean size of particles. Also, the coefficient $C$ for feature extraction was kept constant due to the invariability of random noise in the different simulations. Since the density of the tracer increases, there is a higher probability of overlapping particle images, making the detection more difficult. This aspect is evident when analysing both the trend of the normalised number of identified particles, $N_{pi} = n_{pi}/n_p$, and the reliability, $E_r$ (Fig. 2). The global threshold binarization shows the highest reliability for all the particle density used, but it is necessary to point out that this is coupled with a low value of $N_{pi}$, less than 0.7, for all the simulations reaching approximately $10^{-3}$ with 16,000 particles. The algorithms of dynamic threshold and feature extraction show similar values of reliability, equal to 1.0 for the lowest density and approximately 0.4 for the highest one, but with the feature extraction method showing the highest number of particles detected, in particular when the particle image density decreases.

3.2. Particle image size

Similarly to the increase of density, the growth of the particle image, due to large particle size, high magnification factor, or high diffraction limited image particles, leads to an increasing difficulty in detecting particles. The mean radius of particle image, $r_p$, was varied from 1 to 8 pixels (Table 2), with 2000 particles per image. Noise was added to the image fixing the mean and standard deviation to 10 and 1 DN, respectively. The particle identification parameters for the different algorithms are reported in Table 2. In this test, the minimum distance between features was set constant to 4 pixels for all the radii used, due to the same tracer density ($\rho \approx 3.1 \times 10^{-2}$ ppp) in the different simulations. The minimum particle distance, $d_{pmin}$, for the last four simulations, was increased in order to take into account the growing size of particles. It is important to note that this determines a progressive decrease of the ratio between the particles identified and the features extracted. For the smallest particles ($r_p = 1$ pixel) this ratio is equal to 1.0, while for the largest particles ($r_p = 8$ pixels), is less than 0.5. Nevertheless, the introduction of $d_{pmin}$ helped the feature extraction algorithm to avoid multiple barycentres associated to the same particle.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$r_p$ (pixel)</th>
<th>Base threshold (DN)</th>
<th>Contrast threshold (DN)</th>
<th>C</th>
<th>$d_{fmin} = L$ (pixel)</th>
<th>$d_{pmin}$ (pixel)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S2–1</td>
<td>1</td>
<td>51</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>S2–2</td>
<td>2</td>
<td>62</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>S2–3</td>
<td>3</td>
<td>69</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>S2–4</td>
<td>4</td>
<td>86</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>S2–5</td>
<td>5</td>
<td>97</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>S2–6</td>
<td>6</td>
<td>118</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>S2–7</td>
<td>7</td>
<td>127</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>S2–8</td>
<td>8</td>
<td>135</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Fig. 2. Tracer density influence on (a) the normalised number of identified particles, $N_{pi}$, and (b) the reliability, $E_r$. 

Table 2

Mean radius of particle image and principal parameters of the identification algorithms (global threshold, dynamic threshold, feature extraction) applied for the simulation series S2.
Fig. 3 shows the results of the different simulations and, as expected, these are similar to the previous test showing a decrease in the particle identification performances for all the algorithms applied. However, it is necessary to point out that, for all the algorithms, the number of particles identified reaches a peak at a mean radius of particle image of 2 pixels, and decreases at \( r_p = 1 \) pixel.

As in the previous test, the global threshold method guarantees the highest reliability, but always providing the lowest number of particles identified and, in the worst case, reaching a value of \( N_{pi} \) less than 0.1. Overall, the feature extraction method performs better than the threshold approaches, providing the highest number of particles correctly identified for all the mean particle sizes.

Analysing the trends of the reliability of all the algorithms, it emerges that there is a strong decrease in performances for \( 5 < r_p < 6 \) pixels, with a reduction of 40% in the worst case. This can be associated to a critical overlap distance. In order to investigate this aspect, the calculation of the 2D mean distance of the nearest neighbour particles was conducted. Only particles within the light sheet were considered. The value of this distance was approximately 5.2 pixels. Therefore, the critical overlap distance is exactly the mean radius of particles. If the mean particle distance is lower than the particle radius, then the reliability of the identification algorithm has a strong decay.
3.3. Background mean noise

After analysing the effects of seeding density and particle image mean radius, these parameters were fixed at 2000 particles per image and 3 pixels, while the mean noise of background, $F$, was added with steps of 20 DN starting from a dark background ($F = 0$ DN), and reaching the highest value of 140 DN (Table 3). The standard deviation of the Gaussian noise was set constant for all the simulations and equal to 1 DN. As can be seen from Table 3 the base threshold for the binarization methods increases with noise. The feature extraction parameters were instead maintained constant for all the simulation series $S_3$ (Table 3). The results of this test are shown in Fig. 4. As in the previous tests, the feature extraction and the dynamic thresholding methods show nearly the same number of particles identified, which is approximately 150% higher than that of the global threshold. All the algorithms are not affected by the changes of the background mean noise until 80 DN. As the mean background noise increases, the contrast between particles and background decreases for the lightest particles and there is consequently a reduction in the reliability of all the algorithms. However, it is important to note that the performances of the feature extraction and the dynamic thresholding degrade more slowly than the global threshold, which shows a reliability that reaches 0.5 for the noisiest image.

3.4. Background standard deviation noise

This test is particularly remarkable, because the presence of random noise is common in real PTV images. The standard deviation of the Gaussian background noise, $\sigma_g$, was varied from 0 to 14 DN as shown in Table 4, maintaining a constant value of the mean background noise to 10 DN. The image density and the mean particle size were the same as in the previous test: $n_p = 2000$, $r_p = 3$ pixels. The parameters used for the detection algorithms are summarised in Table 4. For all the simulation series $S_4$, the base threshold of the binarization algorithms was set constant (85 DN), because the random noise generated did not affect the overall grey level distribution. On the other hand, the coefficient $C$, and consequently $\tilde{z}_P$, was increased for the noisiest images so as to avoid false detections. The results of this test are shown in Fig. 5. The global threshold method shows the highest reliability but, as in the previous tests, it gives poor results in terms of $N_{pi}$, which is always lower than 0.4. The feature extraction shows the best results in terms of particles correctly identified, but it seems to be more influenced by the presence of random noise than the other algorithms. Starting from a standard deviation of 6 DN there is a negative trend for both the reliability and the number of identified particles. Although the coefficient $C$ is increased, a greater amount of false positive particles are detected, which is probably due to the presence of intensity gradients associated to the random noise.

### Table 4

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$\sigma_g$ (DN)</th>
<th>Base threshold (DN)</th>
<th>Contrast threshold (DN)</th>
<th>$C$</th>
<th>$d_{min} - L$ (pixel)</th>
<th>$d_{max}$ (pixel)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_4-1$</td>
<td>0</td>
<td>85</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$S_4-2$</td>
<td>2</td>
<td>85</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$S_4-3$</td>
<td>4</td>
<td>85</td>
<td>5</td>
<td>$10^{-3}$</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$S_4-4$</td>
<td>6</td>
<td>85</td>
<td>5</td>
<td>$5 \times 10^{-3}$</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$S_4-5$</td>
<td>8</td>
<td>85</td>
<td>5</td>
<td>$5 \times 10^{-3}$</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$S_4-6$</td>
<td>10</td>
<td>85</td>
<td>5</td>
<td>$10^{-2}$</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$S_4-7$</td>
<td>12</td>
<td>85</td>
<td>5</td>
<td>$10^{-2}$</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>$S_4-8$</td>
<td>14</td>
<td>85</td>
<td>5</td>
<td>$10^{-2}$</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>

**Fig. 5.** Background standard deviation noise influence on (a) the normalised number of identified particles, $N_{pi}$, and (b) the reliability, $E_r$. 

---

3.5. Time performances

In addition to the detection ability, it is of practical interest to evaluate the speed performance of a given particle identification method. Specifically, for the different algorithms proposed the speed measurement was carried out considering test case S1. Fig. 6 shows the processing time, $t_n$, as a function of the number of particles $n_p$, with $t_n$ being the actual time normalised by the time required by the global threshold method for $n_p = 16,000$ ($t = 0.242$ s). For all the algorithms, the processing time seems to increase closely proportional (coefficient of determination $\approx 1$) to $n_p^2$. 

Fig. 6. Normalised processing time for the three identification algorithms with increasing number of particles.

Fig. 7. Feature extraction results relative to the application on two consecutive images of a ventricular flow. (a) First image of the set; (b) identified particles; (c) velocity field obtained by solving optical flow equation and enlarged view of it.
The feature extraction scheme, without the calculation of the velocity field, has processing time of about five times longer than that of the global threshold method. The dynamic threshold, instead, shows the highest computational time of all the identification algorithms. This processing time is of three order of magnitude higher, which is probably due to the iteration approach method.

3.6. Particle identification using real data

A test on real data was carried out by using two experimental images provided by Querzoli et al. [25], relative to the ventricular flow (Reynolds number approximately equal to 8000) simulated by means of a laboratory model. The working fluid inside the ventricle was distilled water, seeded with neutrally buoyant particles. A high-speed digital camera with a spatial resolution of 1280 × 1024 pixels at 8-bit greyscale was used for data acquisition at 250 frames/s. Fig. 7a shows the first image acquired while Fig. 7b reports the related computed barycentres with the feature extraction algorithm, using the following parameters: \( C = 5 \times 10^{-3} \), \( d_{f_{\text{min}}} = 4 \) pixels, \( d_{p_{\text{min}}} = 1 \) pixel. The number of features extracted was 6098, while the number of detected particles was 5814. The discarded barycentres are multiples of the same particle, and this is probably due to not perfectly spherical particles, which are a consequence of refraction effects near the ventricular border.

The dynamic and the global threshold binarization algorithms were also applied to the same image using a threshold (73 DN) detected with the Otsu method and for the dynamic binarization a contrast threshold of 5 DN. The DT algorithm detects 5047 particles, which is comparable with the FE scheme, while the GT algorithm identifies the lowest number of particles, 2333.

The further advantage of the proposed technique is its ability to provide a predictor velocity field that can be used to facilitate the subsequent tracking phase. Fig. 7c reports the velocity field obtained by solving system (3) using a pyramidal approach with three levels and an integration window of 21 pixels. The figure shows the presence of the diastolic jet during the filling phase of the ventricle. This determines the formation of a vortex ring [25], whose middle section is clearly visible. The capability of the FE algorithm to work with high seeding density is clear from the enlarged view in Fig. 7c, thus encouraging and facilitating the use of two-frame tracking algorithms [9,12].

4. Conclusion

A feature based approach for the identification of particle images in PTV technique has been proposed. The method presented is based on the solution of the optical flow equation using a sum-of-squared-difference method [14]. Performance tests have been conducted via the development and utilisation of synthetic images with a variety of generated image parameters. The obtained results show that the new approach provides high reliability coupled with a great number of particles identified. A few problems arise when increasing the standard deviation of the background noise. Nevertheless, the overall performances are comparable with the consolidated and widely used binarization method of dynamic threshold [9]. On the other hand, the feature extraction procedure has lower time consumption and the possibility to provide a velocity vector field that can be used as a predictor for the final step of particle pairing in particle tracking algorithms.

Acknowledgements

This work was partially supported by MIUR (Italian Ministry of Education, University and Research) and Sapienza, University of Rome. The algorithm presented in this paper is available online on http://w3.uniroma1.it/shindler/software.php. We thank Giorgio Querzoli and Stefania Fortini for providing the experimental images. We would also like to thank the reviewers for their valuable comments to improve the quality of this paper.

Appendix A

The first step for the generation of synthetic images is the definition of their size, which is set in this work to 256 × 256 pixels. Each pixel has 256 grey levels (8-bit). The intensity, \( F \), of the background is set constant for the whole image. In order
to obtain realistic PTV images, it is necessary to simulate the random noise of images, caused principally by the amplification and digitization of the output electric signal from the sensor. In a first approximation, this noise can be considered independent from the grey level of the pixel, with a Gaussian distribution and additive. If \( g(x, y) \) indicates the generic digital number (DN) of the pixel with coordinates \( x, y \), due to the random noise with a standard deviation, \( \sigma_g \), the image background intensity, \( I_{bg} \), results:

\[
I_{bg}(x, y) = F + g(x, y).
\]  

(11)

The next phase consists in defining the intensity, \( I(x, y) \), caused by the light scattering from the particle. Using the Fraunhofer approximation for the far field \([26]\), it can be seen that the particle intensity distribution formed on the sensor is described by the Airy function, represented by a disc and several rings of decreasing intensity. In practice, the Airy function can be approximated with a normalised Gaussian function \([20,27]\):

\[
I(x, y) = I_0 \exp \left( - \frac{(x - x_p)^2 + (y - y_p)^2}{r_p^2} \right).
\]

(12)

where \( x_p, y_p \) are the coordinates of the particle barycentre and \( r_p \) the radius of the generic particle generated randomly with a Gaussian distribution with mean \( r_p \) and standard deviation \( \sigma_p \), which is set constant and equal to 1 pixel for all the generated images. In order to simulate the laser-light sheet illumination, the maximum intensity (\( I_0 \)) is defined as a function of the depth location (\( z_p \)) of the particle:

\[
I_0 = 255 \exp \left( - \frac{z_p^2}{2\sigma_z^2} \right).
\]

(13)

where \( 2\sigma_z \) is the thickness of the laser-light sheet set, in this work, to 5 pixels. If the particle location (\( x_p, y_p, z_p \)) and particle size (\( r_p \)) are known, the particle image can be written on the image by using the above equations. As an initial condition, the particles are randomly generated with a uniform distribution within a certain volume. Fig. 8 shows a synthetic image generated with the parameters of the simulation S1–6 (Table 1).

References