Numerical simulation of laminar to turbulent nanofluid flow and heat transfer over a backward-facing step

Hussein Togun a,b,⇑, M.R. Safaei c, Rad Sadri a, S.N. Kazi a, A. Badarudin a, K. Hooman d, E. Sadeghinezhad a

a Department of Mechanical Engineering, University of Malaya, 50603 Kuala Lumpur, Malaysia
b Department of Mechanical Engineering, University of Thi-Qar, 64001 Nassiriya, Iraq
c Young Researchers and Elite Club, Mashhad Branch, Islamic Azad University, Mashhad, Iran
d School of Mechanical and Mining Engineering, The University of Queensland, QLD 4072, Australia

abstract

This paper presents a numerical study of heat transfer to turbulent and laminar Cu/water flow over a backward-facing step. Mathematical model based on finite volume method with a FORTRAN code is used to solve the continuity, momentum, energy and turbulence equations. Turbulence was modeled by the shear stress transport (SST) $K-\omega$ Model. In this simulation, three volume fractions of nanofluid (0%, 2% and 4%), a varying Reynolds number from 50 to 200 for the laminar range and 5000 to 20,000 for the turbulent range, an expansion ratio of 2 and constant heat flux of 4000 W/m$^2$ were considered. The results show the effect of nanofluid volume fraction on enhancing the Nusselt number in the laminar and turbulent ranges. The effect of expansion ratio was clearly observed at the downstream inlet region where the peak of the Nusselt number profile was referred to as enhanced heat transfer due to the generated recirculation flow. An increase of pressure drop was evident with an increasing Reynolds number and decreasing nanofluid volume fraction, while the maximum pressure drop was detected in the downstream inlet region. A rising Reynolds number caused an increasing Nusselt number, and the highest heat transfer augmentation in the present investigation was about 26% and 36% for turbulent and laminar range, respectively compared with pure water.

1. Introduction

Flow over a backward-facing step generates recirculation zones and forms vortices due to the separation flow obtained from the adverse pressure gradients in the fluid flow. The phenomena of flow separation are found in different applications such as heat exchangers, nuclear reactors, power plants, cooling devices, etc. In the past decades, a number of works have been done on this phenomena and its effect on heat transfer rate. The pioneer investigators Boelter [1], Ede et al. [2], Seban [3], Abbot and Kline [4], Seban [5], Filetti and Kays [6], Goldstein et al. [7], Durst and Whitelaw [8], and Brederode and Bradshaw [9] developed experimental and theoretical methods of studying separation flow that takes place due to changes in the cross section of the passage. With advances in measurement devices and CFD software, the researchers have identified detailed information regarding the structure of separation flow and recirculation zone. Armaly et al. [10] employed a Laser Doppler Anemometer...
to measure the velocity distribution and reattachment length for air flow over a backward-facing step. They investigated the
laminar, transition, and turbulent range domains and the obtained results were in good agreement with the experimental
and numerical findings. Particle tracking velocimetry was employed for studying the laminar separation flow in a forward-
facing step by Stuer et al. [11]. The experimental results demonstrated an increase in distance between the breakthroughs
in span as the Reynolds number decreased. It was also noticed that the transverse direction of separation was slow, compared
to the short time scale over flow visualization. The study of the fluid flow of two non-Newtonian liquids in sudden expansion
with viscoelastic polyacrylamide (PAA) solutions and a purely viscous shear-thinning liquid performed by Pak et al. [12]. The
Reynolds number was varied from 10 to 35,000 with an expansion ratio of 2–2.667; according to the results from the laminar
range, the reattachment length of non-Newtonian fluid was shorter compared to the Newtonian fluid and two to three times
shorter for the turbulent range than water. The effects of step height on heat transfer and turbulent flow characteristics were
presented numerically by Nie and Armaly [13]. Uniform heat flux was maintained at the downstream region of the passage with
Re = 28,000. It was found that an increase in step height caused the primary and secondary recirculation zones to enlarge.
Increase Nusselt number noticed with increase blockage ratio and Reynolds number. K Hannafer et al. [14] carried out a numerical
study on the heat transfer and laminar mixed convection of pulsatile flow over a backward-facing step with the help of the finite
element method. Based on the results, by increasing the Reynolds number, the heat transfer rate amplified while the thickness
of the thermal boundary layer reduced. In contrast, Chen et al. [15] numerically studied heat transfer and turbulent forced
convection flow over a backward-facing step. The results attained revealed enhanced heat transfer in response to an increase
in step height. The effect of backward- and forward-facing steps on turbulent mixed-convection flow over a flat plate was investi-
gated by Abu-Mulaweh [16]. Increased turbulence intensity occurred from the temperature fluctuations at the downstream
region due to the introduction of the backward- and forward-facing steps. Hussein et al. [17] presented experimental study of
turbulent separation flow in concentric annular passage with sudden expansion. They found maximum augmentation of heat
transfer about 18% at step height 18.5 mm in compared to without step. In general, separation and reattachment flow are
adopted in several experimental and numerical studies [18–24].

More recently, the majority of studies have been utilizing nanofluid because of its higher thermal conductivity compared
to normal fluid [25]. Abu Nada [26] is a pioneer in research on laminar nanofluid flow over a backward-facing step with Cu,
Ag, Al2O3, CuO, and TiO2 nanofluid, volume fractions between 0.05 and 0.2 and Reynolds numbers ranging from 200 to 600.
An investigation of findings signifies that the Nusselt number increased with the volume fraction and Reynolds number.
Later, Kherbeet et al. [27] presented a numerical investigation of heat transfer and laminar nanofluid flow over a micro-scale

<table>
<thead>
<tr>
<th>Nomenclature</th>
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<tbody>
<tr>
<td>$C_p$ specific heat capacity (J kg$^{-1}$ K$^{-1}$)</td>
</tr>
<tr>
<td>$E$ total energy (J kg$^{-1}$)</td>
</tr>
<tr>
<td>$h$ heat transfer coefficient (W m$^{-2}$ K$^{-1}$)</td>
</tr>
<tr>
<td>$k$ turbulence kinetic energy (m$^2$ s$^{-2}$)</td>
</tr>
<tr>
<td>Nu Nusselt number</td>
</tr>
<tr>
<td>$P$ pressure (Pa)</td>
</tr>
<tr>
<td>Pr Prandtl number</td>
</tr>
<tr>
<td>$q$ heat flux (W m$^{-2}$)</td>
</tr>
<tr>
<td>Re Reynolds number</td>
</tr>
<tr>
<td>$T$ temperature (K)</td>
</tr>
<tr>
<td>$u$ velocity component (m s$^{-1}$)</td>
</tr>
<tr>
<td>$x, y$ spatial coordination (m)</td>
</tr>
<tr>
<td>$y$ distance to the next surface (m)</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Greek symbols</th>
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<tbody>
<tr>
<td>$\lambda$ thermal conductivity (W m$^{-1}$ K$^{-1}$)</td>
</tr>
<tr>
<td>$\mu$ dynamic viscosity (Pa s)</td>
</tr>
<tr>
<td>$\nu$ kinematic viscosity (m$^2$ s$^{-1}$)</td>
</tr>
<tr>
<td>$\rho$ density (kg m$^{-3}$)</td>
</tr>
<tr>
<td>$\sigma$ turbulent Prandtl number</td>
</tr>
<tr>
<td>$\tau$ wall shear stress (Pa)</td>
</tr>
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<table>
<thead>
<tr>
<th>Subscripts</th>
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<tbody>
<tr>
<td>eff effective</td>
</tr>
<tr>
<td>f base fluid</td>
</tr>
<tr>
<td>$i, j$ components</td>
</tr>
<tr>
<td>m mixture</td>
</tr>
<tr>
<td>P nanoparticles</td>
</tr>
<tr>
<td>t turbulent</td>
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</table>
backward-facing step. The Reynolds numbers ranged from 0.01 to 0.5, nanoparticle types comprised Al$_2$O$_3$, CuO, SiO$_2$, and ZnO, and the expansion ratio was 2. An increasing Reynolds number and volume fraction seemed to lead to an increasing Nusselt number; the highest Nusselt number value was obtained with SiO$_2$.

Additional last investigations concern nanofluid flow over a backward-facing step for the laminar range [28–35], but such work with respect to the turbulent regime, in particular, is still not entirely understood. Due to the Cu has higher thermal conductivity and many experimental investigations done with good improvement in thermal performance then used in this simulation. The aim of the present paper is therefore to study the enhancement of heat transfer regarding both turbulent and laminar Cu/water flow over a backward-facing step.

2. Numerical model

2.1. Description of geometry

The geometry and flow domain applied in this study are illustrated in Fig. 1. The geometrical dimensions were 1.25 cm inlet diameter, 200 cm upstream length, 2.5 cm outlet diameter and 150 cm downstream length for an expansion ratio of 2. The downstream wall was heated while all other walls were insulated. The Reynolds numbers varied from 50 to 200 for the laminar and 5000 to 20,000 for the turbulent regimes. The working fluids were pure water or Cu/water at different volume fractions.

2.2. Governing equations

The governing equations of continuity, momentum and energy were solved for turbulent flow at the rectangular coordinates under the hypotheses of steady-state, two-dimensional, incompressible, turbulent, and constant flow condition properties [36]:

\[
\frac{\partial}{\partial x_i} (\rho u_i) = 0 \tag{1}
\]

\[
\frac{\partial}{\partial x_i} (\rho u_i u_j) = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_i} \left[ \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial u_l}{\partial x_l} \right) \right] + \frac{\partial}{\partial x_j} (-\rho u_i u_j) \tag{2}
\]

\[
\frac{\partial}{\partial x_i} \rho u_i (\rho E) + P = \frac{\partial}{\partial x_j} \left[ \left( \frac{c_p u_i}{P_{T_i}} \right) \frac{\partial T}{\partial x_j} + u_i (\tau_{ij})_{\text{eff}} \right] \tag{3}
\]

\[
E = C_p T + \left( \frac{u^2}{2} \right) \text{ and } (\tau_{ij})_{\text{eff}} \text{ represents the deviatoric stress tensor as determined by Eq. (4).}
\]

\[
(\tau_{ij})_{\text{eff}} = \mu_{\text{eff}} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \mu_{\text{eff}} \frac{\partial u_i}{\partial x_j} \delta_{ij} \tag{4}
\]

The SST K–\omega Model equations, as developed by Menter [37], can be written as Eqs. (5) and (6):

\[
\frac{\partial}{\partial x_i} (\rho k u_i) = \frac{\partial}{\partial x_j} \left( \Gamma_k \frac{\partial k}{\partial x_j} \right) + G_k - Y_k + S_k \tag{5}
\]

\[
\frac{\partial}{\partial x_i} \left( \rho \omega u_i \right) = \frac{\partial}{\partial x_j} \left( \Gamma_{\omega} \frac{\partial \omega}{\partial x_j} \right) + G_{\omega} - Y_\omega + D_\omega + S_\omega \tag{6}
\]

where $G_k$ is defined as the production of turbulent kinetic energy because of the mean velocity gradients, and $G_{\omega}$ is defined as the generation of $\omega$.

\[
G_k = \min(G_k, 10 \beta^* k_{\text{eff}}) \tag{7}
\]

Fig. 1. Geometry domain.
For $G_k = -\rho U\overline{w}_i(\partial u_i/\partial x)$, where $G_k$ is a function of $G_{\omega}$.

\[
G_{\omega} = \frac{\alpha}{\nu} G_k
\]

where $\beta^*$ is a model constant, and $\alpha$ could be given by Eq. (9).

\[
\alpha = \infty \left( \frac{\alpha^* + \text{Re}_k/\text{Re}_\omega}{1 + \text{Re}_k/\text{Re}_\omega} \right)
\]

where $\text{Re}_\omega = 2.95$, $\alpha$ is define by Eq. (10):

\[
\alpha = F_1 \alpha_{\infty,1} + (1 - F_1) \alpha_{\infty,2}
\]

\[
\alpha_{\infty,1} = \frac{\beta_{1,1}}{\beta_{1,0}^*} \frac{\kappa^2}{\sigma_{\omega,1} \sqrt{\beta^*_\omega}}
\]

\[
\alpha_{\infty,2} = \frac{\beta_{1,2}}{\beta_{1,0}^*} \frac{\kappa^2}{\sigma_{\omega,2} \sqrt{\beta^*_\omega}}
\]

The value of constants are $\kappa = 0.41$ and $\beta_i = 0.072$. $\alpha = \alpha_{\infty} = 1$, for high Reynolds number.

The effective of diffusivity $C_k$ and $C_{\omega}$ can be represented by Eqs. (13) and (14):

\[
C_k = \mu + \frac{\mu_t}{\sigma_k}
\]

\[
C_{\omega} = \mu + \frac{\mu_t}{\sigma_{\omega}}
\]

The turbulent Prandtl numbers of $k$ and $\omega$ are represented by Eqs. (15) and (16):

\[
\sigma_k = \frac{1}{F_1 / \sigma_{k,1} + (1 - F_1) / \sigma_{k,2}}
\]

\[
\sigma_{\omega} = \frac{1}{F_1 / \sigma_{\omega,1} + (1 - F_1) / \sigma_{\omega,2}}
\]

where $\mu_t$ is given by Eq. (17):

\[
\mu_t = \infty \frac{\rho k}{\alpha}
\]

$\alpha^*$ is the coefficient for the damping of turbulent viscosity and is computed from:

\[
\alpha^* = \infty \left( \frac{\alpha_{k}^* + \text{Re}_\omega/\text{Re}_k}{1 + \text{Re}_\omega/\text{Re}_k} \right)
\]

The coefficients in Eq. (18) are: $\text{Re}_\omega = \rho k/\mu \omega$, $\text{Re}_k = 6$, $\alpha_{k}^* = \beta_1/3$ and $\beta_i = 0.072$.

Also, for high Reynolds numbers $\alpha^* = \alpha_{\omega}^* = 1$.

The blending equation ($F_1$) is calculated by:

\[
F_1 = \tan(\Phi_1^*)
\]

\[
\Phi_1 = \min \left[ \max \left( \frac{\sqrt{k}}{0.09 \omega y} \frac{500\mu}{\rho y^2 \omega} \frac{4\rho k}{\sigma_{\omega,2} D_\omega^x y^2} \right) \right]
\]

\[
D_\omega^x = \max \left[ 2\rho \frac{1}{\sigma_{\omega,2}} \frac{1}{\sigma_{\omega,2}} \frac{1}{\sigma_{\omega,2}} \frac{\partial k}{\partial \omega} \frac{\partial \omega}{\partial \omega} \right] 10^{-10}
\]

where $D_\omega^x$ is the positive portion of the cross-diffusion term. $Y_k$ and $Y_{\omega}$ show the dissipation of $k$ and $\omega$ due to turbulence:

\[
Y_k = \rho \beta^* k \omega
\]

\[
Y_{\omega} = \rho \beta^* \omega^2
\]

\[
\beta_i = F_1 \beta_{1,1} + (1 - F_1) \beta_{1,2}
\]

where $S_k$ and $S_{\omega}$ correspond to the possible source terms and $D_\omega$ is defined as the cross-diffusion term and is computed from Eq. (25):
\[ D_{\text{ct}} = 2(1 - F_1)\rho\sigma_{\text{ct}} \frac{1}{\omega} \frac{\partial k}{\partial x_j} \frac{\partial \omega}{\partial x_i} \]  

(25)

The values of the constant model are shown in Table 1 as used by [38].

For the two-dimensional conduction model and steady state with heated downstream wall, Eq. (26) may be used:

\[
\frac{\partial}{\partial x_i} \left( \lambda \frac{\partial T}{\partial x_i} \right) = 0
\]

(26)

**Table 1**

The constant of \( K-\omega \) Model [38].

<table>
<thead>
<tr>
<th>( \sigma_{1,2} )</th>
<th>( \sigma_{3,2} )</th>
<th>( x_{\gamma} )</th>
<th>( x_{\omega} )</th>
<th>( \beta_i )</th>
<th>( x_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.176</td>
<td>1</td>
<td>2</td>
<td>1.168</td>
<td>0.072</td>
<td>1.9</td>
</tr>
</tbody>
</table>

**Table 2**

Properties of nanofluid [41,50].

<table>
<thead>
<tr>
<th>Property</th>
<th>Water</th>
<th>Cu</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_p ) (J/kg K)</td>
<td>4179</td>
<td>3830</td>
</tr>
<tr>
<td>( \rho ) (kg/m³)</td>
<td>997.1</td>
<td>8954</td>
</tr>
<tr>
<td>( K ) (W/m K)</td>
<td>0.6</td>
<td>400</td>
</tr>
<tr>
<td>( \mu ) (Pa s)</td>
<td>( 8.91 \times 10^{-4} )</td>
<td>( - )</td>
</tr>
<tr>
<td>( \alpha ) (K⁻¹)</td>
<td>( 2.1 \times 10^{-4} )</td>
<td>( 1.67 \times 10^{-5} )</td>
</tr>
</tbody>
</table>

**Table 3**

Grid independency tests.

<table>
<thead>
<tr>
<th>Number of grids</th>
<th>3000 × 8</th>
<th>4000 × 9</th>
<th>5000 × 10</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3A – For laminar regime</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Nusselt number for Re = 50 and pure water (( \phi = 0 ))</td>
<td>115.64018</td>
<td>115.6147</td>
<td>115.60996</td>
</tr>
<tr>
<td></td>
<td>5000 × 10</td>
<td>6000 × 12</td>
<td>7000 × 14</td>
</tr>
<tr>
<td><strong>3B – For the turbulence RNG K-( \omega ) model</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Nusselt number for Re = 5000 and pure water (( \phi = 0 ))</td>
<td>2483.3585</td>
<td>2483.3635</td>
<td>2483.3621</td>
</tr>
</tbody>
</table>

**Fig. 2.** Variation of X-velocity at X = 207.44 mm, Re = 175.
3. Thermophysical properties of the nanofluid

The physical properties of the governing equations are as per Ramiar et al. [39], Khanafer and Vafai [40] and Cho et al. [41].

\[ \rho_m = \phi \rho_p + (1 - \phi) \rho_f \] (27)

An accurate equation for calculating the effective heat capacitance is [39–41]:

\[ (\rho c_p)_m = (1 - \phi)(\rho c_p)_f + \phi(\rho c_p)_p \] (28)

Chon et al.’s [42] correlation, which considers the Brownian motion and mean diameter of the nanoparticles, is used to calculate the effective thermal conductivity:

\[ \frac{k_{nf}}{k_f} = 1 + 64.7 \phi^{0.746} \left( \frac{d_p}{d_p} \right)^{0.369} \left( \frac{k_p}{k_f} \right)^{0.7476} Pr^{0.0955} Re^{1.2321} \] (29)

Pr and Re are defined as:

\[ Pr = \frac{\mu_f}{\rho_f T_f} \quad \text{and} \quad Re = \frac{\rho_f k_b T}{3 \pi \mu L_f} \]

where \( L_f \) is the mean free path of water, \( k_b \) is the Boltzmann Constant (\( 1.3807 \times 10^{-23} \) J K\(^{-1}\)) and \( \mu \) is the temperature-dependent viscosity of the base fluid as calculated with the following equation:

\[ \mu = O \times 10^{P T} \] (30)

where \( O, P, \) and \( Q \) are constants. For water, these are equal to \( 2.414 \times 10^{-5}, 247.8 \) and \( 140 \), respectively [43].

**Table 4**

Comparison of the separation points with previous work [51].

<table>
<thead>
<tr>
<th>Re</th>
<th>Reattachment length/step height (Jongebloed [51] work, RNG K−ω turbulence model)</th>
<th>Reattachment length/step height (present work, K−ω turbulence model)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7000</td>
<td>6.92</td>
<td>6.87</td>
</tr>
<tr>
<td>11,400</td>
<td>7.01</td>
<td>6.98</td>
</tr>
<tr>
<td>17,799</td>
<td>8.00</td>
<td>7.99</td>
</tr>
</tbody>
</table>

**Fig. 3.** Comparison Nusselt number with [52].
Masoumi et al. [44] have recently developed a new equation for predicting nanofluid effective viscosity that is a function of temperature, mean nanoparticle diameter, nanoparticle volume fraction, nanoparticle density and the base fluid’s physical properties. Eq. (31) is adopted for calculating the nanofluid effective viscosity:

\[
\mu_{\text{eff}} = \mu_{\text{bf}} + \mu_{\text{app}} \quad (31)
\]

where \( \mu_{\text{bf}} \) and \( \mu_{\text{app}} \) are the base fluid and apparent viscosity, respectively. The apparent viscosity is characterized by Eq. (32):

\[
\mu_{\text{app}} = \frac{\rho V_B d_p^2}{72 C \delta} \quad (32)
\]

where \( C \) depends on the base fluid’s viscosity, the nanoparticles’ mean diameter, \( d \), depends on the mean diameter and volume fractions of the nanoparticles, and \( V_B \) is the Brownian velocity of the nanoparticles which depends on temperature, diameter and the density of particles equation (33):

\[
C = \mu_{\text{bf}}^{-1} (c_1 \phi + c_2) d_p + (c_3 \phi + c_4) \quad (33)
\]

where \( c_1 = -1.133 \times 10^{-6}, c_2 = -2.771 \times 10^{-6}, c_3 = 9.0 \times 10^{-8} \) and \( c_4 = -3.93 \times 10^{-7} \) [41].

Fig. 4. Distribution of surface Nusselt number at different volume fraction and (A) Re = 50, (B) Re = 100, (C) Re = 200.
In this simulation, the nanofluid’s thermophysical properties based on the properties of water (as base fluid) and copper (as nanoparticles) are calculated by equations (27)–(33) corresponding to volume fractions $\phi = 2\%$ and $\phi = 4\%$ (Table 2) [41,50]. Volume fraction clearly has an influence on thermal conductivity due to increased surface area of nanoparticles dispersed in water. Furthermore, the base fluid viscosity apparently decreases with temperature, contributing to the intensifying Brownian motion along with temperature [43].

The dimensionless pumping power is calculated by the nanofluid flow rate and the pressure drop across the passage.

$$P_{\text{pump}} = \frac{Q \cdot \Delta P}{\mu^3 \rho^{-2} d_h^{-1}}$$

where $\mu$ and $\rho$ are represent viscosity and density of nanofluid, respectively, $Q$ is the flow rate and $\Delta P$ pressure drop across the passage, $d_h$ hydraulic diameter.

The Nusselt number can be determined using the Eq. (35).

$$\text{Nu} = \frac{h_{nf} d_h}{k_{nf}}$$

where $h_{nf}$ and $k_{nf}$ are define the heat transfer coefficient and thermal conductivity of nanofluid, respectively.

**Fig. 5.** Distribution of surface Nusselt number at different volume fraction and (A) Re = 5000, (B) Re = 10,000, (C) Re = 20,000.
4. Numerical procedure

The governing equations were solved with a computer program using FORTRAN code according to associated boundary and initial conditions. The finite volume method [45,46] was applied to discretize the governing equations within the computational domain, while the SIMPLE algorithm [47–49] assisted with linking the pressure and velocity fields. The diffusion terms in the momentum equations and convective terms were estimated by selecting the Second Order Central Difference and Second Order Upwind differencing, respectively, resulting in greater solution stability.

Non-uniform quadrilateral grids were utilized for meshing the solution domain. The meshing process strategy was highly concentrated near the step and step corners in order to ensure numerical simulation accuracy and to conserve both grid size and computational time. The residual sum for each variable was computed and stored after each iteration, thus recording the convergence history. The convergence criterion necessitated that the maximum relative mass residual based on the inlet mass be smaller than $1 \times 10^{-3}$.

Fig. 6. Effect of laminar Reynolds number on surface Nusselt number for (A) Vol. = 0%, (B) Vol. = 2%, (C) Vol. = 4%.
5. Grid independence

Structured non-uniform grid distributions were performed to discretize the computation domain. Due to the significance that velocity and temperature gradients have near the walls, the number of elements increased there. Different grid sizes were adopted, and Tables 3a and 3b show two cases of investigations performed for selected grids in this simulation.

6. Numerical procedure validation

6.1. Laminar forced convection validation

The laminar forced convection of single-phase nanofluids in a 2D horizontal backward-facing step at an expansion ratio of 2 and step height of 4.8 mm was studied by Al-Aswadi et al. [35]. Calculations were done for $50 \leq \text{Re} \leq 175$ while the volume fraction of nanoparticles was kept fixed at $\phi = 0.05$. The computed $X$-velocity of Cu/water nanofluid at Re = 175 was contrasted with the work of Al-aswadi et al. [35] in Fig. 2.

![Fig. 7. Effect of turbulent Reynolds number on surface Nusselt number for (A) Vol. = 0%, (B) Vol. = 2%, (C) Vol. = 4%.](image_url)
The present results demonstrate very good agreement with the results of Al-aswadi et al. [35]. Therefore, the current numerical procedure can be used with confidence to simulate laminar forced convection flows.

6.2. Turbulent forced convection validation

The present numerical procedure for solving turbulent forced convection was validated against the existing results of Jongebloed [51] for turbulent forced convection in a backward-facing step. In this work, laminar and turbulent forced convection was simulated.

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**Fig. 8.** Average Nusselt number with different Reynolds number for laminar range.

**Fig. 9.** Average Nusselt number with different Reynolds number for turbulent range.
Convection heat transfer of air in a backward-facing step with an expansion ratio of 1.942 and step height of 4.9 m was studied numerically by finite volume method. Investigations were done at $400 \leq \text{Re} \leq 1200$ for the laminar regime and $6600 \leq \text{Re} \leq 24480$ for the turbulent regime. For $\text{Re} = 7000$, 11,400, and 17,799, the predicted reattachment length/step height ($X_1/S$) is shown in Table 4 and the results were compared to Jongebloed's [51]. Table 4 indicates reasonable agreement between the results of the present work and those of Jongebloed [51]. The small discrepancies seen in this table may be due to the differences between the turbulence models used. For more validation results, investigation study of heat transfer and fluid flow over vertical double forward facing step which conducted by Tuqa et al. [52] used. In Fig. 3 the present results compared with the simulation results of [52] for case 1 at $T = 310$ K where satisfy agreement found with our code.

![Streamline of velocity at expansion ratio 2 with volume fraction 4%](image)

**Fig. 10.** Streamline of velocity at expansion ratio 2 with volume fraction 4% (A) $\text{Re} = 50$, (B) $\text{Re} = 100$, (C) $\text{Re} = 200$. 
7. Results and discussion

7.1. Effect of nanofluid volume fraction

Fig. 4(a)–(c) shows the variation in surface Nusselt number for the laminar range of Reynolds numbers 50, 100, and 200 and the volume fractions of Cu 0%, 2%, and 4%. The Nusselt number profile increased suddenly at the downstream inlet region.

Fig. 11. Streamline of velocity at expansion ratio 2 with volume fraction 4% (A) Re = 5000, (B) Re = 10,000, (C) Re = 20,000.
and then decreased gradually up to the exit due to the recirculation flow created by the backward-facing step. It is also clear that the Nusselt number increased at higher nanofluid volume fractions due to increase of thermal conductivity of fluid by disperse nanoparticles. Nusselt number variations in the turbulent range at Reynolds numbers 5000, 10,000, and 20,000, and various nanofluid volume fractions are presented in Fig. 5(a)–(c). Generally, the results portray a rise in Nusselt number at the downstream inlet region, and it was also observed that an increasing nanofluid volume fraction caused the Nusselt number to increase.

7.2. Effect of the Reynolds number

The effects of the Reynolds number on the local Nusselt number for the laminar and turbulent ranges are presented in Figs. 6(a)–(c) and 7(a)–(c), respectively. With the increase of Reynolds number, the Nusselt number increased in the laminar and turbulent ranges. For pure water, the effect of the Reynolds number in the turbulent regime is obvious because clearly, the difference in Nusselt number at various Re values is significant in comparison with nanofluid for the laminar range.

![Isothermal Streamline at expansion ratio 2 with volume fraction 4%](image)

Fig. 12. Isothermal Streamline at expansion ratio 2 with volume fraction 4% (A) Re = 50, (B) Re = 100, (C) Re = 200.
Fig. 13. Isothermal Streamline at expansion ratio 2 with volume fraction 4% (A) Re = 5000, (B) Re = 10,000, (C) Re = 20,000.
7.3. Average Nusselt number

In Figs. 8 and 9 the variation of average Nusselt number with the Reynolds number at different nanofluid volume fractions can be seen. For all cases, the average Nusselt number augmented as the Reynolds number and nanofluid volume fractions increased. The highest Nusselt number was obtained at 4% volume fraction and 20,000 Reynolds number. Generally, the

Fig. 14. Pressure drop with different Reynolds number and volume fraction for laminar range.

Fig. 15. Pressure drop with different Reynolds number and volume fraction for turbulent range.
maximum ratio of enhancement heat transfer to nanofluid was about 26% and 36% for turbulent and laminar range, respectively compared with pure water due to increase of intensity convection of enhanced conductivity nanofluid.

7.4. Streamlines

The streamlines at the beginning of expansion for the laminar and turbulent regimes at 4% volume fraction and Reynolds number ranging from 50 to 200 and 5000 to 20,000, respectively, are presented in Figs. 10(a)–(c) and 11(a)–(c). Obviously, the recirculation region expanded with the increase of Reynolds number for the laminar and turbulent ranges, while the largest recirculation region was obtained at Re = 20,000, which corresponds to the greatest heat transfer improvement. For more clarification, isothermal streamlines are presented in Figs. 12(a)–(c) and 13(a)–(c) Reynolds number ranging from 50 to 200 and 5000 to 20,000, respectively.

7.5. Pressure drop

The pressure drop variation with axial distance for different Reynolds numbers and nanofluid volume fractions is presented in Figs. 14 and 15. According to the results, the pressure drop intensified as the Reynolds number increased and nanofluid volume fraction declined. Generally, the highest pressure drop occurred at the downstream inlet region due to recirculation flow which caused the improvement of heat transfer.

8. Conclusion

The shear stress transport (SST) K–ω Model with boundary conditions, three nanofluid volume fractions and Reynolds numbers ranging from 50 to 200 for the laminar regime and 5000 to 20,000 for the turbulent regime, at a constant heat flux of 4000 W/m² and an expansion ratio fixed at 2, was considered. Calculations show that the heat transfer augmented when the volume fraction and Reynolds number were increased. It was additionally observed that the recirculation flow as created by the backward-facing step enhanced heat transfer. The velocity streamline illustrated the recirculation and separation region. The maximum heat transfer increase was obtained at 4% volume fraction and Reynolds number of 20,000, while the greatest pressure drop at the downstream inlet region indicated the heat transfer improvement.

Acknowledgements

The authors gratefully acknowledge the High-impact Research Grant UM.C/625/1/HIR/MOHE/ENG/46, UMRG RP012D-13AET, IPPP/PV113/2011A, and University of Malaya, Malaysia, for the support in conducting this research.

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