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Composite Sliding Mode Control of Induction Motors Using Singular Perturbation Theory

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Abstract – *In this paper, a composite sliding mode control-observer approach is presented for the induction motor drive. This approach, based on the singular perturbation theory, decomposes the original system into separate slow and fast subsystems and permits that separate slow and fast control and observer can be designed for each subsystem and then combined into a composite control and observer for the original system. The controller design uses the sliding mode technique and is divided in two phases: slow control and fast control so that a final composite control is obtained. In addition and assuming that only the fast states are available; a two time scale sliding mode observer design is proposed for which a stability analysis is easily made. The simulations results validate the performance of the proposed approach.* ***Copyright © 2012 Praise Worthy Prize S.r.l. - All rights reserved.***

Keywords: *Induction Motor, Singular Perturbation, Sliding Mode Control, Observer, Stability*

Nomenclature

I.M.	Induction motor
ω , ω^*	Electrical rotor speed and reference rotor speed
v_{sd} , v_{sq}	Stator voltages in the synchronous rotating frame
i_{sd} , i_{sq}	Stator currents in the synchronous rotating frame
ϕ_{sd} , ϕ_{sq}	Stator fluxes in the synchronous rotating frame
ω_s , ω_{sl}	Synchronous and slip frequencies
L_s , L_r	Stator and rotor inductances
R_s , R_r	Stator and rotor resistances
T_s , T_r	Stator and rotor time constants
M , σ	Mutual inductance and leakage factor
J	Moment of rotor inertia
f	Coefficient of viscous friction
p	Number of pole pairs
T_{em} , T_L	Electromagnetic and load torques
t	Slow time scale (real time)
τ	Fast time scale $\tau = (t - t_0)/\varepsilon$
S_c	Sliding mode control surface
$S_{c,s}$	Slow sliding mode control surface
$S_{c,f}$	Fast sliding mode control surface
S	Sliding mode observer surface
$\hat{x}$, x^*	Estimated and reference value of x

I. Introduction

In the past two decades, the variable structure control (VSC) strategy using the sliding mode concept has been widely studied and developed for control and state estimation problems since the works of Utkin [1]-[3].

For induction motor drive, this control technique has many good properties to offer such as insensitivity to parameter variations, external disturbance rejection and fast dynamic response [4]-[7]. Furthermore, sliding mode observers have been used for estimating the states of the control system.

Sliding mode observers, also, have the same robust features as the sliding mode control [8]-[12].

In other hand, singular perturbation theory provides the mean to decompose two time scale systems into slow and fast subsystems of lower order described in separate time scales, which greatly simplify their structural analysis and any subsequent control design[13]-[15].

Then, the control (and/or observer) design may be done for each lower order subsystem, and the combined results yield to a composite control (and/or observer) for the original system.

So, the idea of combining singular perturbation theory and sliding mode technique constitutes a good possibility to achieve classical control objectives for systems having unmodeled or parasitic dynamics and parametric uncertainties [16]-[20]. Such a structure needs to have information about all state variables of the process. So, it is necessary to estimate the inaccessible states of the process by using a state observer [21]-[24].

Recently, singular perturbation theory has been widely used in observers for sensorless control drives, because it greatly simplifies the observer design [25]-[27]. This paper is organized as follows: In section II, we briefly review the two time scale approach based on the singular perturbation theory.

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The general design of a two time scale sliding mode observer is presented in section III. In section IV, the two time scale sliding mode control of induction motors is briefly reviewed. In section V, we present the design of the proposed two time scale sliding mode observer for the induction motors. In that section, a study of stability analysis of this observer is made via singular perturbation method and Lyapunov stability theory.

In section VI, and through simulation, the studied observer is associated to the sliding mode composite control of the induction motor where stator fluxes are replaced by those delivered by the observer.

Finally, in section VII, we give some comments and conclusions.

II. Two Time Scale Approach Review

The two time scale approach, based on the singular perturbation theory, can be applied to systems where the state variables can be split into two sets, one having “fast” dynamics, the other having “slow” dynamics. The difference between the two sets of dynamics can be distinguished by the use of a small multiplying scalar ε.

Generally, the scalar parameter ε is the speed ratio of the slow versus fast phenomena. If the slow states are expressed in the t time scale, then, the fast ones will be in the τ time scale defined by:

$$\tau = \left(t - t_0\right) / \varepsilon \tag{1}$$

where t_0 is the initial time. The reader is referred to [13], [14] and [15] for the general theory on singular perturbation.

II.1. Singularly Perturbed Systems

In this paper, we consider the following class of nonlinear singularly perturbed systems described by the so-called standard singularly perturbed form:

$$\begin{aligned} \frac{d}{dt} x &= f_1\left(x\right) + F_1\left(x\right) z + g_1\left(x\right) u, \;\; x\left(t_0\right) = x_0 \\ \varepsilon \frac{d}{dt} z &= f_2\left(x\right) + F_2\left(x\right) z + g_2\left(x\right) u, \;\; z\left(t_0\right) = z_0 \end{aligned} \tag{2}$$

where $x \in \Re^n$ is the slow state, $z \in \Re^m$ is the fast state, $u \in \Re^p$ is the control input, ε is a small positive parameter such that $\varepsilon \in \left[0,\, 1\right]$. The matrices f_1, f_2, F_1, F_2, g_1 and g_2 are assumed to be bounded with their components and analytic real vector fields. It is also assumed that the matrix F_2 is nonsingular for all x.

An additional assumption is that $f_1\left(0\right) = f_2\left(0\right) = 0$ and, for $u = 0$, the origin $\left(x, z\right) = \left(0, 0\right)$ is an isolated equilibrium state.

II.2. Slow Reduced System

In the limiting case, as $\varepsilon \to 0$ in (2), the asymptotically stable fast transient decays ‘instantaneously’ leaving the reduced order model in the t time scale defined by the quasi steady states $x_s\left(t\right)$ and $z_s\left(t\right)$:

$$\frac{d}{dt} x_s = f_s\left(x_s\right) + g_s\left(x_s\right) u_s, \quad x_s\left(t_0\right) = x_0 \tag{3}$$

$$z_s = h\left(x_s\right) = -F_2^{-1}\left(x_s\right)\left[f_2\left(x_s\right) + g_2\left(x_s\right) u_s \right] \tag{4}$$

where x_s, z_s and u_s denote the slow components of the original variables x, z and u, respectively and:

$$\begin{aligned} f_s\left(x_s\right) &= f_1\left(x_s\right) - F_1\left(x_s\right) F_2^{-1}\left(x_s\right) f_2\left(x_s\right) \\ g_s\left(x_s\right) &= g_1\left(x_s\right) - F_1\left(x_s\right) F_2^{-1}\left(x_s\right) g_2\left(x_s\right) \end{aligned}$$

The slow invariant manifold can be defined as:

$$M_\varepsilon = \left\{ z \in D_z \subset \Re^m : z = h\left(x_s\right) \right\}$$

where D_z is closed, bounded and centered in $z = 0$, and the so-called manifold condition:

$$\begin{aligned} &\varepsilon \frac{\partial h}{\partial x}\left[f_1\left(x\right) + F_1\left(x\right) z + g_1\left(x\right) u \right] = \\ &\quad = f_2\left(x\right) + F_2\left(x\right) z + g_2\left(x\right) u \end{aligned}$$

must be satisfied for M_ε to an invariant manifold [15].

II.3. Fast Reduced System

The fast dynamic (also known as boundary layer system) is obtained by transforming the slow time scale t to the fast time scale $\tau = \left(t - t_0\right) / \varepsilon$. We rewrite (2) in the fast time scale τ and introducing the derivation of z from M_ε, i.e., $z_f = z - h\left(x_s\right)$, so:

$$\frac{d\tilde{x}}{d\tau} = \varepsilon\left[f_1\left(\tilde{x}\right) + F_1\left(\tilde{x}\right)\left[z_f + z_s \right] + g_1\left(\tilde{x}\right) u \right] \tag{5}$$

$$\begin{aligned} \frac{dz_f}{d\tau} &= F_2\left(\tilde{x}\right) z_f + g_2\left(\tilde{x}\right)\left(u - u_s\right) + f_2\left(\tilde{x}\right) + \\ &\quad + F_2\left(\tilde{x}\right) h + g_2\left(\tilde{x}\right) u_s - \frac{\partial h}{\partial \tilde{x}} \frac{d\tilde{x}}{d\tau} \end{aligned} \tag{6}$$

where $z_f\left(0\right) = z_0 - h\left(x_0\right)$, $\tilde{x}\left(0\right) = x_0$ and $u_f = u - u_s$ is the fast control, and again examine the limit as $\varepsilon \to 0$.

Then $dx/d\tau = 0$, that is $x = \text{constant}$ in the fast time scale. So, the only fast variations are the variation of z from its quasi steady state z_s. Making $\varepsilon = 0$ in (5) and (6), we obtain an $O(\varepsilon)$ approximation of the fast subsystem:

$$\frac{dz_f}{d\tau} = F_2(\tilde{x}) z_f + g_2(\tilde{x}) u_f,\ z_f(0) = z_0 - h(x_0) \quad (7)$$

II.4. Composite Control

The fast and slow control laws can be combined into a composite control structure:

$$u(x,z) = u_s(x) + u_f(z - h(x)) \quad (8)$$

where u_s and u_f denotes the slow and fast components of the control law, respectively.

III. Two Time Scale Sliding Mode Observer

Now, consider the above continuous nonlinear singularly perturbed system of (2) which can be expressed as follows:

$$\begin{cases} \dot{x} = f_x(x,z,u,\varepsilon) \\ \varepsilon\dot{z} = f_z(x,z,u,\varepsilon) \end{cases} \quad (9)$$

where f_x and f_z are assumed to be bounded and analytic real vector fields, and consider a vector of measurement that is linearly related to the fast state vector as:

$$y = z,\quad y \in \Re^m$$

It is also assumed that the above system is controllable and observable [23]. Consequently, the observer design may be considered for the state observation of the slow variables from the measurement of the fast variables.

III.1. Sliding Mode Observer Design

By structure, observer based on sliding mode technique is very similar to the standard full order observer with replacement of the linear corrective terms by a discontinuous function [11], [23] and [27].

The corresponding sliding mode observer for the system of (9) can be written as a replica of the system with an additional nonlinear auxiliary input term as follows:

$$\begin{cases} \dot{\hat{x}} = f_x(\hat{x},z,u,\varepsilon) + G_x\Gamma_s \\ \varepsilon\dot{\hat{z}} = f_z(\hat{x},z,u,\varepsilon) + G_z\Gamma_s \end{cases} \quad (10)$$

where $\Gamma_s = sign(S(y,\hat{y}))$ is the switching function.

G_x and G_z are the observer gains with $(n\times m)$ and $(m\times m)$ dimensions respectively, to be determined.

The observer sliding surface S can be chosen as a linear function of $(y-\hat{y})$ as given in [11] and [27], so:

$$S(y,\hat{y}) = \Lambda(y-\hat{y}) \quad (11)$$

where $(y-\hat{y})^T = \left[(y_1-\hat{y}_1)\ \ (y_2-\hat{y}_2)\ \ \ldots\ \ (y_m-\hat{y}_m)\right]$ and Λ is $(n\times m)$ gain matrix to be specified.

The error dynamics is calculated by subtracting (10) from (9):

$$\begin{cases} \dot{e}_x = f_x(x,z,u,\varepsilon) - f_x(\hat{x},z,u,\varepsilon) - G_x\Gamma_s \\ \varepsilon\,\dot{e}_z = f_z(x,z,u,\varepsilon) - f_z(\hat{x},z,u,\varepsilon) - G_z\Gamma_s \end{cases} \quad (12)$$

or:

$$\begin{cases} \dot{e}_x = \Delta f_x - G_x\Gamma_s \\ \varepsilon\,\dot{e}_z = \Delta f_z - G_z\Gamma_s \end{cases} \quad (13)$$

where:

$$e_x = x - \hat{x},\ e_z = z - \hat{z}$$

$$\Delta f_x = f_x(x,z,u,\varepsilon) - f_x(\hat{x},z,u,\varepsilon)$$

$$\Delta f_z = f_z(x,z,u,\varepsilon) - f_z(\hat{x},z,u,\varepsilon)$$

Since (13) is a singularly perturbed system, the observer design can be based on sequential application of resulted subsystems of (13) by applying singular perturbation methodology.

III.2. Stability Analysis in the Fast Time Scale

For fast error dynamic subsystem, the associated time scale is defined by $\tau = (t-t_0)/\varepsilon$, then (13) can be transformed into:

$$\begin{cases} \dfrac{de_x}{d\tau} = \varepsilon(\Delta f_x - G_x\Gamma_s) \\ \dfrac{de_z}{d\tau} = \Delta f_z - G_z\Gamma_s \end{cases} \quad (14)$$

Setting $\varepsilon = 0$ in (14), it yields:

$$\frac{de_z}{d\tau} = \Delta f_z - G_z\Gamma_s \quad (15)$$

In this time scale, the stability analysis consists of determining G_z so that in this time scale (τ), the surface

$S(\tau)=0$ is attractive.

It can be shown that when sliding mode occurs on $S(\tau)$, the equivalent value of the discontinuous observer maxillary input is found by solving the Eq. (15) for $G_z\Gamma_s$ after insuring zero for $de_z/d\tau$:

$$G_z\Gamma_s = \Delta f_z \tag{16}$$

and the equivalent switching vector is obtained as:

$$\tilde{\Gamma}_s = G_z^{-1}\Delta f_z \tag{17}$$

III.3. Stability Analysis in the Slow Time Scale

Slow error dynamic subsystem can be found by making $\varepsilon=0$ in (16), so:

$$\frac{de_x}{dt} = \Delta f_x - G_x\Gamma_s \tag{18}$$

$$0 = \Delta f_z - G_z\Gamma_s \tag{19}$$

From (19), the equivalent switching vector can be found:

$$\tilde{\Gamma}_s = G_z^{-1}\Delta f_z$$

Therefore, by appropriate choice of G_x, the desired rate of convergence $e_x \to 0$ can be obtained.

IV. Two Time Scale Sliding Mode Control of induction motors

The design of the classical sliding mode control consists generally of two stages: design of the switching surface S_c and design of the sliding mode controller [2], [3]. The control law used is of the type:

$$u = u_{eq} + u_N$$

where u_{eq} is the equivalent control which acts when the system is restricted to $S_c=0$, while u_N is the discontinuous part of the control u acting when $S_c \neq 0$.

The sliding mode control should be chosen such that the candidate Lyapunov function satisfies the Lyapunov stability criteria. The two time scale sliding mode control for the system (2) is designed in two steps [14]-[17].

First, the sliding mode controllers for each reduced subsystem are designed separately. Then, they are combined to obtain a composite control for the complete system. In this work, the composite control with sliding mode is considered as a case study. The reader is referred to [16]-[20] for more information on the singularly perturbed sliding mode control design.

IV.1. Induction Motor Model

The classical state space model of the induction motor expressed in the (d,q) axis rotating reference frame with $(i_{sd},i_{sq},\phi_{sd},\phi_{sq},\omega)$ as state variables and $(v_{sd},v_{sq},\omega_{sl})$ as control variables is [28]:

$$\begin{cases} \sigma L_s \dfrac{d}{dt} i_{sd} = -L_s\left(\dfrac{1}{T_s}+\dfrac{1}{T_r}\right) i_{sd} + \dfrac{1}{T_r}\phi_{sd} + \omega\phi_{sq} + \\ \qquad + \sigma L_s i_{sq}\omega_{sl} + v_{sd} \\ \sigma L_s \dfrac{d}{dt} i_{sq} = -L_s\left(\dfrac{1}{T_s}+\dfrac{1}{T_r}\right) i_{sq} - \omega\phi_{sd} + \dfrac{1}{T_r}\phi_{sq} + \\ \qquad - \sigma L_s i_{sd}\omega_{sl} + v_{sq} \\ \dfrac{d}{dt}\phi_{sd} = -R_s i_{sd} + \omega\phi_{sq} + \phi_{sq}\omega_{sl} + v_{sd} \\ \dfrac{d}{dt}\phi_{sq} = -R_s i_{sq} - \omega\phi_{sd} - \phi_{sd}\omega_{sl} + v_{sq} \\ \dfrac{d\omega}{dt} = \dfrac{p}{J}\left(T_{em}-T_L\right) - \dfrac{f}{J}\omega \end{cases} \tag{20}$$

where ω_{sl} is the slip frequency $\omega_{sl}=\omega_s-\omega$, $T_s=L_s/R_s$, $T_r=L_r/R_r$ and $\sigma = 1-M^2/(L_sL_r)$.

The electromagnetic torque expressed in terms of the state variables is:

$$T_{em} = p\left(\phi_{sd} i_{sq} - \phi_{sq} i_{sd}\right) \tag{21}$$

IV.2. Singularly Perturbed Induction Motor Model

Based on the well known of the induction machine model dynamics [21], [25]-[27], the slow variables are $(\omega,\phi_{sd},\phi_{sq})$ and the fast variables are (i_{sd},i_{sq}).

Therefore, the corresponding standard singularly perturbed form with $\varepsilon=\sigma$, $x=(\omega,\phi_{sd},\phi_{sq})^T$, $z=(L_s i_{sd}, L_s i_{sq})^T$ and $u=(v_{sd},v_{sq},\omega_{sl})^T$ is:

$$\begin{cases} \dot{x}_1 = k\left(x_2 z_2 - x_3 z_1\right) - \dfrac{f}{J}x_1 - \dfrac{p}{J}T_L \\ \dot{x}_2 = -\alpha z_1 + x_1x_3 + x_3u_3 + u_1 \\ \dot{x}_3 = -\alpha z_2 - x_1x_2 - x_2u_3 + u_2 \\ \varepsilon\dot{z}_1 = -(\alpha+\beta)z_1 + \beta x_2 + x_1x_3 + \varepsilon z_2u_3 + u_1 \\ \varepsilon\dot{z}_2 = -(\alpha+\beta)z_2 + \beta x_3 - x_1x_2 - \varepsilon z_1u_3 + u_2 \end{cases} \tag{22}$$

with:

$$\alpha = R_s / L_s \;;\beta = R_r / L_r \;;k = p^2 / \left(JL_s\right)$$

IV.3. Fast Reduced Subsystem

Following the methodology presented in Section II, the $O(\varepsilon)$ approximation of the exact fast subsystem is given by:

$$\frac{d}{d\tau}\begin{bmatrix} z_{1f} \\ z_{2f} \end{bmatrix} = -\begin{bmatrix} \alpha+\beta & 0 \\ 0 & \alpha+\beta \end{bmatrix}\begin{bmatrix} z_{1f} \\ z_{2f} \end{bmatrix} + \begin{bmatrix} u_{1f} \\ u_{2f} \end{bmatrix} \quad (23)$$

or:

$$\frac{d}{d\tau} z_f = -A_f z_f + u_f \quad (24)$$

IV.4. Slow Reduced Subsystem

For the slow reduced subsystem, we can obtain:

$$\begin{bmatrix} \dot{x}_{1s} \\ \dot{x}_{2s} \\ \dot{x}_{3s} \end{bmatrix} = \begin{bmatrix} -\frac{f}{J}x_{1s} - \lambda\left(x_{2s}^2 + x_{3s}^2\right) - \frac{p}{J}T_L \\ \delta\left(x_{1s}x_{3s} - \alpha\, x_{2s}\right) \\ -\delta\left(x_{1s}x_{2s} + \alpha\, x_{3s}\right) \end{bmatrix} + \\ + \begin{bmatrix} -\lambda\, x_{3s} & \lambda\, x_{2s} & 0 \\ \delta & 0 & x_{3s} \\ 0 & \delta & -x_{2s} \end{bmatrix}\begin{bmatrix} u_{1s} \\ u_{2s} \\ u_{3s} \end{bmatrix} \quad (25)$$

or:

$$\dot{x}_s = f_s\left(x_s\right) + g_s\left(x_s\right)u_s \quad (26)$$

with:

$$\delta = \frac{\beta}{\alpha+\beta}\;,\; \lambda = \frac{k}{\alpha+\beta}$$

and:

$$\begin{bmatrix} z_{1s} \\ z_{2s} \end{bmatrix} = \frac{1}{\alpha+\beta}\left\{\begin{bmatrix} \beta & x_{1s} \\ -x_{1s} & \beta \end{bmatrix}\begin{bmatrix} x_{2s} \\ x_{3s} \end{bmatrix} + \begin{bmatrix} u_{1s} \\ u_{2s} \end{bmatrix}\right\} \quad (27)$$

IV.5. Composite Control

The composite sliding mode control for the induction motor is made on the basis of the decomposed subsystems (23) and (25). Following the procedure described in [14] and [16], the fast and slow control laws can be easily formulated. For the design of the fast control u_f, the proposed sliding mode control surface was:

$$S_{c,f}\left(z_f\right) = \begin{bmatrix} S_{1f} \\ S_{2f} \end{bmatrix} = \begin{bmatrix} k_{1f}z_{1f} \\ k_{2f}z_{2f} \end{bmatrix} \quad (28)$$

where k_{1f} and k_{2f} are positive constants chosen in order to assure proper stability proprieties of the fast subsystem. For the design of the slow control, we use again the sliding mode concept. For this control, another sliding mode control surface must be proposed. We have used the sliding surface given by:

$$S_{c,s}\left(x_x\right) = \begin{bmatrix} S_{1s} \\ S_{2s} \\ S_{3s} \end{bmatrix} = \begin{bmatrix} k_{1s}\left(x_{1s} - x_{1d}\right) \\ k_{2s}\left(x_{2s} - x_{2d}\right) \\ k_{3s}\left(x_{3s} - x_{3d}\right) \end{bmatrix} \quad (29)$$

where x_{1d} ,x_{2d} and x_{3d} are the reference angular speed, and d-q reference fluxes. k_{1s}, k_{2s} and k_{3s} are positive constants that allow to ensure proper stability performances of the closed loop system.

For both fast and slow control laws, the same demonstration used in [21] can be applied to carry out its stability analysis. Finally, the composite control can be synthesized as:

$$u = u_f + u_s \quad (30)$$

V. Two Time Scale Sliding Mode Observer Design

For induction motor, rotor speed and stator currents are easily measured but stator fluxes are rather difficult to measure. In fact, different observer structures have been proposed to estimate those fluxes from rotor speed and state currents [21] and [27]. In this paper, we use a sequential methodology for designing a sliding mode observer for induction motor drive using singular perturbation theory. The two time scale decomposition of the original system of the observer error dynamics into separate slow and fast subsystems permits a simple design and sequential determination of the observer gains.

V.1. Singularly Perturbed Observer

With reference to the above singularly perturbed induction motor model of (22), and considering the measured stator currents as the system outputs, the corresponding sliding mode observer can be constructed as follows:

$$\begin{cases} \dot{\hat{x}}_1 = k\left(\hat{x}_2 z_2 - \hat{x}_3 z_1\right) - \frac{f}{J}x_1 - \frac{p}{J}T_L + G_{x1}\Gamma_s + \\ \quad + q_{x1}\left(x_1 - \hat{x}_1\right) \\ \dot{\hat{x}}_2 = -\alpha\, z_1 + x_1\hat{x}_3 + \hat{x}_3 u_3 + u_1 + G_{x2}\Gamma_s \\ \dot{\hat{x}}_3 = -\alpha\, z_2 - x_1\hat{x}_2 - \hat{x}_2 u_3 + u_2 + G_{x3}\Gamma_s \\ \varepsilon\,\dot{\hat{z}}_1 = -\left(\alpha+\beta\right)z_1 + \beta\,\hat{x}_2 + x_1\hat{x}_3 + \varepsilon\, z_2 u_3 + \\ \quad + u_1 + G_{z1}\Gamma_s \\ \varepsilon\,\dot{\hat{z}}_2 = -\left(\alpha+\beta\right)z_2 + \beta\,\hat{x}_3 - x_1\hat{x}_2 - \varepsilon\, z_1 u_3 + \\ \quad + u_2 + G_{z2}\Gamma_s \end{cases} \quad (31)$$

where $\hat{x}_i$ and $\hat{z}_j$ are the estimation of x_i and z_j for $i \in \{1,2,3\}$ and $j \in \{1,2\}$. G_{x1}, G_{x2}, G_{x3}, G_{z1}, G_{z2} and q_{x1} are the observer gains. The switching vector Γ_s is:

$$\Gamma_s = \begin{bmatrix} sign(s_1) \\ sign(s_2) \end{bmatrix} \tag{32}$$

with:

$$S = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \Lambda^{-1} \begin{bmatrix} z_1 - \hat{z}_1 \\ z_2 - \hat{z}_2 \end{bmatrix} \tag{33}$$

and:

$$\Lambda = \begin{bmatrix} \beta & x_1 \\ -x_1 & \beta \end{bmatrix} \tag{34}$$

The choice of the discontinuous function Γ_s is made to get a simple observer gain synthesis as we will see after. Setting $e_{x_i} = x_i - \hat{x}_i$ and $e_{z_j} = z_j - \hat{z}_j$ for $i \in \{1,2,3\}$ and $j \in \{1,2\}$, the estimation error dynamics are:

$$\begin{cases} \dfrac{de_{x_1}}{dt} = k\left(z_2 e_{x_2} - z_1 e_{x_3}\right) - G_{x1}\Gamma_s - q_{x1} e_{x_1} \\ \dfrac{de_{x_2}}{dt} = \left(x_1 + u_3\right) e_{x_2} - G_{x2}\Gamma_s \\ \dfrac{de_{x_3}}{dt} = -\left(x_1 + u_3\right) e_{x_3} - G_{x3}\Gamma_s \\ \varepsilon \dfrac{de_{z_1}}{dt} = \beta e_{x_2} + x_1 e_{x_3} - G_{z1}\Gamma_s \\ \varepsilon \dfrac{de_{z_2}}{dt} = \beta e_{x_3} - x_1 e_{x_2} - G_{z2}\Gamma_s \end{cases} \tag{35}$$

Exploiting the time properties of multi time scales systems of (35), $\left(e_{z_1}, e_{z_2}\right)$ are fast variables and $\left(e_{x_1}, e_{x_2}, e_{x_3}\right)$ are slow variables. So, the stability analysis of the above system consists of determining G_{z1} and G_{z2} to ensure the attractiveness of the sliding surface $S = 0$ in the fast time scale.

Thereafter G_{x1}, G_{x2} and G_{x3} are determined, such that the reduced order system obtained when $S = \dot{S} = 0$ is locally stable.

V.2. Fast Reduced Order Error Dynamics

From singular perturbation theory, the fast reduced order system of the observation errors can be obtained by introducing the fast time scale τ :

$$\tau = \left(t - t_0\right) / \varepsilon$$

System of Eqs. (35) gives:

$$\begin{cases} \dfrac{de_{x_1}}{d\tau} = \varepsilon\left(k\left(z_2 e_{x_2} - z_1 e_{x_3}\right) - G_{x1}\Gamma_s - q_{x1} e_{x_1}\right) \\ \dfrac{de_{x_2}}{d\tau} = \varepsilon\left(\left(x_1 + u_3\right) e_{x_2} - G_{x2}\Gamma_s\right) \\ \dfrac{de_{x_3}}{d\tau} = \varepsilon\left(-\left(x_1 + u_3\right) e_{x_3} - G_{x3}\Gamma_s\right) \\ \dfrac{de_{z_1}}{d\tau} = \beta e_{x_2} + x_1 e_{x_3} - G_{z1}\Gamma_s \\ \dfrac{de_{z_2}}{d\tau} = \beta e_{x_3} - x_1 e_{x_2} - G_{z2}\Gamma_s \end{cases} \tag{36}$$

Making $\varepsilon = 0$ in the above system, it yields:

$$\begin{cases} \dfrac{de_{z_1}}{d\tau} = \beta e_{x_2} + x_1 e_{x_3} - G_{z1}\Gamma_s \\ \dfrac{de_{z_2}}{d\tau} = \beta e_{x_3} - x_1 e_{x_2} - G_{z2}\Gamma_s \end{cases} \tag{37}$$

or:

$$\frac{d}{d\tau}\left[e_z\right] = \Lambda \left\{ \begin{bmatrix} e_{x_2} \\ e_{x_3} \end{bmatrix} - \Lambda^{-1}\left[G_z\right]\Gamma_s \right\} \tag{38}$$

By appropriate choice of the observer gain terms G_{z1} and G_{z2}, sliding mode occurs in (37) along the manifold $S = e_z = 0$.

Proposition (1): Assume that e_{x_2} and e_{x_3} are bounded in this time, and consider system (37) with the following observer gain matrices:

$$\begin{bmatrix} G_{z1} \\ G_{z2} \end{bmatrix} = \Lambda\Phi \tag{39}$$

where:

$$\Phi = \begin{bmatrix} \varphi_1 & 0 \\ 0 & \varphi_2 \end{bmatrix} \text{ and } \varphi_1, \varphi_2 > 0$$

The attractivity condition of sliding surface $S(\tau) = 0$ is given by:

$$S^T \left(\frac{dS}{d\tau}\right) < 0 \tag{40}$$

In this time scale $\dfrac{dx_i}{d\tau} = 0$ for $i = 1,2,3$ so:

$$S^T \left(\frac{dS}{d\tau}\right) = S^T \left\{ \begin{bmatrix} e_{x_2} \\ e_{x_3} \end{bmatrix} - \Phi\Gamma_s \right\} \tag{41}$$

Thus, (40) is verified with the set defined by the following inequalities:

$$\begin{cases} \varphi_1 > \left|e_{x_2}\right|_{max} \\ \varphi_2 > \left|e_{x_3}\right|_{max} \end{cases} \tag{42}$$

According to the equivalent control method, the system in sliding mode behaves as if $G_z\Gamma_s$ is replaced by its equivalent values $\left(G_z\Gamma_s\right)_{eq}$, which can be calculated from the subsystem (37) assuming $e_z = 0$, $\dot{e}_z = 0$. If $S(\tau) = 0$ the equivalent switching vector $\tilde{\Gamma}_s$ is obtained as:

$$\tilde{\Gamma}_s = \Phi^{-1}\begin{bmatrix} e_{x_2} \\ e_{x_3} \end{bmatrix} \tag{43}$$

V.3. Slow Reduced Order Error Dynamics

For slow error dynamics, we use the system (36) and setting $\varepsilon = 0$.

So, we can write:

$$\begin{bmatrix} \dot{e}_{x_1} \\ \dot{e}_{x_2} \\ \dot{e}_{x_3} \end{bmatrix} = \begin{bmatrix} -q_1 & kz_2 & -kz_1 \\ 0 & \left(x_1+u_3\right) & 0 \\ 0 & 0 & -\left(x_1+u_3\right) \end{bmatrix}\begin{bmatrix} e_{x_1} \\ e_{x_2} \\ e_{x_3} \end{bmatrix} + \\ -\begin{bmatrix} G_{x1} \\ G_{x2} \\ G_{x3} \end{bmatrix}\Gamma_s \tag{44}$$

with:

$$\begin{bmatrix} \beta & x_1 \\ -x_1 & \beta \end{bmatrix}\begin{bmatrix} e_{x_2} \\ e_{x_3} \end{bmatrix} - \begin{bmatrix} G_{z1} \\ G_{z2} \end{bmatrix}\Gamma_s = 0 \tag{45}$$

From Eq. (45), we can get the equivalent switching vector $\tilde{\Gamma}_s$ as:

$$\tilde{\Gamma}_s = \Phi^{-1}\begin{bmatrix} e_{x_2} \\ e_{x_3} \end{bmatrix} \tag{46}$$

In this time scale and according to the equivalent control method, we can replace Γ_s by $\tilde{\Gamma}_s$, and with:

$$G_{x1} = k\left[z_2 \;\; -z_1\right]\Phi \tag{47}$$

and:

$$\begin{bmatrix} G_{x2} \\ G_{x3} \end{bmatrix} = \begin{bmatrix} q_1 & \left(x_1+u_3\right) \\ -\left(x_1+u_3\right) & q_2 \end{bmatrix}\Phi \tag{48}$$

System (44) can be written as the following system:

$$\begin{bmatrix} \dot{e}_{x_1} \\ \dot{e}_{x_2} \\ \dot{e}_{x_3} \end{bmatrix} = \begin{bmatrix} -q_1 & 0 & 0 \\ 0 & -q_2 & 0 \\ 0 & 0 & -q_3 \end{bmatrix}\begin{bmatrix} e_{x_1} \\ e_{x_2} \\ e_{x_3} \end{bmatrix} \tag{49}$$

which is stable for $q_1, q_2, q_3 > 0$.

VI. Simulations Results

The performances of the proposed control observer scheme for the induction motor model in closed loop system developed in the previous sections were studied through simulations.

Some simulations were carried out when the motor is started without torque load. The controller should smoothly regulate the angular speed at 300 rad/s, keep the stator d-component flux ϕ_{sd} at its rated value 1.0 Wb and align the stator flux with the d-axis (i.e. constrain ϕ_{qs} to 0). The control law gains were chosen as k_{1f} = k_{2f} $= k_{1s}$ $= k_{2s}$ = k_{3s} =1.

Following the design considerations of section (V), the observer gains are $\varphi_1 = \varphi_2 = 500$ and $q_1 = q_2 = q_3 = 10$.

The problem of chattering is remedied by replacing the switching function by a continuous one in the sliding surface neighborhood. Fig. 1 shows the rotor speed response of the motor; a very good speed regulation is obtained. In Fig. 2 is shown the slip frequency. The corresponding composite controls are shown in Fig. 3. Fig. 4 shows the stator fluxes responses; after a short initial time, they converge to their desired values.

Stator currents are shown in Fig. 5. These results show that the composite sliding mode control with the proposed observer can track the reference command accurately and quickly.

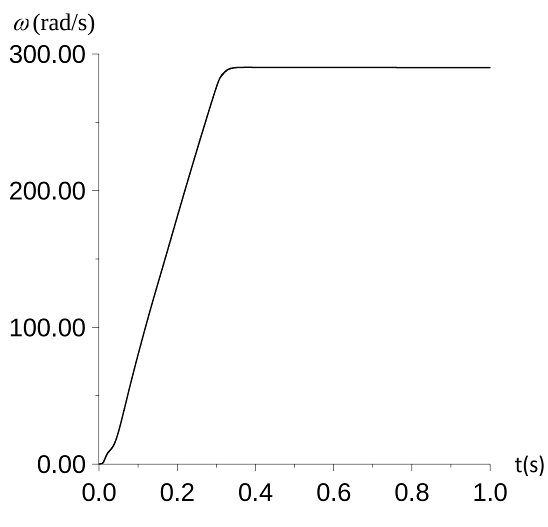


Fig. 1. Angular speed

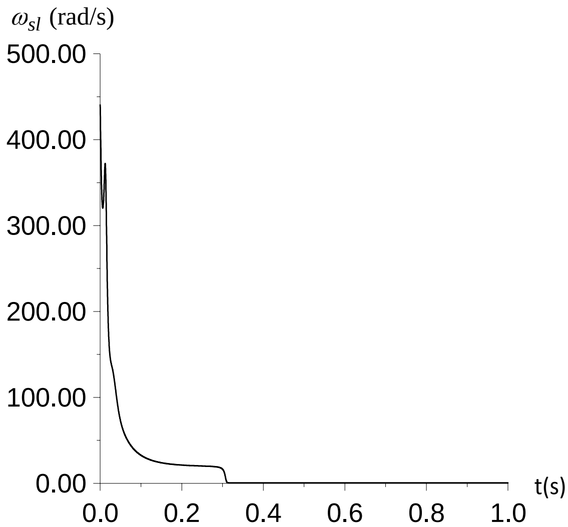


Fig. 2. Composite slip frequency input

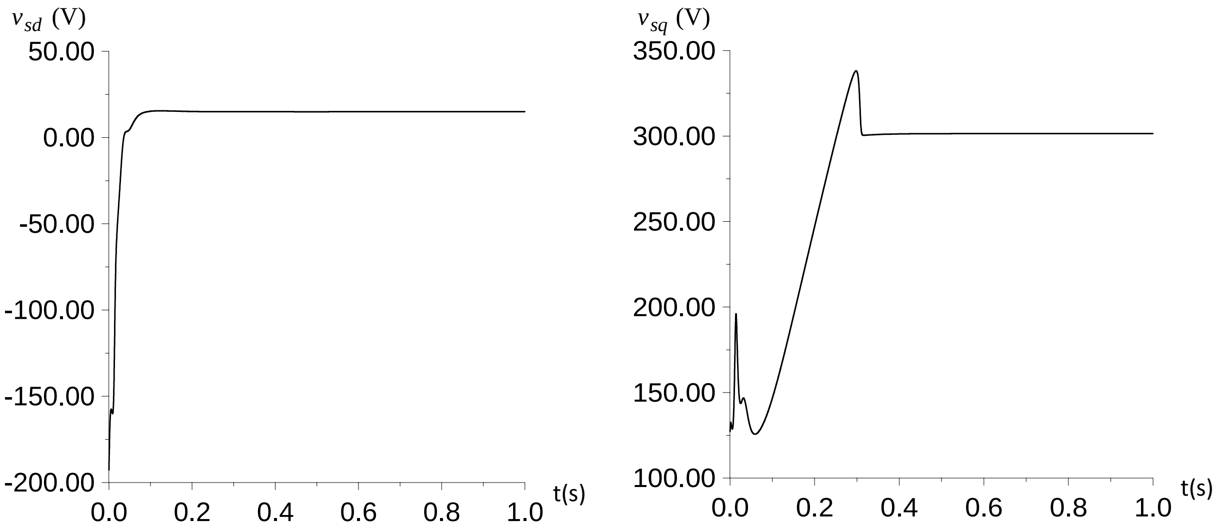


Fig. 3. Composite control voltage inputs

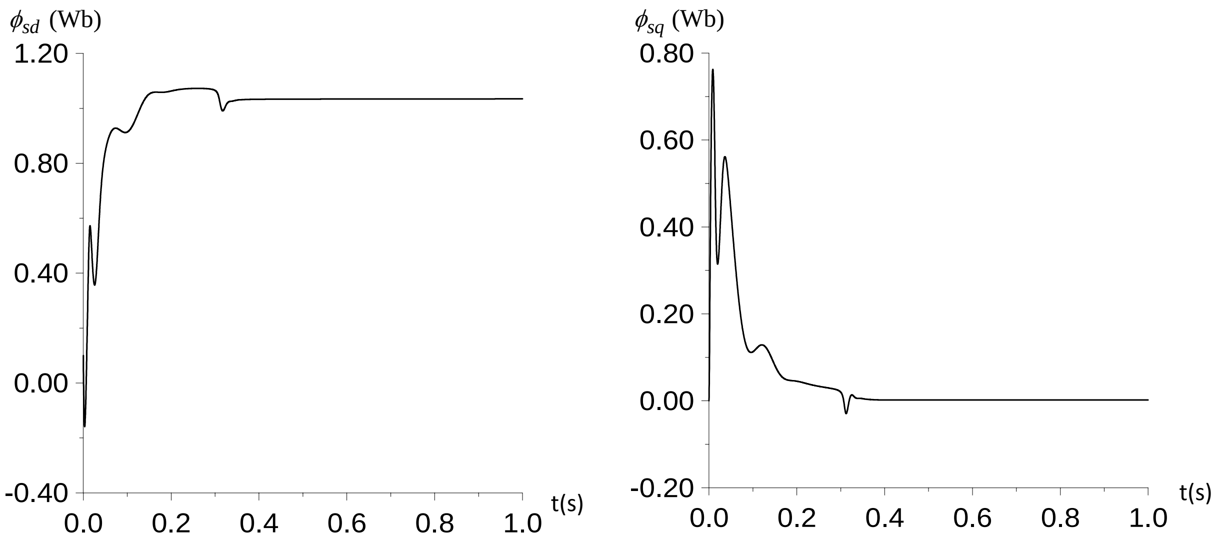
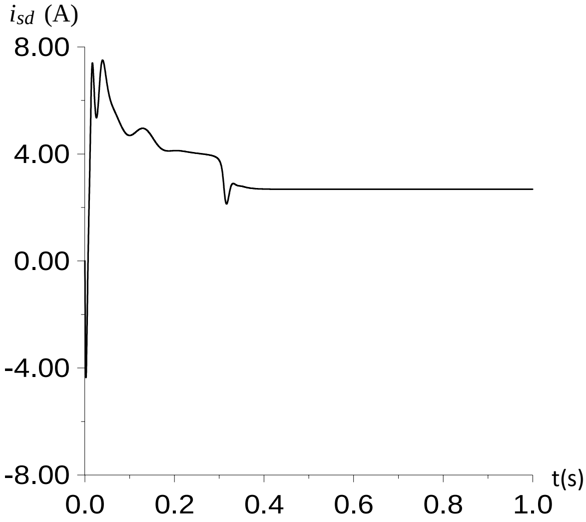


Fig. 4. d-q stator fluxes



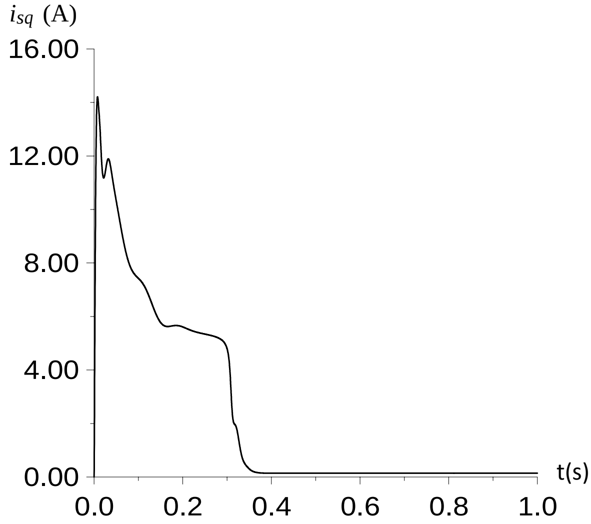


Fig. 5. d-q stator currents

VII. Conclusion

In this paper, we have shown singular perturbation theory to be an effective tool in the analysis of induction motors control-observer problems. Using the assumption of separate time scales, a full order observer has been easily designed in order to estimate the slow variables (stator fluxes) under the assumption that only the fast variables (stator currents) and rotor speed are available for measurement.

It has been shown by the simulation results that this controller-observer scheme is may be useful in controlling induction motors with rotor speed and motor fluxes in order to obtain high dynamic performance.

Sensitivity of the control-observer structure to torque disturbances and uncertainties in the electrical and mechanical parameters are under investigation.

Appendix

TABLE A1
INDUCTION MOTOR NOMINAL PARAMETERS

1.5 kW	220/380 V	3.68 / 6.31 A
$N = 1420$ rpm	$R_s = 4.85\ \Omega$	$R_r = 4.805\ \Omega$
$M = 0.258$	$L_s = 0.274$ H	$L_r = 0.274$ H
$p = 2$	$J = 0.031\ \text{kg} \cdot \text{m}^2$	$f = 0.00114$ N×m×s/rd

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