Averaged Discrete-Time Method for PWM DC-DC Converters Operating in DCVM

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Abstract

A discrete-time model for pulse-width modulated (PWM) converters operating in the discontinuous capacitor voltage mode (DCVM) is formulated. It leads to the exact discrete-time mathematical representation of the values of the output and internal signals. A one-cycle-average (OCA) discrete-time model is introduced. This model provides the exact discrete-time mathematical representation of the averaged values of the output signal. This model also provides the average values of other internal signals with little increase in simulation time. A comparison of this model to other existing models is presented through a numerical example of boost converter. Detailed simulation results confirm the better accuracy and speed of the proposed method.

Keywords: Pulse-width modulation (PWM), power converter, discrete-time modeling, one cycle-averaging, sampled data model.

Introduction

PWM converters are widely used for operating switch controlled systems. These systems are usually operated in two modes of operation, namely: continuous and discontinuous conduction modes [4, 6]. The DCM of operation typically occurs in dc/dc converters at light load. For low power applications, many designers prefer to operate in the DCM in order to avoid the reverse recovery problem of the diode. DCM operation has also been considered a possible solution to the right-half plane (RHP) zero problem encountered in buck-boost and boost derived topologies. In single-phase ac/dc converters with active power factor correction (PFC), the input inductor current becomes discontinuous in the vicinity of the voltage zero crossing; some PFC circuits are even purposely designed to operate in DCM over the entire line cycle in order to
simplify the control. Proper analytical models for DCM operation of PWM converters are therefore essential for the analysis and design of converters in a variety of applications (see [9] and references therein). These modes of operation are also very much useful for efficiently extracting maximum power from the photovoltaic panel (PV) which is another main application [10]. These power converters are connected between the PV and load or bus.

The boundary between the CCM and DCM depends on the ripple current in the inductor or the ripple voltage in the capacitor. The PWM converter operating in DCM is characterized by two main interesting aspects. One is discontinuous inductor current mode (DICM), which is usually regarded as DCM. The other one is discontinuous capacitor voltage mode (DCVM). These modes differ in the fact that the switching instants are internally operated in some subinterval of the switching period to control the fall to zero of an internal variable such as inductor current (DICM) or capacitor voltage (DCVM). In DCVM, voltage is clamped to zero during a part of the switching cycle, while \( i_L > 0 \) for all \( t \) [6]. DCVM has its unique characteristics, such as soft turn-off switching, low switch-current stress, and low input-current ripple [3].

Due to the variety of applications of PWM converters operating in DCM, there is a need for an accurate model for the analysis and design of such converters. Many efforts have been taken in this view for past three decades [6, 9]. Averaging methods are sometimes used to produce approximate continuous-time models for PWM systems by neglecting the switching period of the switches and the sampling period of the controller [2]. In averaging process the ripple in the current or in the voltage is also not considered. To overcome the above disadvantage, the sampled-data modeling techniques are adapted. This provides the most accurate result, which replicates the actual behavior of PWM systems and is also suitable for digital control process.

Sampled-data models allow us to focus on cycle-to-cycle behavior, ignoring intra cycle ripples. This makes them effective in general simulation, analysis and design. These models predict the values of signals at the beginning of each switching period, which most of the times represent peaks or valleys of the signals rather than average values. To better understand the average behavior of the system, a discrete-time model for the OCA signals was presented in [1].

In this paper, a sampled-data model for PWM converters operating in DCVM is formulated. This gives the exact discrete-time mathematical representation of the values of the output and internal signals. A discrete-time model to provide the one-cycle-average (OCA) signals of PWM converters operating in DCVM is proposed. This model provides the exact discrete-time mathematical representation of the averaged values of the output signal. It also provides the average values of other internal signals with little increase in simulation time. The main motivation for the new model is based on the fact that, in many power electronic applications, it is the average values of the voltage and current rather than their instantaneous values that are of greatest interest. Numerical simulations show the accuracy of the propose model.
Existing Averaged models
Different averaging methods for PWM converters are used for analysis and design.

Switched Model
The DCVM PWM converter can be described by

\[
\begin{align*}
    \dot{x}(t) &= \begin{cases} 
        A_1 x(t) + B_1 u(t), & t \in \tau_1 \\
        A_2 x(t) + B_2 u(t), & t \in \tau_2 \\
        A_3 x(t) + B_3 u(t), & t \in \tau_3 
    \end{cases} \\
    y(t) &= \begin{cases} 
        C_1 x(t), & t \in \tau_1 \\
        C_2 x(t), & t \in \tau_2 \\
        C_3 x(t), & t \in \tau_3 
    \end{cases}
\end{align*}
\]  

(1)

(2)

Where \( u \in \mathbb{R}^m \) is the input vector, \( x \in \mathbb{R}^n \) is the state vector, and \( y \in \mathbb{R}^p \) is the output vector. The system switches between three topologies, \((A_1, B_1, C_1), (A_2, B_2, C_2), \) and \((A_3, B_3, C_3)\) as shown in Figure 2, with switching intervals determined by

\[
\begin{align*}
    \tau_1 &:= kT \leq t < kT + d_1^1 T \\
    \tau_2 &:= kT + d_1^1 T \leq t < kT + (d_1^1 + d_1^2) T \\
    \tau_3 &:= kT + (d_1^1 + d_1^2) T \leq t < kT + T
\end{align*}
\]
Figure 2: Ideal waveforms of voltage and current in DCVM

Where $T$ is the switch period, $(d^1_k + d^2_k) \in [0, 1]$ are the switch duty ratios, which are computed as shown in the paper [6] and $k$ is the discrete-time index. All auxiliary inputs will be assumed to be piecewise constants, i.e. $u(t) = u_k$ for all $t \in [kT, (k + 1)T)$. This assumption is not necessary and is made for convenience only; more general cases would only require more complex notations.

**State-space Average Model (SSA)**

The SSA mathematical model of the boost converter is given as

$$
\dot{x}(t) = \left[ d_1 A_1 + d_2 A_2 + (1 - d_1 - d_2) A_3 \right] x_1 + \left[ d_1 B_1 + d_2 B_2 + (1 - d_1 - d_2) B_3 \right] u(t)
$$

**Conventional Discrete-Time Model**

The conventional discrete-time mode (CDTM) is given by

$$
x_{k+1} = A(d^1_k, d^2_k)x_k + B(d^1_k, d^2_k)u_k
$$

Where the input nonlinearities $A(d^1, d^2)$ and $B(d^1, d^2)$ are given by

$$
A(d^1, d^2) := \Phi_3 \Phi_2 \Phi_1 \\
B(d^1, d^2) := \Phi_3 (\Phi_2 \Gamma_1 + \Gamma_2) + \Gamma_3
$$

The arguments $d^1 T, d^2 T, (1 - d^1 - d^2) T$ for $(\Phi_1, \Phi^*_1, \Gamma_1, \Gamma^*_1)$, $(\Phi_2, \Phi^*_2, \Gamma_2$ and $\Gamma^*_2)$ and $(\Phi_3, \Phi^*_3, \Gamma_3$ and $\Gamma^*_3$), respectively are omitted from the above equations for notation simplicity. Where

$$
\Phi_i(t) := e^{A_i t} \\
\Gamma_i(t) := \int_0^t e^{A_i \tau} b_i d\tau
$$

**Proposed Model**

The OCA representation of the output signal [1] is given by
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The signal, \( y^*(t) \) is used to develop a new discrete-time model for PWM converters operating in DCVM. This model provides the basis for discrete-time simulation of the averaged value of any state in the DCVM PWM system, even during transient non-periodic operating conditions.

**OCA Discrete-Time Model**

It is desired to compute, without approximation, the evolution of all system variables at the sampling instants, \( t = kT \) assuming three different topologies for the system. Since the state and output equations (1) - (2) are piecewise-linear with respect to time \( t \), the desired discrete-time model can be obtained symbolically. Using the notation, \( x_k : x(kT) \) and \( y_k^c : y^*(kT) \), the result is the OCA large signal model

\[
x_{k+1} = A(d_k^1, d_k^2) x_k + B(d_k^1, d_k^2) u_k
\]

\[
y_{k+1}^c = C(d_k^1, d_k^2) x_k + D(d_k^1, d_k^2) u_k
\]

Where the input nonlinearities \( A(d^1, d^2), B(d^1, d^2), C(d^1, d^2) \) and \( D(d^1, d^2) \) are given by

\[
A(d^1, d^2) = \Phi_3 \Phi_2 \Phi_1
\]

\[
B(d^1, d^2) = \Phi_3(\Phi_2 \Gamma_1 + \Gamma_2) + \Gamma_3
\]

\[
C(d^1, d^2) = C_1 \Phi_1^* + C_2 \Phi_2^* \Phi_1 + C_3 \Phi_3^* \Phi_2 \Phi_1
\]

\[
D(d^1, d^2) = C_1 \Gamma_1^* + C_2(\Phi_2 \Gamma_1 + \Gamma_2^*) + C_3(\Phi_3^*(\Phi_2 \Gamma_1 + \Gamma_2) + \Gamma_3^*)
\]

The arguments \( d^1 T, d^2 T, (1 - d^1 - d^2) T \) for \( (\Phi_1, \Phi_1^*, \Gamma_1 \) and \( \Gamma_1^* \), \( (\Phi_2, \Phi_2^*, \Gamma_2 \) and \( \Gamma_2^* \) ) and \( (\Phi_3, \Phi_3^*, \Gamma_3 \) and \( \Gamma_3^* \)), respectively are omitted from the above equations for notation simplicity.

Where

\[
\Phi_i(t) = e^{A_i t}
\]

\[
\Gamma_i(t) = \int_0^t e^{A_i \tau} b_i d\tau
\]

\[
\Phi_i^*(t) = \frac{1}{T} \int_0^T \Phi_i(\tau) d\tau
\]

\[
\Gamma_i^*(t) = \frac{1}{T} \int_0^T \Gamma_i(\tau) d\tau.
\]

Note that the averaging operation adds "sensor" dynamics to the system; as a consequence, the large-signal model (6) - (7) is not in standard state-space form. By defining the augmented state vector \( x^* \in \mathbb{R}^{n+p} \) such that

\[
x_{k+1}^* = \begin{bmatrix} x_{k+1} \\ C(d_k^1, d_k^2) x_k + D(d_k^1, d_k^2) u_k \end{bmatrix}
\]
An equivalent (but standard form) representation of the OCA large-signal model is given by:

\[ x_{k+1}^* = A^*(d_{k}^1, d_{k}^2)x_k^* + B^*(d_{k}^1, d_{k}^2)u_k \]

\[ y_k^* = C^*x_k^* \]  

Where

\[ A^*(d^1, d^2) := \begin{bmatrix} A(d^1, d^2) & 0_{n \times p} \\ C(d^1, d^2) & 0_{p \times p} \end{bmatrix} \]

\[ B^*(d^1, d^2) := \begin{bmatrix} B(d^1, d^2) \\ D(d^1, d^2) \end{bmatrix} \]

\[ C^*(d^1, d^2) := \begin{bmatrix} 0_{p \times n} \\ l_{p \times p} \end{bmatrix} \]

Note that not only the OCA values of output signal will be available but also the values of the signals (without averaging) at the beginning of every switching period as well.

**Numerical Example**

Consider a Boost converter shown in Figure 1. The input is \( u = V_g \) and state variables are \( x_1 = i_L \) and \( x_2 = v_C \). Where \( R = 5 \Omega \), \( L = 100 \mu H \), \( C = 1 \mu F \), \( V_g = 5 \, V \), \( T = 100 \mu s \), and \( D = 0.3 \).

**Figure 3:** boost converter during the first stage (switch \( S_1 \) is on and switch \( S_2 \) is off)

**Figure 4:** boost converter during the second stage (both the switches \( S_1 \) and \( S_2 \) are on)
The three periodical switching operating stages and the state equation are given below. In the resulting operating mode, these three operating stages are periodically switched and they are looped correspondingly. The state space matrices $A_1, A_2, A_3, B_1, B_2, B_3, C_1, C_2,$ and $C_3$ are determined from the circuits shown in Figures 3 - 5, and are defined as

$$A_1 = \begin{bmatrix} 0 & 0 & \frac{1}{RC} \\ 0 & -\frac{1}{RC} & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$

$$C_1 = [0 \ 1]$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_2 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$

$$C_2 = [0 \ 1]$$

$$A_3 = \begin{bmatrix} 0 & -\frac{1}{L} & \frac{1}{RC} \\ \frac{1}{C} & -\frac{1}{L} & -\frac{1}{RC} \end{bmatrix}$$

$$B_3 = \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix}$$

$$C_3 = [0 \ 1]$$

**Table 1:** Simulation times for boost converter in DCVM

<table>
<thead>
<tr>
<th>Method</th>
<th>Normalized Simulation Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switched</td>
<td>96.95</td>
</tr>
<tr>
<td>SSA</td>
<td>15.48</td>
</tr>
<tr>
<td>CDTM</td>
<td>1</td>
</tr>
<tr>
<td>DCVM OCA</td>
<td>9.19</td>
</tr>
</tbody>
</table>

All simulations were performed using Matlab on a personal computer (Intel Core 2 Duo) running Microsoft Window XP. The results of switched mode, DCM PWM converter operating in DCVM conventional discrete-time modeling (CDTM) and one-cycle averaging (OCA) for the boost converter are shown in Figure 6. In the figure, current and voltage waveforms are represented. It should be noted that no approximation is made in deriving the new discrete-time model. Consequently, the steady-state values of output voltage are $v_c = 5.6525V$ for conventional discrete-time
model (CDTM), $v_C = 9.1884V$ for state-space average model (SSA), and $v_C = 6.0159V$ for one-cycle averaging model (OCA). It is obvious that the steady state value computed by the proposed method is by far closer to the value of interest to PWM converter design. Table 1 summarize the normalized simulation time for boost converter operating in DCVM for different averaging methods.

**Conclusion**

This manuscript presents a new discrete-time model and a one-cycle-average discrete-time model for PWM converters operating in DCVM. These models provide the exact discrete-time mathematical representation of the instantaneous / averaged values of the output signal respectively. The DCVM OCA model proposed here also provides the average values of other internal signals with little increase in simulation time. A comparison of this model to other existing models is presented through a numerical example of boost converter. The numerical simulation results confirm the accuracy and speed of the OCA discrete-time model for PWM converter operating in the DCVM without any approximation.

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Figure 6: Simulation comparison of various models for boost converter operating in DCVM

References


