Using the Hamming distance to extend TOPSIS in a fuzzy environment

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A B S T R A C T

Considering the fact that, in some cases, determining precisely the exact value of attributes is difficult and that their values can be considered as fuzzy data, this paper extends the TOPSIS method for dealing with fuzzy data, and an algorithm for determining the best choice among all possible choices when the data are fuzzy is also presented. In this approach, to identify the fuzzy ideal solution and fuzzy negative ideal solution, one of the Yager indices which is used for ordering fuzzy quantities in [0, 1] is applied. Using Yager's index leads to a procedure for choosing fuzzy ideal and negative ideal solutions directly from the data for observed alternatives. Then, the Hamming distance is proposed for calculating the distance between two triangular fuzzy numbers. Finally, an application is given, to clarify the main results developed in the paper.

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1. Introduction

Decision making is the process of selecting a possible course of action from all of the alternatives. In almost all such problems, a multiplicity of criteria for judging the alternative are pervasive. That is, for many such problems, the decision maker wants to attain more than one or goal in selecting the course of action while satisfying the constraints dictated by environment processes, and resources [1]. Multi-criteria decision making (MCDM) may be considered a complex and dynamic process including one managerial level and one engineering level [2]. The managerial level defines the goals, and chooses the final “optimal” alternative. The multi-criteria nature of decisions is emphasized at this managerial level, at which public officials called “decision makers” have the power to accept or reject the solution proposed by the engineering level. These decision makers, who provide the preference structure, are “off line” from the optimization procedure done at the engineering level. In MCDM problems, there does not necessarily exist a solution that optimizes all objective functions, and then the concept which is called Pareto optimal solution (or efficient solution) is introduced. Usually, there exist a number of Pareto optimal solutions, which are considered as candidate final decision making solutions. It is an issue how decision makers decide on one from the set of Pareto optimal solutions as the final solution (see, for more details, [3]). A MCDM problem can be concisely expressed in matrix format as

\[ C_1 \quad C_2 \quad \ldots \quad C_n \]
\[ A_1 \]
\[ x_{11} \quad x_{12} \quad \ldots \quad x_{1n} \]
\[ A_2 \]
\[ x_{21} \quad x_{22} \quad \ldots \quad x_{2n} \]
\[ A_m \]
\[ x_{m1} \quad x_{m2} \quad \ldots \quad x_{mn} \]
\[ W = [w_1, w_2, \ldots, w_n] \]

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where $A_1, A_2, \ldots, A_m$ are possible alternatives among which decision makers have to choose, $C_1, C_2, \ldots, C_n$ are criteria with which alternative performance are measured, $x_{ij}$ is the rating of the alternative $A_i$ with respect to criterion $C_j$, $w_j$ is the weight of criterion $C_j$.

The main steps of multiple-criteria decision making are the following:

(a) establishing system evaluation criteria that relate system capabilities to goals;
(b) developing alternative systems for attaining the goals (generating alternatives);
(c) evaluating alternatives in terms of criteria (the values of the criterion functions);
(d) applying a normative multi-criteria analysis method;
(e) accepting one alternative as “optimal” (preferred);
(f) if the final solution is not acceptable, gathering new information and going into the next iteration of multi-criteria optimization.

Steps (a) and (e) are performed at the upper level, where decision makers have the central role, and the other steps are mostly engineering tasks. For step (d), a decision maker should express his/her preferences in terms of the relative importance of criteria, and one approach is to introduce criteria weights. This weights in MCDM do not have a clear economic significance, but their use provides the opportunity to model the actual aspects of decision making (the preference structure).

The Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) method is presented in [4], with reference to [5]. The basic principle is that the chosen alternative should have the shortest distance from the ideal solution and the farthest distance form the negative ideal solution. The process of TOPSIS, the performance ratings and the weights of the criteria are given as exact values. In real-world situations, because of incomplete or non-obtainable information, the data (attributes) are often not so deterministic; therefore they are usually fuzzy/imprecise. Therefore, some researchers try to use the TOPSIS method with fuzzy/imprecise data. For example, Tsaur et al. [6] first convert a fuzzy MCDM problem into a crisp problem via centroid defuzzification and then solve the nonfuzzy MCDM problem using the TOPSIS method. Chen and Tzeng [7] transform a fuzzy MCDM problem into a non-fuzzy MCDM using a fuzzy integral. In [3] Chu proposed a fuzzy TOPSIS approach for selecting plant location, where the ratings of various alternative locations under various criteria and the weights of various criteria are assessed in linguistic terms represented by fuzzy numbers. In the proposed method, the ratings and weights assigned by decision makers are averaged and normalized onto a comparable scale. The membership function of each normalized weighted rating can be developed by interval arithmetic of fuzzy numbers. Byun and Lee [8] provide a decision support system for the selection of a rapid prototyping process using the modified TOPSIS method. Recently, in some research, the TOPSIS method is considered for extension. For example, Chen [9] extends the concept of TOPSIS to develop a methodology for solving multi-person multi-criteria decision making problems in a fuzzy environment. Abo-Sinna et al. [10] extend the TOPSIS method to solve multi-objective nonlinear programming problems. Also, Jahanshahloo et al. [11, 12] extend the TOPSIS method for decision making problems with interval and fuzzy data. In this paper, we further extended the concept of the TOPSIS method to develop a methodology for solving multi-criteria decision making problems in a fuzzy environment. According to the concept of TOPSIS, to identify the fuzzy positive ideal solution and fuzzy negative ideal solution we use a method for ordering normalized fuzzy numbers proposed in [13, 14]. And then, the Hamming distance is proposed for calculating the distance between two fuzzy numbers, and using this distance method, we can calculate the distance of each alternative from the fuzzy positive ideal solution and fuzzy negative ideal solution, and then a closeness coefficient of each alternative is defined for determining the ranking order of all alternatives.

The paper unfolds as follows. The next section briefly introduces some preliminary definitions concerning fuzzy numbers and linguistic variables. Section 3 addresses a suggested method for extending TOPSIS in a fuzzy environment. A brief discussion about the proposed method, with an application, is given in Section 4. Conclusions appear in Section 5.

### 2. Preliminaries

Let $X$ be a classical set of objects, called the universe, whose generic elements are denoted by $x$. The membership in a crisp subset of $X$ is often viewed as characteristic function $\mu_A$ from $X$ to $\{0, 1\}$ such that

$$
\mu_A(x) = \begin{cases} 
1 & \text{if and only if } x \in A, \\
0 & \text{otherwise}
\end{cases}
$$

where $\{0, 1\}$ is called a valuation set.

**Definition 1 (Fuzzy Set).** If the valuation set is allowed to be the real interval $[0, 1]$, $A$ is called a fuzzy set and denoted by $\tilde{A}$ and $\mu_\tilde{A}(x)$ is the degree of membership of $x$ in $\tilde{A}$.

**Definition 2 (α-level set or α-cut).** The $\alpha$-cut of a fuzzy set $\tilde{A}$ is a crisp subset of $X$ and is denoted by [15]

$$
[\tilde{A}]_\alpha = \{x \mid \mu_\tilde{A}(x) \geq \alpha\}
$$

where $\mu_\tilde{A}(x)$ is the membership function of $\tilde{A}$ and $\alpha \in [0, 1]$.

The lower and upper points of any $\alpha$-cut, $[\tilde{A}]_\alpha^L$ and $[\tilde{A}]_\alpha^U$, are represented by $\inf [\tilde{A}]_\alpha$ and $\sup [\tilde{A}]_\alpha$ respectively and we suppose that both are finite. For convenience, we show $\inf [\tilde{A}]_\alpha$ with $[\tilde{A}]_\alpha^L$ and $\sup [\tilde{A}]_\alpha$ with $[\tilde{A}]_\alpha^U$ (see Fig. 1).
**Definition 3** (Normality). A fuzzy set \( \tilde{A} \) is normal if and only if \( \sup_x \mu_{\tilde{A}}(x) = 1 \).

**Definition 4** (Convexity). A fuzzy set \( \tilde{A} \) in \( X \) is convex if and only if for every pair of point \( x^1 \) and \( x^2 \) in \( X \), the membership function of \( \tilde{A} \) satisfies the inequality

\[
\mu_{\tilde{A}}(\delta x^1 + (1 - \delta) x^2) \geq \min(\mu_{\tilde{A}}(x^1), \mu_{\tilde{A}}(x^2))
\]

where \( \delta \in [0, 1] \). Alternatively, a fuzzy set is convex if all \( \alpha \)-level sets are convex.

**Definition 5** (Fuzzy Number). A fuzzy number \( \tilde{A} \) is a convex normalized fuzzy set \( \tilde{A} \) of the real line \( \mathbb{R} \) with continuous membership function.

**Definition 6** (Triangular Fuzzy Numbers). The triangular fuzzy numbers can be denoted as \( \tilde{A} = (a, m, n) \), where \( a \) is the central value \( (\mu_{\tilde{A}}(a) = 1) \), \( m \) is the left spread and \( n \) is the right spread. (See Fig. 2.) The membership function of such triangular fuzzy number is defined as

\[
\mu_{\tilde{A}}(x) = \begin{cases} 
0 & \text{if } x \leq a - m \\
\frac{x + m - a}{a} & \text{if } a - m < x \leq a \\
\frac{a + n - x}{a} & \text{if } a < x \leq a + n \\
0 & \text{if } a + n < x.
\end{cases}
\]

**Definition 7** (Multiplication of Triangular Fuzzy Numbers). Suppose that we have two triangular fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \) such that \( \tilde{A} = (a, m, n) \) and \( \tilde{B} = (b, s, r) \); then, the multiplication of the fuzzy numbers \( \tilde{A} \) and \( \tilde{B} \) is defined as follows [15]:

\[
\tilde{A} \cdot \tilde{B} = \begin{cases} 
(ab, as + bm - ms, ar + bn + nr) & \text{if } \tilde{A} > 0, \tilde{B} > 0 \\
(ab, -ar + bm + mr, as - bn + sn) & \text{if } \tilde{A} < 0, \tilde{B} > 0 \\
(ab, -ar - bn - nr, -as - bm + nr) & \text{if } \tilde{A} < 0, \tilde{B} < 0.
\end{cases}
\]

**Definition 8.** A fuzzy number \( \tilde{A} \) is called a positive fuzzy number if \( \mu_{\tilde{A}}(x) = 0 \) for all \( x < 0 \).

**Definition 9.** If \( \tilde{A} \) is a triangular fuzzy number and \( [\tilde{A}]_x^\alpha > 0 \) and \( [\tilde{A}]_x^\alpha \leq 1 \) for \( \alpha \in [0, 1] \), then \( \tilde{A} \) is called a normalized positive triangular fuzzy number.
Table 1
Importance weights as linguistic variables.

<table>
<thead>
<tr>
<th>Importance Weight</th>
<th>Linguistic Variable</th>
<th>Fuzzy Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very low (VL)</td>
<td>(0, 0, 0.1)</td>
<td></td>
</tr>
<tr>
<td>Low (L)</td>
<td>(0.1, 0.1, 0.2)</td>
<td></td>
</tr>
<tr>
<td>Medium low (ML)</td>
<td>(0.3, 0.2, 0.2)</td>
<td></td>
</tr>
<tr>
<td>Medium (M)</td>
<td>(0.5, 0.2, 0.2)</td>
<td></td>
</tr>
<tr>
<td>Medium high (MH)</td>
<td>(0.7, 0.2, 0.2)</td>
<td></td>
</tr>
<tr>
<td>High (H)</td>
<td>(0.9, 0.2, 0.1)</td>
<td></td>
</tr>
<tr>
<td>Very high (VH)</td>
<td>(1.0, 0.1, 0)</td>
<td></td>
</tr>
</tbody>
</table>

**Definition 10** (*Hamming Distance*). Let \( \tilde{A} = (a, m, n) \), \( \tilde{B} = (b, s, r) \) be two triangular fuzzy numbers; then the distance between them is defined using the Hamming distance [16] as

\[
d(\tilde{A}, \tilde{B}) = \int |\mu_A(x) - \mu_B(x)| dx.
\]

**Remark 1.** If \( \tilde{A} = [(\tilde{A})_L^u, (\tilde{A})_U^u], \) then by choosing \( \alpha = 1 \) we can identify the central value of \( \tilde{A} \), and by \( \alpha = 0 \) we can identify the left and right spreads of \( \tilde{A} \).

The importance weights of various criteria are considered as linguistic variables and can be expressed in positive triangular fuzzy numbers as in Table 1, derived from [9].

The concept of linguistic variables is useful in dealing with situations that are too complex or too poorly defined to be reasonably described in conventional quantitative expressions [17].

3. The proposed method

In some real-world situation, determining the exact value of the attributes (data) is difficult and it is also difficult for a decision maker to assign a precise performance rating to such alternatives for the criteria under consideration; so, these data are considered as fuzzy data. In this paper a new and attractive approach, which is to use only the information of the existing data, is proposed. Suppose \( A_1, A_2, \ldots, A_m \) are \( m \) possible alternatives among which decision makers have to choose, \( C_1, C_2, \ldots, C_n \) are criteria with which alternative performances are measured. \( \bar{x}_{ij} \) is the rating of the alternative \( A_i \) with respect to criterion \( C_j \) and is a fuzzy number. A MCDM problem with fuzzy data can be concisely expressed in matrix format (namely, using a fuzzy decision matrix) as

\[
\begin{array}{c|cccc}
C_1 & C_2 & \ldots & C_n \\
A_1 & \bar{x}_{11} & \bar{x}_{12} & \ldots & \bar{x}_{1n} \\
A_2 & \bar{x}_{21} & \bar{x}_{22} & \ldots & \bar{x}_{2n} \\
A_m & \bar{x}_{m1} & \bar{x}_{m2} & \ldots & \bar{x}_{mn} \\
\end{array}
\]

where \( \bar{w}_j \) is the weight of criterion \( C_j \) and is a normalized fuzzy number. Here the importance weights of various criteria are considered as linguistic variables which can be expressed in positive triangular fuzzy numbers (see Table 1). The approach for extending the TOPSIS method fuzzy environment is as follows:

**Step 1.** We identify the evaluation criteria.

**Step 2.** We generate alternatives.

**Step 3.** We evaluate alternatives in terms of criteria (the values of the criterion functions which are fuzzy).

**Step 4.** We identify the weights of criteria.

**Step 5.** We construct the fuzzy decision matrix. In the fuzzy decision matrix, we suppose that each \( \bar{x}_{ij} \) is a triangular fuzzy number, i.e., \( \bar{x}_{ij} = (x_{ij}, \alpha_{ij}, \beta_{ij}) \).

**Step 6.** We calculate the normalized fuzzy decision matrix as follows.

First, for each fuzzy number \( \bar{x}_{ij} = (x_{ij}, \alpha_{ij}, \beta_{ij}) \), we calculate the set of the \( \alpha \)-cut as

\[
\bar{x}_{ij} = [\bar{x}_{ij}^L, \bar{x}_{ij}^U], \quad \alpha \in [0, 1].
\]

Therefore, each fuzzy number \( \bar{x}_{ij} \) is transformed into an interval; now by an approach proposed in Jahanshahloo et al. [11] we can transform this interval into normalized interval as follows:

\[
[\bar{n}_{ij}^L]_w = \left[ \frac{m}{\sqrt{\sum_{i=1}^{m} ((\bar{x}_{ij}^L_x)_w)^2 + ((\bar{x}_{ij}^U_x)_w)^2)} \right], \quad i = 1, \ldots, m, j = 1, \ldots, n;
\]

\[
[\bar{n}_{ij}^U]_w = \left[ \frac{m}{\sqrt{\sum_{i=1}^{m} ((\bar{x}_{ij}^L_x)_w)^2 + ((\bar{x}_{ij}^U_x)_w)^2)} \right], \quad i = 1, \ldots, m, j = 1, \ldots, n;
\]
where, now, interval $[\tilde{n}_{ij}^L, \tilde{n}_{ij}^U]$ is a normalized form of interval $[\tilde{x}_{ij}^L, \tilde{x}_{ij}^U]$. According to Remark 1 we can transform this normalized interval into a fuzzy number such as $\tilde{n}_{ij} = (n_{ij}, a_{ij}, b_{ij})$ such that $n_{ij}$ is obtained when $\alpha = 1$, i.e., $n_{ij} = [\tilde{n}_{ij}^L]_{\alpha=1} = [\tilde{n}_{ij}^U]_{\alpha=1}$, and also by setting $\alpha = 0$ we have $[\tilde{n}_{ij}^L]_{\alpha=0} = n_{ij} - a_{ij}$ and $[\tilde{n}_{ij}^U]_{\alpha=0} = n_{ij} + b_{ij}$; then,

$$a_{ij} = n_{ij} - [\tilde{n}_{ij}^L]_{\alpha=0}$$  
$$b_{ij} = [\tilde{n}_{ij}^U]_{\alpha=0} - n_{ij}$$

and $\tilde{n}_{ij}$ is a normalized positive triangular fuzzy number, i.e., $\tilde{n}_{ij}$ is a normalized form of fuzzy number $\tilde{x}_{ij}$. Now, we can work with these normalized fuzzy numbers.

**Step 7.** By considering the different importance of each criterion, we can construct the weighted normalized fuzzy decision matrix as

$$\tilde{v}_{ij} = \tilde{n}_{ij}, \tilde{w}_{ij}$$  \hspace{1cm} (3.4)

where $\tilde{w}_{ij}$ is the weight of the $j$th attribute or criterion.

**Step 8.** We identify the fuzzy positive ideal solution and fuzzy negative ideal solution. For this purpose we apply a method for ordering normalized fuzzy numbers proposed by Yager [13, 14]. Yager proposed the following index for the purpose of ordering fuzzy quantities in $[0, 1]$:

$$Y(\tilde{A}) = \frac{\int_{\tilde{A}} g(x) \mu_{\tilde{A}}(x) dx}{\int_{\tilde{A}} \mu_{\tilde{A}}(x) dx}$$  \hspace{1cm} (3.5)

where $\tilde{A}$ is a fuzzy number and $g(x)$ measures the importance of value $x$; we will specify $g(x)$ as $g(x) = x$ in the coming discussion [18]. Since each $\tilde{v}_{ij}$ is a normalized fuzzy number, we use (3.5) to obtain the matrix $U = \{u_{ij} | i = 1, \ldots, m; j = 1, \ldots, n\}$, namely, the ordering decision matrix, where $u_{ij} = Y(\tilde{v}_{ij})$.

Assume that for each $j \in J$, $u_{ij} = \max(u_{ij}, i = 1, \ldots, m)$, $u_{ij} = \min(u_{ij}, i = 1, \ldots, m)$, and also, for each $j \in J$, $u_{ij} = \max(u_{ij}, i = 1, \ldots, m)$, $u_{ij} = \min(u_{ij}, i = 1, \ldots, m)$; then we have

$$\tilde{A}^+ = \{\tilde{u}^+_1, \ldots, \tilde{u}^+_n\} = \{(\tilde{v}_{ij} | j \in I), (\tilde{v}_{ij} | j \in J)\}$$  \hspace{1cm} (3.6)

$$\tilde{A}^- = \{\tilde{u}^-_1, \ldots, \tilde{u}^-_n\} = \{(\tilde{v}_{ij} | j \in I), (\tilde{v}_{ij} | j \in J)\}$$  \hspace{1cm} (3.7)

where $I$ is associated with benefit criteria and $J$ is associated with cost criteria.

**Step 9.** We can currently calculate the separation of each alternative from the fuzzy positive ideal solution, using the Hamming distance, as

$$d_i^+ = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{u}_{ij}^+) \hspace{0.5cm} i = 1, \ldots, m$$  \hspace{1cm} (3.8)

and, similarly, we can calculate the separation from the fuzzy negative ideal solution as

$$d_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{u}_{ij}^-) \hspace{0.5cm} i = 1, \ldots, m$$  \hspace{1cm} (3.9)

**Step 10.** We define a closeness coefficient to determine the ranking order of all alternatives once $d_i^+$ and $d_i^-$ for each alternative $A_i$ have been calculated. The relative closeness of the alternative $A_i$ with respect to $\tilde{A}^+$ is defined as

$$\tilde{r}_i = d_i^- / (d_i^+ + d_i^-) \hspace{0.5cm} i = 1, \ldots, m$$  \hspace{1cm} (3.10)

Obviously, an alternative $A_i$ becomes closer to $\tilde{A}^+$ and farther from $\tilde{A}^-$ as $\tilde{r}_i$ approaches 1. Therefore, according to the closeness coefficient, we can determine the ranking order of all alternatives and select the best one from among a set of feasible alternatives.

4. Application

The managerial board of a hospital wants to employ a nurse. There are some applications for that position. All applications will be considered under five main criteria.

These criteria are: $C_1$: demanded salary, $C_2$: average time for accessing the hospital, $C_3$: experience, $C_4$: test and exam, $C_5$: computer and language skills.

The criterion ‘demanded salary’ is the cost attribute for the hospital. The amount that the hospital can pay for a nurse is limited, and lower than 10 is acceptable (the unit for this criterion is 100,000 Rials). The aim of the second criterion is for the nurse to reach the hospital in the case of urgency quickly, and lower than 20 min is acceptable. The ‘experience’ will give an idea of the trade proficiency of candidates, and above 5 is acceptable. This criterion consists of the following three sections:
Table 2
The importance weights.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>c₁</th>
<th>c₂</th>
<th>c₃</th>
<th>c₄</th>
<th>c₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>Importance</td>
<td>L</td>
<td>ML</td>
<td>VH</td>
<td>M</td>
<td>H</td>
</tr>
<tr>
<td>Fuzzy weights</td>
<td>(0.1,0,1,0.2)</td>
<td>(0.3,0,2,0.2)</td>
<td>(1.0,0,1,0)</td>
<td>(0.5,0,2,0.2)</td>
<td>(0.9,0,2,0.1)</td>
</tr>
</tbody>
</table>

Table 3
Fuzzy decision matrix.

<table>
<thead>
<tr>
<th></th>
<th>c₁</th>
<th>c₂</th>
<th>c₃</th>
<th>c₄</th>
<th>c₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>(6.5,1.0,1.1)</td>
<td>(7.3,1.0,1.0)</td>
<td>(9.5,2.0,3.2)</td>
<td>(9.7,3.0,0.5)</td>
<td>(8.3,2.0,1.0)</td>
</tr>
<tr>
<td>A₂</td>
<td>(7.3,1.5,0.8)</td>
<td>(6.2,1.2,0.5)</td>
<td>(9.2,1.0,2.5)</td>
<td>(8.5,2.0,1.0)</td>
<td>(7.9,1.0,1.5)</td>
</tr>
<tr>
<td>A₃</td>
<td>(5.2,1.0,1.5)</td>
<td>(7.1,0.5,1.5)</td>
<td>(8.7,1.5,1.3)</td>
<td>(9.2,3.0,2.0)</td>
<td>(8.7,2.7,1.3)</td>
</tr>
<tr>
<td>A₄</td>
<td>(6.0,0.7,1.2)</td>
<td>(8.5,2.0,1.0)</td>
<td>(7.8,1.0,2.0)</td>
<td>(9.0,1.0,1.0)</td>
<td>(8.0,2.5,1.5)</td>
</tr>
</tbody>
</table>

Fig. 3. The hierarchical structure.

Table 4
Normalized fuzzy decision matrix.

<table>
<thead>
<tr>
<th></th>
<th>c₁</th>
<th>c₂</th>
<th>c₃</th>
<th>c₄</th>
<th>c₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>(0.365,0.063,0.058)</td>
<td>(0.353,0.049,0.048)</td>
<td>(0.381,0.101,0.094)</td>
<td>(0.376,0.105,0.037)</td>
<td>(0.357,0.080,0.051)</td>
</tr>
<tr>
<td>A₂</td>
<td>(0.410,0.091,0.035)</td>
<td>(0.299,0.057,0.025)</td>
<td>(0.369,0.063,0.068)</td>
<td>(0.330,0.067,0.055)</td>
<td>(0.339,0.036,0.074)</td>
</tr>
<tr>
<td>A₃</td>
<td>(0.292,0.061,0.076)</td>
<td>(0.343,0.024,0.072)</td>
<td>(0.349,0.080,0.025)</td>
<td>(0.357,0.106,0.097)</td>
<td>(0.374,0.111,0.065)</td>
</tr>
<tr>
<td>A₄</td>
<td>(0.337,0.046,0.059)</td>
<td>(0.410,0.096,0.049)</td>
<td>(0.313,0.059,0.053)</td>
<td>(0.349,0.025,0.056)</td>
<td>(0.344,0.102,0.073)</td>
</tr>
</tbody>
</table>

1. Number of years for which the candidate has worked. (A precise numeric value.)
2. Ability to perform some critical jobs. (Linguistic variable.)
3. Survey of CV form of each candidate. (Linguistic variable.)

The above mentioned items give us an imprecise value which we consider as a fuzzy number. The criterion of ‘test and exam’, consists of practical and written examinations, and an average score above 7 is acceptable. This criterion consists of the following two sections:
1. A written examination which gives a precise value.
2. A practical examination which gives a linguistic value.

The above mentioned items give us an imprecise value which we consider as a fuzzy number. There can be a need for computer and language skills in extreme situations. Therefore, the managerial board test these and a score lower than 5 is not acceptable.

Every person who fails at least as regards one criterion would be discarded by the managerial board. After primary checking, only four candidates remain, namely, A₁, A₂, A₃, A₄.

Clearly, c₁ and c₂ are cost criteria and c₃, c₄ and c₅ are benefit criteria. Assume that to assess the importance of the criteria, the managerial board use linguistic weighting variables (shown in Table 2). To determine the fuzzy weight of each criterion, we convert these linguistic variables into triangular fuzzy numbers (see the third row in Table 2).

The data for these alternatives are considered as triangular fuzzy numbers and shown in Table 3 as a fuzzy decision matrix.

The hierarchical structure of this problem is shown in Fig. 3.

To apply the extended TOPSIS approach developed, the decision matrix contained in Table 3 needs to be normalized by using Eqs. (3.2) and (3.3). Also the weighted normalized fuzzy decision matrix is calculated using Eq. (3.4). Tables 4 and 5 show the results.

The ordering decision matrix is given in Table 6, in which there corresponds to each component of the weighted normalized fuzzy decision matrix an exact value, from Eq. (3.5), given for the purpose of comparison.

According to Eqs. (3.6) and (3.7) the fuzzy ideal solution and fuzzy negative ideal solution are calculated as in Table 7.
Table 5
Weighted normalized fuzzy decision matrix.

<table>
<thead>
<tr>
<th></th>
<th>c₁</th>
<th>c₂</th>
<th>c₃</th>
<th>c₄</th>
<th>c₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>(0.036, 0.036, 0.090)</td>
<td>(0.106, 0.076, 0.095)</td>
<td>(0.381, 0.129, 0.094)</td>
<td>(0.188, 0.107, 0.101)</td>
<td>(0.321, 0.127, 0.087)</td>
</tr>
<tr>
<td>A₂</td>
<td>(0.410, 0.041, 0.093)</td>
<td>(0.090, 0.066, 0.072)</td>
<td>(0.369, 0.094, 0.068)</td>
<td>(0.165, 0.086, 0.104)</td>
<td>(0.305, 0.093, 0.108)</td>
</tr>
<tr>
<td>A₃</td>
<td>(0.029, 0.029, 0.081)</td>
<td>(0.103, 0.071, 0.105)</td>
<td>(0.349, 0.107, 0.025)</td>
<td>(0.178, 0.103, 0.139)</td>
<td>(0.337, 0.152, 0.102)</td>
</tr>
<tr>
<td>A₄</td>
<td>(0.034, 0.034, 0.085)</td>
<td>(0.123, 0.092, 0.106)</td>
<td>(0.313, 0.084, 0.053)</td>
<td>(0.175, 0.077, 0.109)</td>
<td>(0.310, 0.140, 0.107)</td>
</tr>
</tbody>
</table>

Table 6
Ordering fuzzy decision matrix.

<table>
<thead>
<tr>
<th></th>
<th>c₁</th>
<th>c₂</th>
<th>c₃</th>
<th>c₄</th>
<th>c₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>0.0539</td>
<td>0.1123</td>
<td>0.3693</td>
<td>0.1860</td>
<td>0.3076</td>
</tr>
<tr>
<td>A₂</td>
<td>0.427</td>
<td>0.0920</td>
<td>0.3603</td>
<td>0.1710</td>
<td>0.3100</td>
</tr>
<tr>
<td>A₃</td>
<td>0.0463</td>
<td>0.1143</td>
<td>0.3216</td>
<td>0.1900</td>
<td>0.3203</td>
</tr>
<tr>
<td>A₄</td>
<td>0.0509</td>
<td>0.1276</td>
<td>0.3026</td>
<td>0.1856</td>
<td>0.2989</td>
</tr>
</tbody>
</table>

Table 7
Fuzzy positive ideal solution and fuzzy negative ideal solution.

<table>
<thead>
<tr>
<th></th>
<th>c₁</th>
<th>c₂</th>
<th>c₃</th>
<th>c₄</th>
<th>c₅</th>
</tr>
</thead>
<tbody>
<tr>
<td>˜A⁺</td>
<td>(0.029, 0.029, 0.081)</td>
<td>(0.090, 0.066, 0.072)</td>
<td>(0.381, 0.129, 0.094)</td>
<td>(0.178, 0.103, 0.139)</td>
<td>(0.337, 0.152, 0.102)</td>
</tr>
<tr>
<td>˜A⁻</td>
<td>(0.410, 0.041, 0.093)</td>
<td>(0.123, 0.092, 0.106)</td>
<td>(0.313, 0.084, 0.053)</td>
<td>(0.165, 0.086, 0.104)</td>
<td>(0.310, 0.140, 0.107)</td>
</tr>
</tbody>
</table>

Table 8
Closeness coefficients and ranking.

<table>
<thead>
<tr>
<th></th>
<th>˜d⁺</th>
<th>˜d⁻</th>
<th>˜R</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>A₁</td>
<td>0.09974</td>
<td>0.20087</td>
<td>0.6682</td>
<td>1</td>
</tr>
<tr>
<td>A₂</td>
<td>0.13174</td>
<td>0.17996</td>
<td>0.5774</td>
<td>3</td>
</tr>
<tr>
<td>A₃</td>
<td>0.11967</td>
<td>0.16703</td>
<td>0.5774</td>
<td>3</td>
</tr>
<tr>
<td>A₄</td>
<td>0.25221</td>
<td>0.04056</td>
<td>0.1385</td>
<td>4</td>
</tr>
</tbody>
</table>

Note: All the computations in this example were carried out by a computer program using MAPLE software.

Table 8 shows the distance of each candidate from the fuzzy ideal solution and fuzzy negative solution. In the fourth column of Table 8, calculated closeness coefficients for all alternatives are given. The ranking of the preference order of these alternatives according to the closeness coefficient is shown in the final column of Table 8.

Now, a preference order can be ranked according to the value of ˜R; therefore, the best alternative is the one with the maximum value of ˜R. That is, the best option is the one with the shortest distance to the fuzzy ideal solution and with the longest distance to the fuzzy negative ideal solution. The alternative A₁ having the biggest ˜R is the best candidate for our aims.

5. Conclusion

In real-world situations, because of incomplete or non-obtainable information, the data (attributes) are often not wholly deterministic; therefore they are usually fuzzy/imprecise. Therefore, in this paper the TOPSIS method is extended for dealing with fuzzy data. The concept of the α-cut is used to construct the normalized fuzzy decision matrix. Also, Yager’s index is employed for identification of the fuzzy ideal solution and fuzzy negative ideal solution. Note that in this approach the fuzzy positive ideal solution and fuzzy negative ideal solution are chosen directly from the data for observed alternatives by using Yager’s index. Then, the Hamming distance is proposed for calculating the distance between two triangular fuzzy numbers.

References