Cognitive Radio Game: A Framework for Efficiency, Fairness and QoS Guarantee

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Abstract—In this paper we develop a framework for resource allocation in a secondary spectrum access scenario where a group of Cognitive Radios (CR) access the resources of a primary system. We assume the primary system is a cellular OFDM-based network. We develop the optimum resource allocation strategies which guarantee a level of QoS, defined by minimum rate and the target Bit Error Rate (BER), for the primary system. Using the Game theoretic axiom of fairness, i.e., Nash Bargaining Solutions (NBS), we show that by allocating a priority factor to all players an efficient and fair resource allocation can be achieved. We show how the priority factors are assigned in this scheme and outline a method to select the users who are allowed to share a specific sub-channel.

I. INTRODUCTION

As the demand for high data rate communications increase, traditional Fixed Spectrum Allocation (FSA) methods face a challenge to provide such services satisfactorily. That is why many spectrum regulatory bodies, such as the US FCC are looking at opening the licensing procedures to more flexible approaches such as secondary spectrum access based on deploying Cognitive Radios (CR) [1].

In this paper, we look at a scenario where a licensed spectrum owner, allows a secondary system to use the under-utilized resources, without performance degradation to the primary users. We assume the primary system is a cellular OFDM-based system operating in Uplink. It is also assumed that the secondary system is an ad hoc network of CRs [2].

Secondary spectrum access can happen in a negotiated or opportunistic approach [3]. We assume negotiated secondary access in this work, where a signaling channel should exist to communicate the interactions between the primary and the secondary systems [4]. The specifications of such signaling channel are out of the scope of this paper. We develop a network-assisted, i.e., centralized, resource allocation scheme for such negotiated secondary spectrum access scenario. In this method, both primary and secondary users, who are willing to use the band, send information such as their QoS requirements and Channel State Information (CSI) to the Base Station (BS) of the primary system. The BS allocates the resources to all users (primary and secondary) so that each system’s requirements are met. For the primary system the main requirement is a guaranteed QoS level, specified by a minimum rate and the desired Bit Error Rate (BER) threshold. The secondary system, inherently, can not demand QoS guarantee, however, it tries to find as much transmission opportunity as possible. We analyze this scenario using Game Theory and we call this problem Cognitive Radio Game (CRG). We also discuss the role of fairness in this resource allocation strategy. The main contribution of this paper is to propose a resource allocation method that addresses efficiency, fairness and QoS in a unified framework for a secondary spectrum access scenario. Most proposed approaches in the literature, only address aspects of this framework. We also develop a criterion for selecting the users who share a sub-channel.

Developing optimum resource allocation strategies has been vigorously studied for many different scenarios. Although we propose a novel unified framework, some papers have addressed parts of this problem before. In [5], the authors look for distributed, self-enforcing strategies for coexistence in unlicensed bands. They apply non-cooperative Game Theory methods to find fair and efficient solutions. Similar intelligent power allocation strategies, but without fairness considerations, for unlicensed bands have also been studied in [6]. Instead of solving the corresponding optimization problem, the authors propose to search the strategy space using Genetic Algorithm to identify the optimum behavior. Fairness in optimum bandwidth allocation for elastic services in (wired) broadband networks is studied in [7]. Here, the authors develop both centralized and distributed strategies, using Nash Bargaining Solutions (NBS), and show that the NBS is a proportional fair solution. Another fair multiuser channel allocation scheme using NBS has been proposed in [8]. The proposed framework in [8] addresses efficiency, fairness and complexity of resource allocation where sub-carrier, rate and power is allocated for an OFDMA network. Their results show that by considering fairness in resource allocation the achievable rates are not far from methods that maximize throughput (i.e., sum of rates) without fairness consideration. A Game Theoretic formulation for network assisted resource management in wireless networks has been developed in [9]. Here, the author applies an abstract utility function using sigmoid functions for the optimization purposes for both GSM/GPRS/EGPRS networks and CDMA-based networks. QoS guarantee has also been addressed using...
Game Theory in [10]. Here, the authors analyze equilibrium points and intelligent strategies for coexisting IEEE 802.11e wireless LANs, which requires QoS guarantee. The throughput and delay have been used as the merits of QoS.

The rest of this paper is organized as follows. In section II, we will outline the secondary spectrum access problem in the form of CRG. The problem formulation for the case of two-player, single channel Game has been addressed in section III, while the derivation of optimum solution is presented in section IV. Finally, the conclusions are presented in section V.

II. PROBLEM DEFINITION

Consider a scenario where a primary system coexists with a secondary system in a specific frequency band. The primary system considered here is a cellular OFDM-based system operating in the uplink. The secondary system is an ad hoc network of CRs using OFDM and capable of modifying their transmission characteristics. There are $N$ orthogonal OFDM sub-channels, all with bandwidth $w$. We develop a network-assisted resource allocation method, where the primary users are demanding specific levels of QoS defined by a minimum data rate and a target BER. The secondary system can not demand QoS guarantee. The CRs are communicating point-to-point and we do not consider relaying capability for CRs, for the simplicity of problem formulation. Due to the limited power capability of the handsets and the fact that the primary system’s capacity is interference limited, the primary system tries to minimize the transmission power allocated to users. The secondary network is also keen to minimize its transmission power, so that it imposes less interference to the primary system. Therefore, sum power minimization can be used as the optimization goal to minimize the radiation towards different systems sharing the same spectrum. Weighted sum power minimization is a more general form of this problem which enables us to find efficient rate and power distributions for all users, primary and secondary. The weights are allocated to different users based on their priority in accessing the available resources. In this paper, we focus on showing why this interpretation of power allocation weights is correct and how the weights should be calculated in this paper. The detailed solution to the weighted sum minimization problem, where the weights are allocated as addressed in this paper, can be found in [11].

In our proposed method, the primary and secondary users inform the BS of their Channel State Information (CSI), QoS requirements and power limitations in periodic intervals prior to resource allocations for that period. The BS computes the allocated transmission power based on user priority and channel conditions for all players. The CSI as well as the power allocation results are exchanged between BS and users on a dedicated signaling channel, accurately and instantly. The details of this signaling channel are out of the scope of this paper. Also, it is assumed that channel variations happen in intervals larger than resource allocation period.

We will analyze this scenario using Game Theory and we call this game a Cognitive Radio Game (CRG). We consider each transmitter-receiver pair as a player and assume rationality for all players. We propose to model and analyze the CRG using cooperative Game Theory [12]. The network-assisted resource allocation assumption means that the BS acts as a market place, where the primary and secondary players interact. Here, the transmission of the secondary player might cause unacceptable interference to the primary players. Therefore, the secondary player is only allowed to transmit when the primary system’s requirements are met. We assume that the secondary player has agreed with this condition before starting the game. We also assume the primary player has the appropriate tools to check the behavior of the secondary player. If the secondary player violates his rights, he will lose the opportunity to use the resources and hence he has no incentive to do so. However, the details of monitoring mechanism of the primary player are not discussed in this paper. An advantage of this network-assisted resource allocation method is that sufficient processing power to calculate the optimum strategies is available at the BS. Also the primary player (who is generally the license owner for the operating band) can overlook the security issues, such as who is admitted to use the band and how to charge the secondary users (i.e., the Authentication, Authorization and Accounting (AAA) issues), more easily. The general channel model, for sub-channel $n$, is shown in Fig. 1.

III. PROBLEM FORMULATION

We use a simplified scenario to gain insight into the problem. We consider a two-player Game, where the first player represents the primary system and the second player belongs to the secondary system. We assume that only one sub-channel with bandwidth $w$ is available. This simplified scenario allows us to present intuitions more clearly, while it is extendable to more general scenario of $K$ players coexisting in $N$ sub-channels [11]. It is assumed that both primary and secondary systems use adaptive M-QAM modulation. The order of modulation, i.e., $M$, is determined according to the channel condition for each player and, hence, it could differ for different users. Using the approximate formulation of BER.
for M-QAM [13], we have,

\[ BER_i \approx c_1 \exp\left(\frac{-c_2 \gamma_i}{2 b_i - 1}\right), \tag{1} \]

where \( c_1 = 0.2 \) and \( c_2 = 1.5 \), \( b_i \) is the number of bits per symbol and \( \gamma_i \) is the SNR for user \( i \) at the receiver. The approximation is tight within 1 dB for \( b_i \geq 2 \) and \( BER_i \leq 10^{-5} \) [13]. Hence, we can formulate the \( i^{th} \) user’s rate, \( R_i \), as

\[ R_i = w \log_2\left(1 + \frac{P_i G_{i,i} c_3}{\sigma_i^2 + \sum_{j=1}^{2} P_j G_{j,i}}\right), \tag{2} \]

where \( c_3 = c_2 / \ln(c_1 / \zeta) \), \( \zeta \) is the target BER, \( \sigma_i^2 \) is the variance of the white Gaussian noise and \( G_{i,j} \) is the channel gain between transmitter \( i \) and receiver \( j \) in (2) with a proper choice of \( c_3 \). Using (2), the achievable rate region for the two-player scenario is shown in Fig. 2. The primary player (i.e. player \( I \)) tries to minimize its power in achieving rates no less than \( R_{i,\min}^1 \) with \( BER_i \leq \zeta \). The secondary player tries to access the channel without causing considerable interference to the primary system. It is assumed that the maximum transmission power for the primary player is \( P_{\max}^1 \) and for the secondary player is \( P_{\max}^2 \), which are different, in general. We can formulate the problem as follows

\[ \text{Minimize } \sum_{i=1}^{2} \lambda_i P_i, \tag{3} \]

subject to

\[ R_{i,\min}^1 - w \log_2\left(1 + \frac{P_i G_{i,i} c_3}{\sigma_i^2 + \sum_{j=1, j \neq i}^{2} P_j G_{j,i}}\right) \leq 0, \tag{4} \]

where \( P_i \) is the allocated power and \( \lambda_i \) is the allocated weight to player \( i \), in (3). We will calculate the proper value of the weights in the next section. Guaranteeing a level of QoS for the primary system (i.e., the minimum rate and target BER), as the shaded region in Fig. 2 shows, means that we restrict the maximum achievable rate for the secondary player. Due to non-convexity of this problem, i.e., (3) subject to (4), using Lagrangian multipliers method and directly applying Karush-Kuhn-Tucker (KKT) conditions [14] results in an NP-hard problem. Instead, we propose to use Sequential Quadratic Programming (SQP) [11], which is regarded to be one of the most successful method for solving nonlinearly constrained optimization problems. SQP is an iterative approach, where in step \( k \) the nonlinear programming problem is modeled by a quadratic programming sub-problem at a given approximate solution \( P^k \). The solution of this sub-problem is then used to construct a better approximation, \( P^{k+1} \), for the next step.

The sub-problem reflects the local properties of the original problem, but on the other hand it is easier to be solved. The choice of quadratic sub-problem, i.e., a problem with quadratic objective function and linear constraints, brings the advantage of reflecting the non-linearity of the original problem in its objective function, while being relatively easier to be solved. The power allocation results are broadcast to all users via the dedicated signaling channel. For the primary player, the allocated power is the minimum power required to achieve the target level of QoS for that player. Since this power allocation satisfies the goal of the primary player, i.e., minimizing its transmission power while guaranteeing a level of QoS, he has no incentive to transmit with a higher transmission power. Also, lower transmission power than the SQP solution would result in degradation of QoS for the primary player, and hence, not desirable. For the secondary player the allocated power is an upper transmission power limit. Since transmitting with lower power means lower achievable rates, the rational secondary player will use the maximum allocated transmission power. Hence, the result of the optimization problem (3) subject to (4) provides an equilibrium point for the CRG. Since the solution is unique, this equilibrium point is unique. Also, the equilibrium is Pareto-optimal because none of the users can achieve a better payoff without degrading the other user’s utility. Note that the achieved equilibrium point depends on the weighting factors of the primary and the secondary players in the optimization problem, i.e., \( \lambda_i, i = 1, 2 \), in (3). For each set of weights a separate equilibrium is achieved. Therefore, selection of an optimal operating (i.e., equilibrium) point is equivalent to the selection of a set of proper weights. We find these proper weights as an integrated part of our secondary spectrum access problem in the next section.
IV. OPTIMAL PRIORITY FACTOR CALCULATION

Consider the two-player Game, sharing a sub-channel with the rate region shown in Fig. 2. The feasible rate region in Fig. 2 is parameterized by $\lambda_i$. In other words, depending on the weight assigned by the resource allocation algorithm to each player, different power allocation and, hence, different operating point will result. For instance, if $\lambda_1 \geq \lambda_2$, the sum power minimization problem emphasizes on minimizing $P_1$ rather than $P_2$. This in turn means that the operating point is closer to point $C$ in Fig 2. Therefore, to find the corresponding priority value of each point on the boundary of the rate region, we can “assume” that the power allocation in that point is $\lambda_i P_i$ for $i = 1, 2$. We emphasize again that the actual allocated power to each player is $P_i$.

Point $A$ in Fig. 2 is the point where the primary player achieves its maximum rate, while the secondary player is not allowed to transmit, i.e., maximum priority for the primary player and minimum priority for the secondary player are allocated. Hence, at this point we have

$$\lambda_2 = \lambda_{2\min} = 0,$$  \hfill (5)

$$\lambda_1 = \lambda_{1\max} = \frac{\sigma_1^2 (c_{1\max}/w - 1)}{P_1^{\max} c_{1\max}}. \hfill (6)$$

On the other hand, point $C$ in Fig. 2 is the point where the primary player achieves its minimum guaranteed rate, i.e. the minimum priority for the primary player and the maximum priority for the secondary player as follows,

$$R^{1}_{\min} = w \log_2 (1 + \frac{\lambda_{1\min} P^{1\min}_{1} G_{1,1} c_3}{\sigma_1^2 + \lambda_{2\max} P^{2\max}_{2} G_{2,1}^2}), \hfill (7)$$

$$R^{2}_{\max} = w \log_2 (1 + \frac{\lambda_{2\max} P^{2\min}_{2} G_{2,2}^2}{\sigma_2^2 + \lambda_{1\min} P^{1\min}_{1} G_{1,2}^2}), \hfill (8)$$

where $P^{\min}_{i}$ is the required transmit power for player $i$ to achieve $R^{i\min}$, while player 2 is transmitting with the maximum transmission power, i.e., $P^{2\max}_{2}$. We can solve (7) and (8), for $\lambda_1$ and $\lambda_2$ as follows

$$\lambda_1 = \lambda_{1\min} = f(\lambda_2) = \frac{(2R^{1\min}_{\min}/w - 1)(\sigma_1^2 + \lambda_2 P^{2\max}_{2} G_{2,1}^2)}{P^{1\min}_{1} G_{1,1} c_3}, \hfill (9)$$

$$\lambda_2 = \lambda_{2\max} = \frac{(2R^{2\max}_{\max}/w - 1)\times}{\frac{\sigma_1^2 G_{1,1} c_3 + \sigma_2^2 G_{1,2}^2 (2R^{1\min}_{\min}/w - 1)}{[G_{1,1} G_{2,2} (2R^{1\min}_{\min}/w - 1) - G_{1,2} G_{2,1}^2 (2R^{2\max}_{\max}/w - 1)]}}. \hfill (10)$$

By replacing $\lambda_2$ given by (10) in (9), the priority factor for player $i$ can be evaluated.

A. Sub-Channel Sharing

The priority factors given by (9) and (10), correspond to the extreme case where the primary player only maintains its minimum required rate, but the secondary player achieves the maximum possible rate. In other words, it is not possible to allow the secondary user to transmit higher powers since this will prohibit the primary system to guarantee QoS. Keeping this in mind, closer investigation of (10) provides useful intuition into sharing a sub-channel for secondary spectrum access. In order to have positive weights for power allocation, the denominator of (10) should be positive, hence,

$$G_{1,1} G_{2,2} > G_{1,2} G_{2,1} \left(\frac{(2R^{2\max}_{\max}/w - 1)(2R^{1\min}_{\min}/w - 1)}{c_3^2}\right). \hfill (11)$$

This is an interesting generalization of the similar relations between direct and cross channel gains as reported in [5] and [15]. Both [5] and [15] have shown that in a shared spectrum band, if multiplication of cross channel gains are higher that multiplication of direct channel gains between a pair of transmitter-receivers, the optimum rate can be achieved by orthogonal transmission rather than spreading the power over the entire band. Here, (11) shows that the same concept is true in the case that a specific QoS level for the primary system is guaranteed. Therefore, the channel gains and the level of QoS form a trade off, to decide whether or not to share a channel. If channel conditions are desirable (high direct channel gains compared to cross channel gains) a higher level of QoS can be guaranteed. Therefore, inequality (11) provides a decision mechanism for selecting which users can share a specific sub-band in a generalized scenario as discussed in [11].

B. Fairness Considerations

In this section we exploit the fairness axioms of Game Theory to evaluate the priority factors to be used in our optimization problem. Among different cooperative Game solutions, Nash Bargaining solution (NBS) provides a fair, unique and Pareto-optimal result [12]. In a $K$-player Game, the non-empty, bounded set $F$ denotes the feasible payoff allocations set, and $v \in \mathbb{R}^K$ is the vector of payoffs at the agreement point. The vector $v$ represents the set of payoffs for all players that can be achieved without cooperation. In other words, the rational players require achieving payoffs not less than the agreement point, in order to cooperate. The allocation vector (in $\mathbb{R}^K$) for the bargaining problem $(F,v)$ is denoted by $\phi(F,v)$. In the proposed CRG, $F$ corresponds to the set of all achievable rates, i.e., the rate region in Fig. 2, and $v$ is the vector of allocated rates to all players. It has been shown that there exists a unique NBS for any given bargaining problem, which can be calculated by [12]

$$\phi(F,v) \in \arg \max_{x \in F} \prod_{x \geq F} (x_i - v_i). \hfill (12)$$

The NBS is also the point where “egalitarian” and “utilitarian” solutions of the bargaining problem coincide [12]. For our two-player Game, the allocation $x$ in $F$ that is weakly efficient and satisfies the equal gain condition

$$R_1 - R_{\min,1} = R_2, \hfill (13)$$

is called the egalitarian solution. Also for this two-player bargaining problem the point that satisfies

$$R_1 + R_2 = \max_{R' \in F} (R'_1 + R'_2), \hfill (14)$$
is called a utilitarian solution. The unique solution point that satisfies both egalitarian and utilitarian solutions is the NBS. Condition (14) dictates that the operating point should be on the boundary of the feasible rate region. Therefore, the intersection of the line (13) with the boundary of the feasible rate region gives the unique fair Pareto-optimal solution, i.e., NBS. We denote the NBS point by $R_{i,NBS}$ and $P_{i,NBS}$, $i = 1, 2$. Therefore, we can find the corresponding priority factors as

$$
\lambda_1 = \lambda_{1,NBS} = \left(\frac{2R_{1,NBS}}{w} - 1\right) \times \frac{[\sigma^2 G_{1,1}G_{2,2}c_3 + \sigma^2 G_{1,1}G_{1,2}c_3(2R_{NBS}/w - 1)]}{G_{1,1}G_{2,2}c_3 - G_{1,2}G_{2,1}(2R_{NBS}/w - 1)(2R_{NBS}/w - 1)},
$$

$$
\lambda_2 = \lambda_{2,NBS} = \left(\frac{2R_{2,NBS}}{w} - 1\right) / P_{2,NBS}^2 \times \frac{[\sigma^2 G_{1,1}c_3 + \sigma^2 G_{1,2}(2R_{NBS}/w - 1)]}{G_{1,1}G_{2,2}c_3 - G_{1,2}G_{2,1}(2R_{NBS}/w - 1)(2R_{NBS}/w - 1)}.
$$

Therefore, we have completed the solution of CRG. Using (15) and (16) we can find proper weight values which will be used in (3) subject to (4). The optimization problem, then, is solved using SQP approach [11]. The results are the optimum strategies, i.e., power allocation for the players. As verified by the results in Fig. 3, the optimum operating point is indeed on the boundary of feasible rate region and maximizes (12).

V. CONCLUSIONS

In this paper, we addressed the resource allocation in secondary spectrum access problem. Such problem arises in many applications where the license owner of a band tries to gain benefit from the unutilized resources, without imposing interference to the primary users. The weighted sum power minimization problem is defined as the optimization goal for the proposed CRG. We showed that by allocating a priority factor to each player, any point on the boundary of feasible rate region can be achieved. In general, there are infinite NE points on the boundary of capacity region. The primary BS has a degree of freedom to select the operating point among all Pareto-optimal solutions. Fairness can be used as an extra requirement that results in a unique solution, i.e., NBS. We outlined the framework for such efficient and fair resource allocation method that guarantees the required QoS of primary system.

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