ANALYTICAL MODEL FOR HEAT TRANSFER IN AN UNDERGROUND AIR TUNNEL

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Abstract—A simplified analytical model is developed to determine the energy performance of an underground air tunnel. The model assumes that the air tunnel-ground system reaches periodic and quasi-steady state behavior after some days of operation. The model can predict the air temperature variation along the air tunnel for any hour of the day. It can also determine the daily mean and amplitude of the total cooling/heating effect of the tunnel. Parametric analysis is conducted to determine the effect of tunnel hydraulic diameter and air flow rate on the heat transfer between ground and air inside the tunnel. The model is validated against measured data. Copyright © 1996 Elsevier Science Ltd.

Underground air tunnel Air temperature Soil temperature Cooling effect Parametric analysis

NOMENCLATURE

\( a \) = Constant defined in equation (19)
\( A \) = Cross-section area of tube (m²)
\( c_p \) = Specific heat of fluid (J/kg·K)
\( d \) = Depth of tube (m)
\( D \) = Pipe diameter (m)
\( D_h \) = Hydraulic diameter (m)
\( h \) = Convective heat transfer conductance (W/m²/K)
\( k_s \) = Soil thermal conductivity (W/m/K)
\( k_t \) = Thermal conductivity of tube material (W/m/K)
\( l \) = Thickness of soil layer disturbed by tube (m)
\( L \) = Length of tube (m)
\( P \) = Tube perimeter (m)
\( q \) = Heat flux (W/m²)
\( Q \) = Total heat transfer (W)
\( r_o \) = Radius of buried spherical structure (m)
\( s \) = Streamwise coordinate along tube (m)
\( t \) = Time (s)
\( t_t \) = Thickness of tube material (m)
\( T_f \) = Fluid temperature inside tube (K)
\( T_s \) = Soil temperature (K)
\( T_a \) = Ambient air temperature (K)
\( u \) = Fluid enthalpy (J/kg)
\( U_s \) = U-value of soil layer of thickness \( l \) and tube material (W/m²/K)
\( V_f \) = Fluid velocity inside tube (m/s)
\( x \) = Coordinate system (m)

Greek symbols

\( \alpha \) = Constant defined in equation (22)
\( \lambda \) = Constant defined in equation (23)
\( \kappa_s \) = Soil thermal diffusivity (m²/s)
\( \rho \) = Soil density (kg/m³)
\( \omega \) = Angular frequency (rad/s)

Subscripts

\( f \) = Fluid (i.e. air)
\( m \) = Mean of temperature variation
\( s \) = Soil
\( v \) = Amplitude of temperature variation

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1. INTRODUCTION

The use of enthalpy transfer between air and below grade earth for cooling during the summer and for heating during the winter was known in ancient times. In southern Tunisia and eastern Spain [1], complete underground houses were built to cope with the relatively hot and arid climates. In northern China [2], large houses were excavated to avoid severe, cold winter conditions. Even in places where geological conditions are not suitable for excavation, ancient architects used other techniques to give relief from the heat, especially in desert and semi-desert climates. Beside the well known wind-towers of Egypt, Saudi Arabia and Pakistan, the Iranians [3] used buried tunnels to cool air caused to flow by wind towers. Currently, the use of the earth as an energy conservation technique started to increase in Europe and America, particularly after the 1973 oil crisis.

The earth heat transfer technique is based on the fact that soil a few feet below grade can be used as a heat sink or as a heat source. In fact, energy exchange between a below-grade, building envelope and the surrounding earth can be smaller than that between an above-grade envelope and the surrounding ambient air in many parts of the world for a significant part of the year. This is due to the fact that the seasonal temperature variations are greatly reduced in the ground and that the phase lag between the soil temperature and the ambient air temperature increases with depth. For example, the earth temperature for light dry soil at a depth of about 3 m (10 ft) varies by approx. ±3°C (±5°F) from the mean soil temperature, which is approximately equal to the mean annual air temperature, and has a phase lag of about 75 days behind the ambient air temperature [4]. Therefore, in winter, the ground temperature increases with increasing depth, so that layers some distance below the surface can be used as a building heat source. However, in summer, the ground temperature decreases with increasing depth, allowing use of the earth as a heat sink.

Eckert [5] showed, by very simplistic calculations, the potential of the earth to be used as an energy source, i.e. a heat sink or a heat store. To provide a feeling for the amount of energy exchanged between a buried structure and the earth, Eckert made the assumption that the structure has the shape of a sphere with radius \( r_0 \) maintained at a constant temperature \( T_0 \). Thus, the steady radial heat flux \( Q \) from the surface of the structure into the ground (conductivity \( k_s \)) with a mean temperature \( T_s \) is given by [6]

\[
Q = 4\pi k_s r_0 (T_0 - T_s)
\]

or, in terms of heat transfer coefficient:

\[
U_s = \frac{Q}{A(T_0 - T_s)} = \frac{k_s}{r_0}
\]

where \( A = 4\pi r_0^2 \) is the surface area of the spherical structure. From this expression for the heat transfer coefficient \( U_s \), two results can be drawn: large structures (large \( r_0 \) value) experience small heat losses or gains per unit area, and small structures transfer heat easily into the ground.

The first result, in addition to the fact that the temperature difference \( (T_0 - T_s) \) is generally much smaller for an underground building than for a building above ground, is the reason that earth sheltered structures efficiently reduce the energy required for heating or cooling. The second result finds its application in underground air tunnels. These can be used to cool air in the summer and to heat it in the winter.

This paper deals with the second application. An analytical model is presented for the heat transfer between an air tunnel and the ground, assuming a periodic variation of both air source and ground temperatures.

Prior work on the subject is mostly based on numerical techniques. Glennie et al. [7] presented a computer simulation to calculate the cooling available on a particular average day of a month; but their experimental results were not reliable. Nordham [8] and Abrams et al. [9] also presented computer calculations based on a series of steady state relationships that had limitations and cannot be considered to be complete. The problem is basically a transient one, since heat removed from air within the tube heats the surrounding earth and reduces the cooling effect relative to the steady state calculation which assumes a constant earth temperature. Dhaliwal and Goswami [10] used a time increment procedure to calculate the temperature along a circular pipe and compared their model with experimental results. Schneider [11] used a finite element technique to determine the
heat loss from an insulated buried pipe. However, Schneider did not apply his results to the problem of air-tunnels. Recently, other models have been proposed [12, 13]. However, these models are based on simplistic assumptions and are not validated against measured data.

In this paper, an analytical model for earth air-tunnel systems is developed. The model is suitable for design calculations and for feasibility studies. The analytical model assumes that, after a few days of operation, the air tunnel ground system reaches a steady, periodic state. Therefore, all the variations are periodic functions of time. Throughout the paper, daily variations are assumed, however the model is valid for other frequencies. The results obtained by the model are tested against experimental data from Dhaliwal and Goswami [10].

2. FORMULATION OF THE PROBLEM

Consider a cylindrical pipe of cross section area $A$ (Fig. 1) through which a fluid (i.e. air) flows with a constant and uniform velocity $V_f$. Assuming that the fluid has constant density $\rho$ and specific heat $c_p$, the bulk fluid temperature coincides with the cross-section average temperature defined as

$$T_f(s) = \frac{1}{A} \int_A T_f \, dA.$$  

Assuming no energy generation within the fluid, the energy conservation equation inside the tube can be written as (Fig. 2):

$$\rho A \frac{\partial u}{\partial t} \, ds = -\rho V_f A \frac{\partial u}{\partial s} \, ds + \left( \int_P q_{sf} \, dP \right) \, ds$$

where $u$ is the fluid enthalpy ($du = c_p \, dT$), $P$ is the tube perimeter, $q_{sf}$ is the convective heat transferred from the surrounding earth to the fluid, $s$ is the streamwise coordinate.

Note that, in equation (4), the axial heat conduction (the term $q_e$ in Fig. 2) was neglected. This can be justified by the fact that, for air, the Peclet number $Pe$ ($Pe = \text{energy convected/energy conducted}$) is large.
After rearrangement, equation (4), which is valid for any cross-section tube shape, can be expressed as

\[ \rho AC_p \frac{\partial T_f}{\partial t} = -\rho V_f AC_p \frac{\partial T_f}{\partial s} + \left[ \int_P h_s(T_s - T_f) \, dP \right]. \]  

(5)

The fluid bulk temperature \( T_f \) is a function of both the time \( t \) and the distance \( s \) along the tube, \( h_s \) is the convective heat transfer coefficient at the surface of the tube, \( T_s \) is the surrounding soil temperature.

Referring to Fig. 3, the buried tube problem can be formulated by the following governing equations.

For the soil: \( \frac{1}{\kappa_s} \frac{\partial T_s}{\partial t} = \Delta T_s \)  

(6)

with the following boundary conditions:

\( T_s = T_0 \) at the surface of the ground

\( -k(\partial T_s/\partial n) = h_s(T_s - T_f) \) at the surface of the tube

where \( \kappa_s \) is the soil thermal diffusivity, \( \Delta \) is the three-dimensional Laplace operator, \( T_0 \) is the ground surface temperature, \( k \) is the effective soil thermal conductivity after the presence of the pipe has had its effect on soil moisture content, \( n \) is the direction of the normal to the tube surface.

For the fluid within the tube:

\[ \frac{\partial T_f}{\partial t} + V_f \frac{\partial T_f}{\partial s} + \int_P h_s(T_f - T_s) \, dP = 0 \]  

(7)

and \( T_f(s = 0, t) = T_0(t) \)

where \( T_0(t) \) is the ambient air temperature.

2.1. Undisturbed soil temperature

A knowledge of the undisturbed soil temperature distribution is required for the calculation of heat transfer between the earth and buried structures. The variation of the temperature with depth depends, however, on the conditions of the soil surface. In the early sixties, Kusuda and Achenbach [14], and later Labs [4], studied earth temperatures at various depths (and at the surface) in some locations in the United States. They found that the undisturbed earth temperature \( T_s^*(y, t) \) at a given depth \( y \) from the surface varies periodically with time and can be approximated with an expression in the form of:

\[ T_s^*(y, t) = T_m + T_v \text{Re}(e^{i\omega t - \delta y}) \]  

(8)

where

\[ \delta = \left( \frac{\omega}{\kappa_s} \right)^{1/2} \]  

(9)

At the ground surface (\( y = 0 \)), temperature \( T_s^*(0, t) \) varies sinusoidally with time:

\[ T_s^*(0, t) = T_m + T_v \text{Re}(e^{i\omega t}) \]  

(10)

where \( T_m \) and \( T_v \) are the mean and the amplitude of ground surface temperature variation and \( \omega \) is the angular frequency of the periodic variation.

For daily variation \( \omega = 2\pi/\text{day} = 7.27220^{-1} \text{rad./s} \). The soil temperature becomes nearly constant at a depth of the same order of magnitude as the modulus of \( \delta^{-1} \). For daily variation, the soil temperature does not exhibit any fluctuations below depths of about 0.20–0.30 m.

3. BELOW GRADE CONDUIT ANALYSIS

In this model, we assume that a tube induces a disturbance effect on the earth temperature only up to a distance, \( \varepsilon \), from the surface of the tube. This layer of soil is treated as an insulation added to the tube. This is a good approximation as long as \( \varepsilon \) is small.
In addition, for daily variations, the heat conduction from the region near the pipe is assumed to follow a quasi-steady-state behavior. In the Appendix, a discussion of this assumption can be found.

For common soils, the depth \( \ell \) is approximately equal to 0.10 m. If soil properties are known, a better value of \( \ell \) can be calculated by using equation (9)

\[
\ell = \| \delta \| = \sqrt{\frac{k_s}{\omega}}.
\]

In these conditions, the governing equations become (recall Fig. 3):

For the tube:

\[
\frac{\partial T_t}{\partial t} + V_t \frac{\partial T_t}{\partial x} + \frac{P h_t}{\rho C_p A} (T_t - T_s) = 0 \tag{11}
\]

and

\[
T_t(0, t) = T_s(t)
\]

where \( T_s \) is the time-varying soil temperature at the surface of the tube.

For the soil:

\[
h_t P (T_t - T_s) = U_s P (T_s - T_\infty)
\]

where \( T_\infty \) is the time-varying, undisturbed soil temperature assumed to be uniform around the tube surface (a relatively good assumption as long as the tube diameter and \( \ell \) are small compared to the depth \( d \) at which the tube is buried) and \( U_s \) is the U-value of the soil layer of thickness \( \ell \), including the thermal resistance of the tube material. For a circular tube, \( U_s \) is given by

\[
U_s = \left( \frac{2}{D} \right) \left\{ \ln \left( \frac{\ell + D/2 + t_t}{D/2 + t_t} \right) + \frac{1}{\ln \left( \frac{D/2 + t_t}{D/2} \right)} \right\}^{-1}
\]

(13)

where \( t_t \) is the tube material thickness and \( k_t \) is the thermal conductivity of the tube material.

Although equation (12) appears to be the same as a steady state equation, note that all temperatures vary with time \( t \) and spatial coordinate \( x \).

From equation (12), the soil temperature at the surface of the tube can be expressed as:

\[
T_s = \frac{h_t T_t + U_s T_\infty}{h_t + U_s}.
\]

(14)

Then, equation (11) becomes, after substitution of equation (14):

\[
\frac{\partial T_t}{\partial t} + V_t \frac{\partial T_t}{\partial x} + \frac{4h_t U_s}{\rho C_p D_h (U_s + h_t)} (T_t - T_\infty) = 0
\]

(15)

where \( D_h = 4A/P \) is the hydraulic diameter of the tube.

![Fig. 3. Soil-air thermal system with dimensions shown.](image-url)
The steady-periodic solution of equation (15) can be expressed as

$$T_i = T_{mf} + \text{Re}(T_{vr} e^{i\omega t})$$

where $T_{mf}$ is the mean fluid temperature, $T_{vr}$ is the amplitude of the fluid temperature variation and $\omega$ is the angular frequency of the periodic variation of temperature. Values for $T_{mf}$ and $T_{vr}$ are calculated in the next section.

4. FLUID TEMPERATURE SOLUTION

4.1. Expression for the fluid temperature amplitude, $T_{vr}$

From equation (15), it is straightforward to see that the governing equations for $T_{vr}$ are

$$i\omega T_{vr} + V_r \frac{\partial T_{vr}}{\partial x} + \frac{4h_t U_i}{\rho C_p D_h (U_s + h_t)} (T_{vr} - T_{us}) = 0$$

and

$$T_{vr} = T_{us}$$

where $T_{us}$ and $T_{va}$ are the amplitude of the undisturbed soil temperature and the ambient air temperature, respectively.

The convective heat transfer coefficient $h_t$ is dependent upon distance along the tube, $x$. It is relatively higher in the undeveloped flow region near the entry of the tube than downstream. Kays [15] shows that for turbulent flow, $h_t$ can be approximated by:

$$h_t = h_\infty \left(1 + \frac{a}{x}\right)$$

where $h_\infty$ is the heat transfer coefficient far away from the entry and $a$ is a constant depending on the tube geometry.

Substituting equation (19) into equation (17), it follows that:

$$\frac{\partial T_{vr}}{\partial x} + \frac{1}{V_r} \left[ \frac{ah_\infty (x + a)}{(h_\infty + U_s)x + ah_\infty} \right] T_{vr} = \frac{1}{V_r} \left[ \frac{ah_\infty (x + a)}{(U_s + h_\infty)x + ah_\infty} \right] T_{us}.$$  

(20)

The solution of this equation satisfying the initial condition (18) is given by

$$T_{vr} = T_{us} + (T_{vs} - T_{us}) \exp \left[ -\frac{1}{V_r} \left( \frac{\lambda - i\omega}{\lambda} \right) x + \frac{a U_i \lambda^2}{ah_\infty} \ln \left( 1 + \frac{\lambda x}{\lambda a} \right) \right]$$

$$-\frac{i\omega}{V_r} T_{us} \int_0^x \exp \left[ \frac{1}{V_r} \left( (\lambda + i\omega) (x' - x) + \frac{a U_i \lambda^2}{ah_\infty} \ln \left( \frac{\lambda a + ax'}{\lambda a + ax} \right) \right) \right] dx'$$

(21)

where $\lambda$ is a constant defined as

$$\lambda = \frac{4 U_i}{\rho C_p D_h}$$

(22)

and $\lambda$ is another constant defined by:

$$\lambda = \frac{\alpha h_\infty}{U_s + h_\infty}.$$  

(23)

4.2. Expression for the mean fluid temperature $T_{mf}$

The governing equations for $T_{mf}$ are

$$\frac{\partial T_{mf}}{\partial x} + \frac{4h_t U_i}{V_r \rho C_p D_h (U_s + h_t)} (T_{mf} - T_{ms}) = 0$$

(24)

and

$$T_{mf}(0) = T_{ms}.$$  

(25)
The solution of equation (24) can easily be deduced from the expression for \( T_{\text{mf}} \) given in equation (21) by substituting \( \omega = 0 \) and by replacing \( T_{\text{mf}} \) and \( T_{\text{ma}} \), respectively, with \( T_{\text{m}} \). Thus, \( T_{\text{mf}} \) is:

\[
T_{\text{mf}} = T_{\text{ma}} + (T_{\text{ma}} - T_{\text{m}}) \exp \left[ -\frac{1}{V_f} \left( \frac{1}{\lambda x} \ln \left( 1 + \frac{\alpha x}{\lambda a} \right) \right) \right].
\] (26)

5. PARAMETRIC ANALYSIS

The above described model was applied to a circular pipe buried 1.5 m below grade. The ambient/inlet temperature was assumed to vary sinusoidally within a 24 h period, with 19°C as a mean and 9 K as an amplitude. The same assumption was made for the surface soil temperature with a mean of 16°C and an amplitude of 5 K. The other pipe-air-soil system parameters, when not varying, were taken to be:

\[
\begin{align*}
\kappa_s & = 6.48 \times 10^{-7} \text{ M}^2/\text{s} \\
U_s & = 35 \text{ W/m}^2 \text{ K} \\
h_{\infty} & = 16 \text{ W/m}^2 \text{ K} \\
a & = 1.4 \, D_h \text{ (see Ref. [13])} \\
D_h & = 0.20 \text{ m} \\
V_f & = 3.5 \text{ m/s} \\
t & = 0 \text{ at noon} \\
L & = 80 \text{ m (pipe length)}.
\end{align*}
\]

The ambient temperature was assumed to reach its daily maximum at noon which was taken as the origin of time (\( t = 0 \)). Any other time origin can be used.

All the calculations were carried out using equation (16) with \( T_{\text{mf}} \) determined from equation (26) and \( T_{\text{mf}} \) from equation (21).

5.1. Effect of the pipe diameter

Figure 4 shows that the outlet temperature depends significantly upon the pipe diameter. An increase in the diameter results in a higher outlet air temperature. This result can be explained by the fact that the mass of air, contained in the element \( ds \) is proportional to \( D_h^2 \), while the heat

![Effect of Tube Diameter](image_url)

Fig. 4. Effect of tube diameter on air temperature at various positions along the tube.
exchanged between the air and the soil is proportional to \( D_h \). Therefore, as the pipe diameter \( D_h \) increases, less heat is exchanged between a unit mass of air and the surrounding soil, resulting in a smaller decrease of the outlet air temperature.

5.2. Effect of air velocity

From Fig. 5, it can be seen that an increase in air velocity, that is, an increase of the air mass flow rate through the pipe, results in an increase of the outlet temperature, since the air has a shorter residence time in contact with the soil.

6. COOLING EFFECT EVALUATION

The local rate of heat transfer, \( q_c \), from the air to the soil is given by:

\[
q_c = h_f P(T_i - T_s). \tag{27}
\]

For a tube of length \( L \), the total rate of heat transfer,

\[
\dot{Q}_c = \int_0^L q_c \, dx. \tag{28}
\]

One way to determine \( \dot{Q}_c \) is to use equation (11) which can be expressed as follows:

\[
q_c = -\rho c_p A \left[ \frac{\partial T_i}{\partial t} + V_i \frac{\partial T_i}{\partial x} \right]. \tag{29}
\]

Therefore, the total instantaneous "cooling effect" for a buried pipe of length \( L \) is

\[
\dot{Q}_c = -\rho c_p A \left[ \left( \int_0^L T_i \, dx \right) \frac{\partial }{\partial t} \right] + V_i (T_i(L) - T_s). \tag{30}
\]
The expression of $T_r$ is given by equation (12). As a particular case, the daily mean of the total heat transfer $\dot{Q}_{mc}$ is given by

$$\dot{Q}_{mc} = \left( \rho c_p \frac{\pi D^2 V_f}{4} \right) (T_{ms} - T_{ms}) \left( 1 - \exp \left( - \frac{1}{V_f} \left( \lambda L + \frac{a U_t \lambda^2}{\alpha h_1} \ln \left( 1 + \frac{\alpha L}{\lambda a} \right) \right) \right) \right).$$

(31)

In the following sections, the effects of different parameters, such as pipe diameter and air velocity, on the total cooling rate are analyzed. When not taken as variables, the parameter values

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\[
\dot{Q}_{mc} = \left( \rho c_p \frac{\pi D^2 V_f}{4} \right) (T_{ms} - T_{ms}) \left( 1 - \exp \left( - \frac{1}{V_f} \left( \lambda L + \frac{a U_t \lambda^2}{\alpha h_1} \ln \left( 1 + \frac{\alpha L}{\lambda a} \right) \right) \right) \right).
\]

(31)

In the following sections, the effects of different parameters, such as pipe diameter and air velocity, on the total cooling rate are analyzed. When not taken as variables, the parameter values
6.1. Influence of the pipe diameter

Figure 6 presents the mean and the amplitude of the daily cooling rate as a function of pipe diameter. Increasing the diameter improves significantly both the mean and the amplitude of the cooling rate. This result can be explained by the fact that an increase in pipe diameter implies an increase in air mass flowing through the pipe that makes up for the outlet temperature drop. Note that the increase in the cooling rate amplitude is more pronounced than that of the cooling rate mean value. This indicates that the air tunnel system can be effective in reducing cooling peaks.

6.2. Influence of the air velocity

Figure 7 shows that the effect of the air velocity on the mean and the amplitude of the daily cooling rate is similar to that of the pipe diameter. However, the increase in air velocity does not improve the cooling rate as significantly as an increase in the diameter of the pipe.

From the above results, it can be concluded that, to improve the performance of the air tunnel system, an increase in pipe diameter, although increasing the outlet temperature, is preferable to an increase in the air velocity. However, an increase in the air flow rate will result in an increase of the fan power required to move the air along the tunnel.

7. COMPARISON WITH EXPERIMENT

An experiment on an underground air cooling pipe was performed by Dhaliwal et al. [10] starting on 15 September 1982. The soil thermal properties at the site of the experiment were estimated to be:

\[ \alpha_s = 6.45 \times 10^{-7} \text{ m}^2/\text{s} \]
\[ k_s = 1.16 \text{ W/m} \cdot \text{K} . \]

The pipe, of \( D_h = 0.305 \text{ m} \) diameter, was buried at a depth of about \( d = 2.86 \text{ m} \).

![Comparison with Experimental Data](image)

Fig. 8. Comparison of model predictions with experimental data 17 h after the start of the test.
As discussed in Section 2.1, for daily variation (i.e. \( \omega = 7.272 \times 10^{-5} \text{ rad./s} \)), the thickness of the disturbed soil layer, \( \ell' \), is of the same magnitude as \( \delta = \sqrt{[\omega/(\omega/\varepsilon)]^{1/2}} \). Thus,

\[
\ell' = \sqrt{[\omega/(\omega/\varepsilon)]} = 0.094 \text{ m}.
\]

Assuming a pipe thickness of \( t_i = 0.002 \text{ m} \) and a pipe thermal conductivity of \( k_i = 0.33 \text{ W/m} \cdot \text{K} \) (plastic), the \( U_i \)-value is calculated from equation (13) as:

\[ U_i = 14.60 \text{ W/m}^2 \cdot \text{K}. \]

The average air velocity was measured as

\[ V_i = 1.47 \text{ m/s}. \]

The average air properties in the temperature range of 10–30°C can be taken to be

\[
\begin{align*}
\rho &= 1.214 \text{ kg/m}^3 \\
c_p &= 1.205 \times 10^3 \text{ J/kg K} \\
\mu &= 0.0178 \text{ kg/s m} \\
Pr &= 0.65 \\
k_a &= 0.028 \text{ W/m} \cdot \text{K}
\end{align*}
\]

where \( \mu \) is the air viscosity, \( Pr \) is the Prandtl number, and \( k_a \) is the thermal conductivity.

In order to calculate \( h_\infty \), the heat transfer coefficient far away from the pipe entry, the Nusselt number \( \text{Nu}_\infty \) is needed. Recall that \( \text{Nu}_\infty \) is defined as

\[ \text{Nu} = \frac{h_\infty D_h}{k_a}. \]

The Nusselt number of fully developed turbulent flow is given by [13]

\[ \text{Nu}_\infty = 0.023 \Pr^{0.3} \text{Re}^{0.8} \]

where \( \text{Re} \) is the Reynolds number.
From the above equations,
\[ h_\infty = 7.20 \text{ W/m}^2\cdot\text{C}. \]

The inlet air temperature at different times was approximated by a sinusoidal function in the form of equation (7). Using the least-squares technique, it was found that
\[ T_s = 25.5 + 7 \sin \frac{\pi}{12} (t + 5) \text{ (°C)} \]
where \( t \) is the time after the experiment starts, expressed in hours.

The average soil temperature was measured to be
\[ T_m^s = 18.99\text{°C}. \]

No measurement of daily ground surface temperature amplitude \( T_m^s \) was made. However, at the depth \( d = 2.286 \text{ m} \) the soil temperature is equal to the daily averaged ground surface temperature (i.e. \( T_m^s \)).

Finally, using equation (21), the air temperature along the pipe can be calculated [10]. Figures 8–10 show the model predictions and the experiment results at three different times, \( t = 17, 18.5 \) and 20 h, respectively. These figures indicate very good agreement between the measured results and the model predictions. As noted by Dhaliwal et al. [10] in the experiment conducted, the relative humidity did not reach 100% (i.e. there was no condensation), therefore the model developed herein can be applied.

8. SUMMARY

In this paper, an analytical model, by which the heat transfer between an air tunnel and the soil can be analyzed, was developed. The model is valid only when condensation does not occur (i.e. when the relative humidity of the pipe air does not reach 100%). It was shown that an increase in the pipe diameter is thermally preferable to an increase in the air velocity when a given cooling rate is needed. Because of its flexibility, the model developed in this paper is suitable for design calculations and for preliminary feasibility studies of earth air-tunnel systems. Finally, the model was tested and validated against measured data.
REFERENCES


APPENDIX

*The Quasi-Steady-State Approximation for Pipe Wall Heat Flow*

For simplicity of calculation, we will assume that the tube consists of two parallel planes. Then, if $t_i$ and $t_f$ are small enough, the time rate changes of energy inside the tube material and the soil layer are given, respectively, by (see Fig. A1)

$$
\frac{d}{dt} = \frac{1}{\rho c_p} \frac{q_1 - q_0}{t_i}
$$

and

$$
\frac{d}{dt} = \frac{1}{\rho c_p} \frac{q_2 - q_1}{t_f}
$$

For steady-periodic variation $dT/dt = \omega T_s$, ($T_s$ is the amplitude of temperature variation, either inside the tube material or inside the soil layer). Therefore,

$$
\Delta q_t = q_1 - q_0 \approx (\rho c_p) t_i \omega T_s
$$

$$
\Delta q_s = q_2 - q_1 \approx (\rho c_p) t_f \omega T_s
$$

*Order of Magnitude of $\Delta q_t$ and $\Delta q_s$*

For daily variation

$\omega = 7.272 \times 10^{-4}$ rad/s; $t = 0.10$ m; $T_s = 10^\circ$C.

**Tube material**

$t_i = 0.002$ m; $(\rho c_p)_t = 10^4$ J/m$^3$K.

**Soil layer**

$(\rho c_p) \approx 10^5$ J/m$^3$K

![Fig. A1. Heat flux from fluid to soil.](image-url)
Thus
\[ \Delta q_1 \approx 1.5 \text{ W/m}^2 \]
and
\[ \Delta q_2 \approx 7.2 \text{ W/m}^2. \]
The order of magnitude of \( q_0 \) can be found using the following parameters
\[ U_q \approx \frac{k_s}{\tau} \approx 10 \text{ W/m}^2\text{K}; \quad h_s = 10 \text{ W/m}^2\text{K}, \]
\[ T_r = 40^\circ C \quad \text{and} \quad T_r'^\circ = 20^\circ C. \]
Then
\[ T'_r = 30^\circ C \quad \text{[from equation (14)].} \]
Therefore, \( q_0 = h_s(T_r - T'_r) \approx 100 \text{ W/m}^2. \)
So, approximating \( q_1 \) by \( q_0 \) leads to an error of 1.5% and \( q_2 \) by \( q_0 \) about 9%. Thus, the quasi-steady state heat conduction model introduces an uncertainty that is not significant compared to other simplifications of the problem.