Wideband Spectrum Sensing for Cognitive Radio
Via Phase-Field Segmentation

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Abstract—In this paper, we investigate the problem of wideband spectrum sensing for detecting vacant frequency subbands in opportunistic cognitive radio. To this end, we identify the irregularities (discontinuities) in the estimated power spectrum density. More precisely, we propose a new mathematical framework based on phase-field segmentation method, classically used in the image processing community. We show that by properly setting the parameters of the phase-field functional, robustness to fluctuations of the threshold value (caused for instance by estimation errors) used for spectrum sensing can be achieved. Our numerical results indicate that the sensing accuracy is improved, while the computational complexity is reduced, when compared to conventional spectrum sensing methods.

Index Terms—Cognitive radio, wideband spectrum sensing, phase-field functional, edge detection, segmentation, power spectral density.

I. INTRODUCTION

As wireless communication technologies continue to grow, spectrum resources are facing huge demands. Cognitive radio (CR) is proposed as a solution to the problem of spectrum scarcity [1]. CR improves spectrum utilization by allowing secondary users to access opportunistically a licensed band when/where primary users (PU) are absent. To reach this goal, secondary users have to sense the spectrum constantly in order to detect the presence of a primary transmitter signal. Due to channel fading conditions and the so-called hidden terminal problem [2], spectrum sensing is usually imperfect and imposes interference on the primary network. Cooperative spectrum sensing introduced, for instance, in [2]–[6] improves the detection reliability by employing multiple cognitive users, i.e., by increasing diversity. Therefore, spectrum sensing is one of the most important issues in each CR system.

A survey of different spectrum sensing techniques for CR is provided in [3]. A practical scenario occurs when the primary spectrum utilization is such that the secondary user should search over a wide band to identify the locations of vacant frequency bands (also referred to as spectrum holes). In this case, the entire wideband under sensing is usually modeled as a train of consecutive frequency subbands where the power spectral density (PSD) is smooth within each subband but changes abruptly between adjacent subbands depending on the spectrum usage of the primary user. More precisely, this corresponds to some edges in the PSD of a wideband channel, i.e., transitions from an occupied band to an empty band or vice versa. Once these edges are detected, the power within two edges can be estimated and the CR can characterize a frequency band as empty or occupied. In [7], Sadough et al. have shown that wavelets are suitable for modeling ultrawideband channels, whereas in [8], [9], they proposed a wavelet-based channel estimator for wideband channels. In [10], the authors proposed the use of wavelet transform as a mathematical tool to identify and locate the spectrum holes by analyzing the irregularities in the PSD.

In this paper we propose to use the Mumford-Shah segmentation method [11], initially proposed in the image processing community for finding the edges present in an image. In this technique, the edges and segments present in a given image are extracted from the minimization of a specific functional. More precisely, we use the so called phase-field approach [12], [13], as an approximation method for solving the optimization problem in the Mumford-Shah segmentation method. Although the above techniques are initially proposed and used in the image processing community, here, we have used them for finding the edges in the one dimensional PSD of the spectrum under analysis. We will see that the proposed approach provides practical advantages compared to wavelet-based spectrum sensing over wideband channels.

The rest of this paper is organized as follows. Section II explains our main assumptions about the spectrum under analysis and the adopted spectrum sensing method. Section III describes the spectrum sensing method based on phase-field segmentation method. Section IV illustrates, via simulations, the performance and advantages of the proposed method compared to conventional methods. Finally, Section V concludes the paper.

II. SPECTRUM SENSING PROBLEM FORMULATION

Sensing the presence of a primary transmitter inside a given frequency band is usually viewed as a binary hypothesis testing problem with hypothesis \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) defined as:

\[
\begin{align*}
\mathcal{H}_1 & : \text{primary user is in operation} \\
\mathcal{H}_0 & : \text{primary user is not in operation}.
\end{align*}
\]
Obviously, in the above definition, one has to differentiate between the presence (or the absence) of the primary user in reality and from the CR point of view, i.e., the decision made by spectrum sensing. State of the art contributions such as [3] use the above two hypothesis to define the following conditional probabilities.

Miss-detection probability:

\[ P_m = P (H_0^{CN} | H_1^{PN}) , \]  

and false-alarm probability:

\[ P_f = P (H_1^{CN} | H_0^{PN}) \]  

where \( H_1^{PN} \) denotes the actual absence (for \( i = 0 \)) and the actual presence (for \( i = 1 \)) of the primary signal respectively; \( H_i^{CR} \) indicates the decision made based on the received signals during the spectrum sensing process at the cognitive terminal on the absence (for \( i = 0 \)) and on the presence (for \( i = 1 \)) of the primary transmitter signal. Spectrum sensing performance evaluation is usually measured based on the so-called receiver operating characteristic (ROC) curves. These ROC curves plot the probability of miss-detection \( (P_m) \) (the probability that the CR fails to detect the presence of the PU) versus the probability of false-alarm \( (P_f) \) (the probability that the CR decides the PU is in operation whereas it was actually off).

In this paper, spectrum sensing consists in finding the locations and intensities of vacant (also called spectrum holes) and occupied frequency bands, by analyzing the irregularities in the PSD. We assume a wide band in the frequency range \([f_0, f_N]\) with a bandwidth of \( B = f_N - f_0 \) Hz, composed of \( N \) different subbands, where the \( n \)-th subband \( B_n \) corresponds to the frequency range \( f_{n-1} < f < f_n \), for \( n = 1, ..., N \). Moreover, we make the following basic assumptions for the channel spectrum \( S_r (f) \):

- The frequency boundaries \( f_0 \) and \( f_N \) are known to the CR.
- The number of total frequency subbands \( N \) and the locations of the spectrum boundaries \( f_1, f_2, ..., f_{N-1} \) are unknown to the CR. These information remain unchanged within a time burst, but may vary from burst to burst in the presence of slow fading.
- The PSD within each subband \( B_n \) is smooth and almost flat, but exhibits discontinuities from its neighbor subbands.
- The ambient noise is additive and white, with zero mean and two-sided PSD \( S_n (f) = N_0/2, \forall f \).

A typical normalized spectrum \( S_r (f) \) is shown in Fig. 1. Based on the observed signal \( r(t) \) with PSD \( S_r (f) \), the CR has to estimate \( N, \{ f_n \}_{n=1}^{N} \) and \( \{ \alpha^2_n \}_{n=1}^{N} \). Assume that \( \{ f_n \}_{n=1}^{N} \) are the estimates of frequency boundaries of consecutive subbands \( \{ B_n \} \), obtained by using an appropriate method. For instance in [10], these boundaries are obtained by picking the local maximas of the first derivatives of the wavelet transform modulus. Then, the CR has to estimate the PSD levels \( \alpha^2_n \). This is achieved by computing the average PSD within the \( n \)-th subband \( B_n \) for \( n = 1, ..., N \), as:

\[ \alpha^2_n = \frac{1}{f_n - f_{n-1}} \int_{f_{n-1}}^{f_n} S_r (f) df - N_0/2, \]  

where we have assumed that the CR disposes of an estimate of the two-sided noise PSD. The estimated value in (6) is finally compared to a threshold in order to classify the subband as vacant or occupied. The threshold value is usually set so as to satisfy a certain false-alarm probability. Note that inaccurate values for the threshold value affects directly the performance of spectrum sensing since the CR may under/over estimate the number of spectrum holes and cause harmful interference to the primary licensed users.

Next, we propose a spectrum sensing method based on phase-field image segmentation as an alternative to the conventional wavelet-based edge detection.

III. Spectrum Sensing Via Phase-Field Segmentation

A. Principle of Phase-field Edge Detection

As explained previously, wideband spectrum sensing is assimilated to an edge detection problem in an image formed by the PSD \( S_r (f) \) in frequency. Here, we propose to use an alternative technique for finding the boundary edges in the PSD. More precisely, we propose an edge detection based on Mumford-Shah segmentation [11]. In this method, the edge set \( \Gamma_s \) of the channel spectrum \( S_r (f) \) is obtained by minimizing

![Fig. 1. A typical spectrum \( S_r (f) \).](image-url)
the functional \( E(S_r(f), \Gamma_s) \):

\[
E(S_r(f), \Gamma_s) = \int_{\Omega} \frac{\lambda}{2} (S_r(f) - S_r(f))^2 df \\
+ \int_{\Omega - \Gamma_s} \frac{\mu}{2} |\nabla S_r(f)|^2 df + \nu H^{d-1}(\Gamma_s)
\]  

(7)

where \( S_r(f) \) is the smoothed version of the initial PSD \( S_r(f) \) (it is a variable involved in the functional), \( \Gamma_s \) is the edge set, \( \Omega \) is the frequency domain \([f_0, f_N]\), \( \lambda, \mu \) and \( \nu \) are positive weightings that control the balance between data fit, regularization and the length of the edge contour respectively; \( H^{d-1} \) denotes the \((d - 1)\)-dimensional Hausdorff measure where \( d \) is the number of dimensions of the signal under analysis; and \( \nabla(\cdot) \) is the gradient operator. The Mumford-Shah criterion is based on finding the smooth part \( S_r(f) \), and the edge set \( \Gamma_s \) so as to minimize \( E(S_r(f), \Gamma_s) \) of equation 7. Notice that \( S_r(f) \) is actually no more than the estimate of \( S_r(f) \) in the least-square (LS) sense over the domain \( \Omega \). Moreover, \( S_r(f) \) is smooth over the entire spectrum domain except at the discontinuities (i.e., in the set \( \Omega - \Gamma_s \)). The singularity set \( \Gamma_s \) should be small with respect to the \((d - 1)\)-dimensional Hausdorff measure \( H^{d-1} \) which in our case is a scalar \((d = 1) \) [11].

Since direct minimization of the Mumford-Shah functional (7) is very complex, various approximations have been proposed [12], [14]. For instance, Ambrosio and Tortorelli proposed the so-called phase-field approximation in [12], [13]. In this method, an auxiliary variable \( \xi_s(f) \) is introduced into the Mumford-Shah functional which is small on edges and is almost equal to one elsewhere. The phase-field approximation for the Mumford-Shah functional of equation (7) is written as:

\[
E(S_r(f), \xi_s) = \int_{\Omega} \left[ \frac{\lambda}{2} (S_r(f) - S_r(f))^2 + \epsilon |\nabla \xi_s|^2 \right] df \\
+ \frac{\mu}{2} (\xi_s^2 + k_e) |\nabla S_r(f)|^2 + \frac{(1 - \xi_s^2)^2 - \epsilon}{\epsilon} df.
\]  

(8)

Here, \( k_e (k_e < 1) \) is a small positive regularization parameter, \( \epsilon \) controls the width of deep fades in \( \xi_s(f) \), and \( \lambda \) and \( \mu \) are as in equation (7). The last term in equation (8) limits the phase-field parameter \( \xi_s(f) \) to \( \xi_s(f)^2 \approx 1 \) away from discontinuities.

By minimizing equation (8), the edge set \( \xi_s \) and the smooth spectrum \( S_r(f) \) are found jointly. Finally, the boundary estimates \( \{f_n\}_{n=1}^N \) can be find by applying a threshold on \( \xi_s(f) \). Once the boundary set \( \xi_s \) is available, occupied and vacant frequency bands can then be obtained by computing the average PSD within each boundary according to equation (6).

**B. Solution of the Minimization Problem (8)**

Solving optimization problems in which, given some functional, one seeks the function minimizing (or maximizing), is usually performed by means of the Euler-Lagrange equations [15]. The Euler-Lagrange equations associated to \( S_r(f) \) and \( \xi_s(f) \) in the functional of equation (8) are [16]:

\[
S_r(f) = div\left( \frac{\mu}{\lambda} (\xi_s^2 + k_e) \nabla S_r(f) \right)
\]  

(9)

\[
\xi_s(\frac{\mu}{2\epsilon} |\nabla S_r(f)|^2 + \frac{1}{4k_e}) - \Delta \xi_s = \frac{1}{4k_e^2}.
\]  

(10)

where \( div(\cdot) \) denotes the divergence operator. A fast and effective numerical solution to the set of linear partial differential equations like equations (9) and (10) can be obtained through the finite element method (FEM), that we have used in this paper (more details can be found in [16]–[18]).

**IV. SIMULATION RESULTS AND DISCUSSIONS**

**A. Experimental Results**

In this section, we provide numerical results to evaluate the performance achieved by the proposed spectrum sensing technique, in comparison with conventional techniques. Figure 2 depicts \( \xi_s(f) \) and the smoothed estimated spectrum \( S_r(f) \) where the unknown spectrum is that depicted in Fig. 1. In this experiment we have set \( \epsilon = 5 \times 10^{-2}, \lambda = 4 \times 10^4, \mu = 10^4 \) and \( k_e = 5 \times 10^{-2} \).

\[
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**Fig. 2.** Estimated \( \xi_s(f) \) and \( S_r(f) \) from the phase-field method, \( \epsilon = 5 \times 10^{-2}, \lambda = 4 \times 10^4, \mu = 10^4 \) and \( k_e = 5 \times 10^{-2} \).
field spectrum sensing methods, respectively. Concerning the wavelet-based method, we have used Gaussian basis functions with four levels of decompositions so that the best performance (i.e., the largest edge peaks) is achieved [10]. As expected, due to the inherent robustness of the phase-field method, we observe a more stable behavior for the ROC with respect to the value of the edge threshold than the wavelet-based method.

B. Discussions

Although wavelet transform is known to be a powerful mathematical tool for edge detection, our proposed method has several interesting features that makes it a more interesting technique from a practical point of view. Except the robustness of the phase-field technique to the edge detection threshold value (explained above), the main advantages of the phase-field method are provided in what follows.

- The availability of the smoothed spectrum signal $S_r(f)_s$ is an interesting feature. As stated before, the estimation of the PSD levels $\{\alpha_n\}$ is achieved by computing the average PSD within identified subbands limited by two edges in the PSD. In the phase-field method, the PSD levels can be simply set equal to the median value of the noise reduced PSD $S_r(f)_s$ within the desired subband, instead of computing the integral over each subband. Obviously, this reduces the computational load at the CR terminals.
- Obtaining the numerical solution of the proposed functional in equation (8) is simple and fast. Moreover, our simulations showed that the iterative FEM used for solving (9) and (10) converges after only two iterations. Besides, in comparison with wavelet-based methods, it is not necessary to have any concern about the type of wavelet filter, its length, the number of decomposition levels and the complexity of the convolutions necessary to obtain the coefficients of the continuous wavelet transform.
- Practical PSDs may have fluctuations inside subbands, different slope values at edges, etc., so that the assumptions made in Section II about the PSD can not be fulfilled completely. In these cases, the proposed method is more flexible than the wavelet-based method since by properly setting the two parameters $\epsilon$ and $\lambda$ in the phase-field functional of (8), we can adapt the algorithm to the actual shape of the estimated PSD.

V. Conclusion

In this paper, we proposed a spectrum sensing method based on phase-field segmentation for wideband channels in CR systems. By solving the phase-field functional for the estimated PSD, we found jointly a smoothed spectrum and a signal containing the information about the edge locations inside the spectrum. The proposed method has two main interesting features. First, the edge signal provides flexibility on the choice of the threshold value used for edge localization. This led to an inherent robustness of our technique to possible errors on the threshold value selection. Second, the smoothed spectrum can be used for reducing complexity of the estimation of the PSD levels inside each subband, since the median value of the smoothed spectrum can be used instead of integration of the initial noisy PSD over each subband. It was also observed that when the threshold value for edge detection is not accurately selected, the phase-field method outperforms the conventional wavelet-based method. However, under accurate threshold values, although the proposed and the wavelet-based methods perform closely, our method takes the advantage due to its lower computational complexity.

REFERENCES


Fig. 3. Parameter $\xi_s(f)$ evaluated for A) $\epsilon = 5 \times 10^{-3}$ and B) $\epsilon = 20 \times 10^{-2}$. We observe deeper fades, i.e., increased robustness to the choice of the edge detection threshold, for the larger $\epsilon$ value.

Fig. 4. ROC curves obtained by wavelet-based spectrum sensing for an edge threshold value equal to 0.2, 0.3, 0.4, 0.5, 0.6 and 0.8. We observe a large sensitivity of the ROC curve to the choice of the threshold value.
Fig. 5. ROC curves obtained by phase-field spectrum sensing for an edge threshold value equal to 0.2, 0.3, 0.4, 0.6 and 0.8. We observe a stable behavior since the displacement of the ROC curve is small for different threshold values.